WITH ANSWERS

R. R. Turnbull,
54 Park St,
Hull.

WORKS BY CHARLES PENDLEBURY

NEW SCHOOL ARITHMETIC. Complete, with or without Answers. Twenty-sixth Impression. 5s. 6d.: or in two parts, 3s. 6d. each. Answers separately, 6d. net. Key to Part II., 10s. net.

NEW SCHOOL EXAMPLES IN ARITHMETIC. Extracted from the above. Seventeenth Impression. With Answers, 3s. 6d.; without, 3s.

EXERCISES AND EXAMINATION PAPERS IN ARITHMETIC, LOGARITHMS, MENSURATION. Tenth Impression. 3s. 6d. Key, 5s. net.

JUNIOR ARITHMETIC. Sixteenth Impression. Sm. Crown 8vo. With or without Answers, 2s. 9d. Answers separately, 6d. net.

EXAMPLES FROM A JUNIOR ARITHMETIC. Tenth Impression. Sm. Crown 8vo. 1s. 6d. Answers, 6d. net.

NEW "SHILLING" ARITHMETIC. Nineteenth Impression. Sm. Crown 8vo. 1s. 9d.; or with Answers, 2s. 3d. Answers separately, 6d. net.

ARITHMETIC. With 8000 Examples. Complete, with or without Answers. Thirty-third Impression. 5s. 6d. IN TWO PARTS, with or without Answers, 3s. 6d. each. Answers separately, 6d. net.
Key to Part II. Sixth Impression. 8s. 6d. net.

EXAMPLES IN ARITHMETIC. Extracted from the "Arithmetic." Twenty-sixth Impression. With Answers, 3s. 6d.; without, 3s.

REVISION PAPERS IN ARITHMETIC for the Local Examinations, etc. Second Impression. 2s. 6d.

BUSINESS ARITHMETIC. By C. PENDLEBURY, M.A., and W. S. BEARD. With or without Answers. 2s. 6d.

COMMERCIAL ARITHMETIC. Eighteenth Impression. Crown 8vo. 3s.

PREPARATORY ARITHMETIC. Revised and Enlarged [5th] Edition. 3s.

"SHILLING" ARITHMETIC. 856th Thousand. Sm. Crown 8vo. 1s. 9d.; or with Answers, 2s. 3d. Answers separately, 6d. net.

ELEMENTARY ARITHMETIC. Sixteenth Impression. Crown 8vo. With or without Answers. 2s.

NEW CONCRETE ARITHMETIC. Parts I.-III. Paper, 4d. each. Cloth, 5d. each. Parts IV.-VI. Paper, 6d. each; Cloth, 7d. each. Answers, 5d. net each.

GRADUATED ARITHMETIC FOR PRIVATE AND JUNIOR SCHOOLS. In stiff canvas wrapper. Parts I., II., III., 8d. each; IV., V., 9d. each; VI., VII., 10d. each. Answers to Parts I. and II., 5d. net; Parts III.-VII., 5d. net each.

ARITHMETIC FOR THE STANDARDS (Scheme A). Standards I., II., III., sewed, 5d. each; IV., V., VI., sewed, 6d. each. VII., sewed, 8d. Answers to Standards I. and II., 5d. net; III.-VII., 5d. net each.

LONG TOTS AND CROSS TOTS (Simple and Compound). Paper, 6d. net; Answers, 5d. net.

ELEMENTARY TRIGONOMETRY. Ninth Impression. 5s.

LONDON: G. BELL AND SONS, LTD

CAMBRIDGE MATHEMATICAL SERIES

NEW SCHOOL ARITHMETIC

G. BELL AND SONS, LTD.
LONDON: PORTUGAL ST., KINGSWAY
CAMBRIDGE: DEIGHTON, BELL AND CO.
CALCUTTA: LONGMANS, GREEN AND CO.

NEW SCHOOL ARITHMETIC

BY

CHARLES PENDLEBURY, M.A., F.R.A.S.

LATE SENIOR MATHEMATICAL MASTER OF ST. PAUL'S SCHOOL
FORMERLY SCHOLAR OF ST. JOHN'S COLLEGE, CAMBRIDGE
AUTHOR OF "LENSES AND SYSTEMS OF LENSES, TREATED AFTER THE MANNER OF GAUSS"

ASSISTED BY

F. E. ROBINSON, M.A.

ASSISTANT MASTER OF ST. PAUL'S SCHOOL

LONDON
G. BELL AND SONS, LTD.
1928

*** *This Book may be had with or without Answers.*

First published complete, September 1904.
Reprinted 1905, 1906, 1907, 1908, 1909, January and November 1910, 1911, 1912, 1913, 1914, 1916, 1917, 1918, 1920 (June and August), 1921 (June and November), 1922, 1923 (January and November), 1924, 1926, 1927, 1928.

PRINTED IN GREAT BRITAIN BY ROBERT MACLEHOSE AND CO. LTD.
AT THE UNIVERSITY PRESS, GLASGOW

PREFACE

DURING the 18 years that have passed since my original text-book on Arithmetic was first published, there have naturally been some changes in the ideas of how the subject should be taught, and some additional demands due to the increase of scientific requirements.

Some of these changes were formulated in the report issued in 1902 by a committee appointed by the Mathematical Association. The changes there recommended as regards Arithmetic, although not so far-reaching as those advocated in the teaching of Geometry, are yet very important, and I have endeavoured, with the assistance of Mr. Robinson, to embody them in this book.

The basis of the book is the 16th edition of my existing text-book. Much of the old matter has been retained, and much new matter has been added. Some alteration in the arrangement of the sections has also been made. The new feature which will perhaps be the most noticeable, in a casual examination of the book, is the position assigned to the Metric System of Weights and Measures. They are introduced here simultaneously with the corresponding British Weights and Measures and are treated as on practically the same footing. Few, even of the most conservative, will look for any apology for this course. Since the use of the metric system was legalised in

1898, it has grown so rapidly into favour for certain purposes, that it can no longer be consigned to the very subordinate position which it held in former times; and although these measures do not yet enter at all into the daily life of the people, they may do so very intimately before long. However that may be, the time has already come, when a perfect familiarity with the system, and with the methods of using it, is an essential item in an educational course.

The British measures, too, have become somewhat simplified. The Troy pound has disappeared altogether, and one cause of confusion is thereby removed. Other complications, due to the pole and the perch being respectively $5\frac{1}{2}$ yards and $30\frac{1}{4}$ square yards, can easily be avoided by the use of the chain and the square chain. This removes to a great extent another of the troubles which have beset a beginner.

Money and weights, both British and Metric, are moreover introduced before either fractions or decimals, and serve a very useful purpose as objects with which to illustrate the use of fractions and decimals when applied to concrete quantities. The metric measures also serve to illustrate practically the meaning of place values in the common system of numerical notation.

It will be seen that the new method of multiplication is adopted exclusively. It is the right method to use, from a practical as well as from a logical point of view, and even a dull pupil is able to grasp it, and to use it correctly, in a few minutes.

Graphical illustrations are used very freely, especially in the development of the earlier parts of the subject—in the simple rules, G.C.M., L.C.M., fractions and elsewhere. The general use of graphs for purposes of reduction and in other problems is treated in a separate section.

In the sections on approximation and decimalization of money these operations are fully discussed; and in the commercial sections of the book—practice, proportion, interest, etc.—the method of decimalizing money is used very generally. Throughout the book, too, the pupil is shown how, by forming a rough approximation, he may avoid the danger of being satisfied with an absurd result.

Many examining bodies having sanctioned the use of an algebraical symbol in place of a verbal periphrasis, I have availed myself of this freedom wherever the nature of the problem allowed me to do so with advantage.

The section on elementary mensuration, and the new section on logarithms, will be of use to the more advanced student.

I venture to hope that in the new form which the book has now taken it will be found not only to retain all the old characteristics which won much favour for the original book, but to possess also new characteristics of its own which will prove equally acceptable.

Throughout the book everything like an arbitrary rule has been carefully avoided. Such guides and directions as are given for the solution of questions are deduced as the outcome of simple explanation and simple reasoning. I have endeavoured to lead, or at least to accompany, the student rather than to drive him on in front. After every fresh step will be found numerous examples for the student to work out; and those for written work are preceded by others adapted for oral instruction. It is only after a short time spent in talking over the meaning of what is to be done that a young student may generally be trusted with pen, ink and paper. At the same time, by the use of clear and simple language in explaining and discussing the subject

both in theory and practice, and by numerous specimen examples fully worked out, I have attempted to satisfy as far as possible the wants of those who have no oral teacher to aid them.

The examples of all kinds are unusually abundant and at the end of the book is a collection of easy examination papers consisting of seven questions each, followed by a large collection of more difficult problems. A teacher may easily add to any of the examination papers a sufficient number of suitable problems to make up a paper of almost any degree of length and difficulty. Great care has been taken in verifying the answers. I shall be grateful for information of any errors that have escaped my notice.

A large share of the work of revision has been borne by Mr. Robinson. If, in any portion of this book, the arrangement or the methods are better than in the original, the improvement is largely due to his experience, ingenuity and resource.

C. PENDLEBURY.

ST. PAUL'S SCHOOL, LONDON.

CONTENTS.

PART I.

		PAGE
I.	Introductory	1
II.	How Numbers are Expressed	4
III.	Remarks on the First Four Rules	11
IV.	The Roman System of Notation	22
V.	Factors and Prime Numbers	24
	Tests of divisibility	29
VI.	Greatest Common Measure	33
VII.	Least Common Multiple	40
	Miscellaneous Examples	45
VIII.	Measures of Money—British and Foreign	50
IX.	Compound Addition—British Money	58
X.	Compound Subtraction—British Money	62
XI.	Compound Multiplication—British Money	64
XII.	Compound Division—British Money	68
XIII.	Addition, Etc., of Foreign Money	73
	Miscellaneous Examples	75
XIV.	Measures of Weight—British and Metric	78
XV.	Measures of Length—British and Metric	85
	Addition and subtraction of weights and measures	88
	Multiplication and division of weights and measures	93
XVI.	Fractions	97

Introductory; to find the value of a fraction of a given quantity; reduction to lowest terms; comparison of fractions; improper fractions and mixed numbers; addition; subtraction; to multiply by a whole number; to divide by a whole number; to multiply by a fraction; to divide by a fraction; G.C.M. and L.C.M. of two or more fractions

PAGE

XVII. The Use of Brackets 128

XVIII. Complex Fractions 131

XIX. Decimals 133

To multiply a decimal by 10, 100, etc.; to divide a decimal by 10, 100, etc.; addition; subtraction; to multiply a decimal by a whole number; to multiply by a decimal; to divide a decimal by a whole number; to divide by a decimal; remainder after division by a decimal; to express a decimal by an equivalent vulgar fraction; to express certain vulgar fractions as decimals; to express any vulgar fraction as a decimal

XX. Approximation:—Addition, Subtraction, Multiplication, Division 157

XXI. Decimalization of Money 166

XXII. Recurring Decimals 171

To find a vulgar fraction equivalent to a recurring decimal; addition, etc.

XXIII. A Fraction, or Decimal, of a Concrete Quantity . 178

XXIV. Measures of Area—British and Metric . . . 184

Applications of measures of area 188

XXV. Measures of Capacity—British and Metric . . . 193

XXVI. Measures of Angles, Number and Time . . . 197

Miscellaneous Examples 202

PART II.

XXVII. Rectangular Areas 207

XXVIII. Square Roots 213

Prime Factors; General Method; Applications

XXIX. Volume, or Cubical Content 222

XXX. Cube Root by Factors, and by Inspection . . 227

XXXI. Practice 230

Simple Practice: Compound Practice; Bill of Parcels

XXXII. Ratio 238

XXXIII. Proportion 242

Illustration from comparison of thermometric scales

XXXIV. Problems involving Proportion 247

The Unitary Method; Proportion and Equations; Fractional Multipliers and Equations

PAGE

XXXV. PERCENTAGES 260

XXXVI. COMMISSION, RATES AND TAXES, ETC. 266

XXXVII. PROFIT AND LOSS 270

XXXVIII. SIMPLE INTEREST 279
Simple Interest; True Discount and True Present Worth; Practical Discount

XXXIX. COMPOUND INTEREST 295
Miscellaneous examples on Simple and Compound Interest

XL. STOCKS AND SHARES 306

XLI. PROPORTIONAL PARTS AND PARTNERSHIP . . . 320
Proportional Parts; Partnership; Temporary Partnership

XLII. AVERAGES 327

XLIII. MIXTURES 330

XLIV. PROBLEMS 334
Work, Pipes, etc.; Races, Games of Skill, Trains; Clocks and Circular Tracks; Pasture with growing Grass

XLV. METRIC MEASURES 346
British-Metric Equivalents

XLVI. MONETARY SYSTEMS AND EXCHANGE 351

XLVII. EQUATION OF PAYMENTS 359

XLVIII. APPROXIMATION 361
Degree of Accuracy; Approximate Values of Infinite Series and Approximate Fractional Forms; Approximate Square Roots; Surds

XLIX. GRAPHS 376
Graphical Representation; Application to Problems

L. CUBE ROOT, AND HIGHER ROOTS 391

LI. PROPERTIES OF RECURRING DECIMALS 398

LII. ELEMENTARY MENSURATION 405
Areas of Plane Figures; Volumes of Certain Solids; Areas of the Surfaces of Certain Solids

LIII. LOGARITHMS 415

EXAMINATION PAPERS 425

PROBLEMS 449

ANSWERS TO EXAMPLES i

TABLES.

BRITISH MEASURES

For further information the pupil should turn to the pages indicated.

ENGLISH MONEY TABLE. (Page 51.)

4 farthings make 1 penny (1*d.*).
12 pence ..... 1 shilling (1*s.*).
20 shillings ..... 1 pound (£1.).

INDIAN MONEY TABLE.

3 pies make 1 pice.
4 pice ..... 1 anna.
16 annas ..... 1 rupee.

The exchange value of a rupee is about 1*s.* 4*d.*

TROY WEIGHT. (Page 78.)

(Used only for gold, silver, and platinum).

24 grains make 1 pennyweight (1 dwt.).
20 pennyweights ..... 1 ounce (1 oz. tr.).

Apothecaries, in compounding medicines, subdivide a troy ounce into 8 drachms, a drachm into 3 scruples, and a scruple into 20 grains.

AVOIRDUPOIS WEIGHT. (Page 79.)

16 drams make 1 ounce (1 oz.).
16 ounces ..... 1 pound (1 lb.).
28 pounds ..... 1 quarter (1 qr.).
4 quarters or 112 pounds ..... 1 hundredweight (1 cwt.).
20 hundredweights ..... 1 ton.

7000 grains make 1 pound { 437½ grains make 1 oz. avoir. / 480 grains ..... 1 oz. troy.

LINEAR MEASURE. (Page 85.)

12 inches	make	1 foot (1 ft.).
3 feet	.....	1 yard (1 yd.).
22 yards	.....	1 chain.
10 chains	.....	1 furlong (1 fur.).
8 furlongs or 1760 yards	.....	1 mile (1 m.).

Also

100 links	make	1 chain (1 ch.).
5½ yards	.....	1 rod or pole.
4 poles	.....	1 chain.

SQUARE MEASURE. (Page 185.)

144 sq. inches	make	1 sq. foot (1 sq. ft.).
9 sq. feet	.....	1 sq. yard (1 sq. yd.).
484 sq. yards	.....	1 sq. chain
10 sq. chains	.....	1 acre.
640 acres	.....	1 sq. mile (1 sq. m.).

Also

4840 sq. yards	make	1 acre.
30¼ sq. yards	.....	1 sq. pole or perch.
40 perches	.....	1 rood.
4 roods	.....	1 acre.

CUBIC MEASURE. (Page 193.)

1728 cubic inches	make	1 cubic foot.
27 cubic feet	.....	1 cubic yard.

LIQUID MEASURE. (Page 193.)

4 gills	make	1 pint (1 pt.).
2 pints	.....	1 quart (1 qt.).
4 quarts	.....	1 gallon (1 gall.).
36 gallons	.....	1 barrel.

Apothecaries subdivide the pint thus:—60 minims make 1 fluid drachm; 8 fluid drachms make 1 fluid ounce; and 20 fluid ounces make 1 pint.

DRY OR CORN MEASURE. (Page 194.)

4 gills	make	1 pint (1 pt.).
2 pints	.....	1 quart (1 qt.).
4 quarts	.....	1 gallon (1 gall.).
2 gallons	.....	1 peck (1 pk.).
4 pecks	.....	1 bushel (1 bush.).
8 bushels	.....	1 quarter (1 qr.).
4 quarters	.....	1 ton (cargo measurement).

METRIC MEASURES.

The more important measures are in heavier type.

MEASURES OF WEIGHT. (Page 82.)

10 milligrams (mg.)	= 1 **centigram** (cg.)
10 centigrams (cg.)	= 1 *decigram* (dg.)
10 *decigrams* (dg.)	= 1 **GRAM** (g.)
10 grams (g.)	= 1 *decagram* (Dg.)
10 *decagrams* (Dg.)	= 1 *hectogram* (Hg.)
10 *hectograms* (Hg.)	= 1 **kilogram** (Kg.)
100 kilograms, or 10^5 grams	= 1 **quintal** (q.)
1000 kilograms	= 1 **tonne** (t.)

MEASURES OF LENGTH. (Page 87.)

10 millimetres (mm.)	= 1 **centimetre** (cm.)
10 centimetres	= 1 *decimetre* (dm.)
10 *decimetres*	= 1 **METRE** (m.)
10 metres	= 1 *decametre* (Dm.)
10 *decametres*	= 1 *hectometre* (Hm.)
10 *hectometres*	= 1 **kilometre** (Km.)

MEASURES OF AREA. (Page 186.)

100 sq. millimetres (sq. mm.)	= 1 sq. centimetre (sq. cm.)
100 sq. centimetres	= 1 sq. **decimetre** (sq. dm.)
100 sq. decimetres	= 1 sq. metre (sq. m.) = 1 **centiare.**
100 sq. *metres*	= 1 sq. **decametre** (sq. Dm.) = 1 **are.**
100 sq. *decametres*	= 1 sq. *hectometre* (sq. Hm.) = 1 **hectare.**
100 sq. *hectometres*	= 1 sq *kilometre* (sq. Km.)

LAND MEASURE. (Page 186.)

100 centiares	= 1 **ARE.**
100 ares	= 1 **hectare.**
100 hectares	= 1 sq. **kilometre.**

MEASURES OF VOLUME. (Page 194.)

1000 cubic millimetres (c.mm.)	= 1 **cubic centimetre** (c.cm.)
1000 cubic centimetres	= 1 **cubic decimetre** (c.dm.)
1000 cubic decimetres	= 1 **cubic metre** (c.m.) = 1 **stere.**

MEASURES OF CAPACITY. (Page 195.)

10 centilitres (cl.)	= 1 **decilitre** (dl.)
10 decilitres	= 1 **LITRE** (l.)
10 litres	= 1 **decalitre** (Dl.)
10 decalitres	= 1 **hectolitre** (Hl.)

MEASURES OF ANGLES, NUMBER AND TIME

MEASURES OF ANGLES. (Page 197.)

60 seconds (60″) make 1 minute (1′).
60 minutes ..... 1 degree (1°).
90 degrees ..... 1 right angle.

MEASURES OF NUMBER. (Page 197.)

12 units make 1 dozen.
20 units ..... 1 score.
12 dozen ..... 1 gross.
12 gross ..... 1 great gross.
Also 24 sheets of paper ..... 1 quire.
20 quires ..... 1 ream.
10 reams ..... 1 bale.

MEASURES OF TIME. (Page 199.)

60 seconds (60^s) make 1 minute (1^m).
60 minutes ..... 1 hour.
24 hours ..... 1 day.
7 days ..... 1 week.
28, 29, 30, or 31 days ..... 1 calendar month.
12 calendar months ..... 1 year.
365 days ..... 1 common year.
366 days ..... 1 leap year.
100 years ..... 1 century.

BRITISH-METRIC EQUIVALENTS.

The diagram on the next page enables us to obtain at once the metric equivalents of British measures and the British equivalents of metric measures. It is based upon the following approximate relations:

1000 in. = 2540 cm.	1000 pts. = 570 litres.
1000 mi. = 1610 Km.	1000 cwt. = 510 quintals.
1000 yds. = 915 metres.	1000 lb. = 455 Kg.
1000 sq. yds. = 835 sq. m.	1000 ac. = 405 hectares.

1000 bush. = 365 Hl.

To find how many hectolitres are equivalent to 35 bushels, for example, we measure 35 divisions up the left-hand vertical border, beginning at the bottom corner. This brings us to the point X. Through X we now draw a horizontal line to the right until we reach the "bushels to hectolitres" line at E. Counting the number of divisions between X and E, we find approximately the number of hectolitres equivalent to 35 bushels, namely 13.

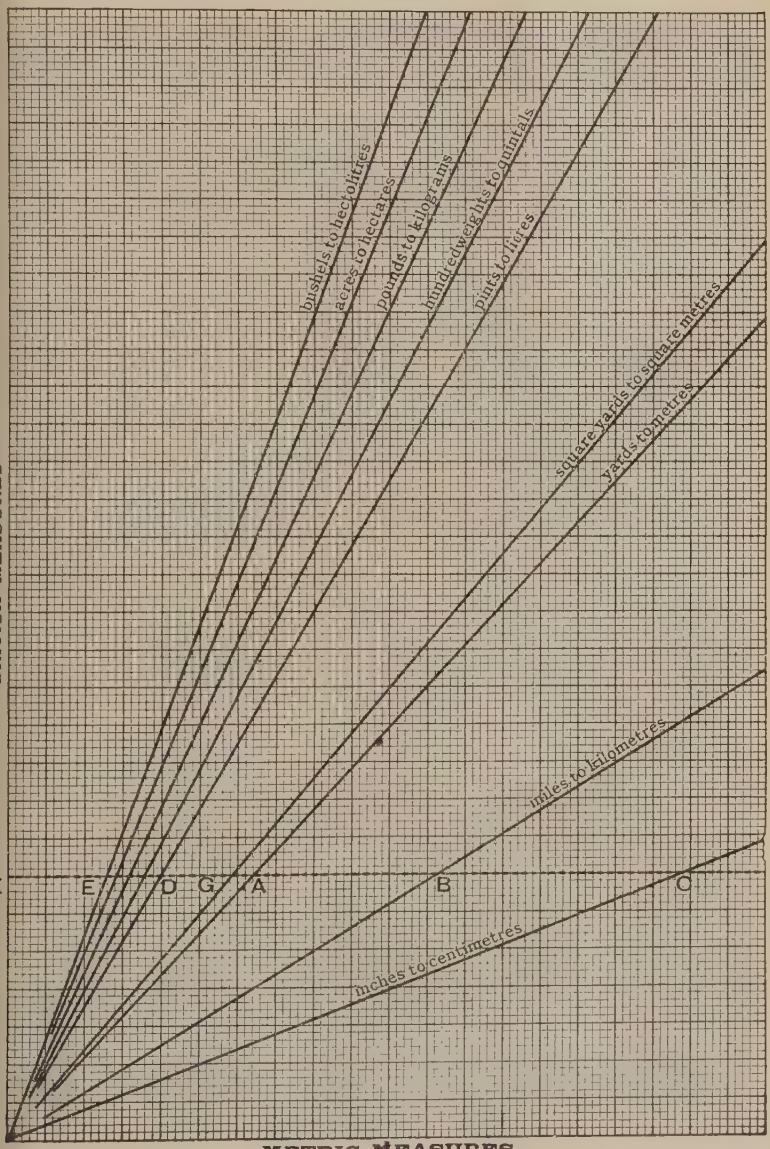
Similarly, counting the divisions between X and D, G, A, B, and C respectively, we see that approximately

35 pints = 20 litres,	35 miles = 56 Km.,
35 sq. yds. = 29 sq. metres,	and 35 inches = 89 cm.
35 yards = 32 metres,	

Conversely, we can find the British equivalent to a metric measure by counting along the bottom border a number of divisions equal to the number of metric units, and then following the vertical line to the point where it crosses the proper diagonal. To find the number of lb. equivalent to 50 Kg., for example, we count 50 divisions from the left along the bottom border, and we then follow the vertical until it crosses the "pounds to kilograms" line. This it does at about the 110th division. Thus we find that 50 Kg. = 110 lb.

When the measures are large we may suppose each of the divisions in the diagram to represent 2 or more units, but the results will be less accurate. It will be better in such cases to use a larger diagram, which the student can make for himself by using squared paper divided into millimetres.

Diagram for changing British Measures into Metric and Metric Measures into British.



ARITHMETIC.

PART I.

I. INTRODUCTORY.

1. A **unit** is a single thing; as *one boy, one penny, one book.*

2. Quantities *of the same kind*, or **like** quantities, are such as may have the same unit; for example, *five horses* and *eight horses* are like quantities, the unit, one horse, being the same for both.

3. A **number** is a unit, such as *one* girl, or a collection of units of the same kind, as *six* pennies. In the latter case the *unit* is *one* penny.

4. ARITHMETIC is a part of the SCIENCE of numbers and of the ART of using them.

5. A number such as *four* or *five*, not attached to any particular things or units, is called an **abstract number.**

6. A number of particular units, such as *four horses*, or *five men*, is called a **concrete number.**

7. All numbers are written by means of the symbols

0, 1, 2, 3, 4, 5, 6, 7, 8, 9,

which are called **digits**; they are named respectively, *nought* or *zero*, *one* or *unity*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine.*

Each of these digits denotes one unit more than the digit which precedes it. Thus 2 denotes one unit more than 1; 3 denotes one unit more than 2; 8 denotes one unit more than 7, and so on; 0 denoting the absence of the units or things referred to, *i.e.* that there are *none* of them.

8. ADDITION is the process of finding a *single* number to represent the same as two or more given numbers together, the numbers being of the same kind.

They must be *of the same kind*, for we may add 1 marble to 2 marbles, getting 3 marbles altogether; but we cannot add 1 marble to 2 shillings, and express the result as a *single* number.

The result of addition is called the **sum** or **amount** of the given numbers.

The mark + placed between two numbers denotes that they are to be added together, and is called the **sign of addition**, or **plus**. Thus $3 + 4$ denotes that 4 is to be added to 3. In words $3 + 4$ is read thus; three *plus* four.

9. For the word *equals*, or the words *is equal to*, we use the sign =, which is called the **sign of equality**. Thus $3 + 1 = 4$, means that 3 *plus* 1, or 3 with 1 added to it, *is equal to* 4.

We will frequently use the sign $\therefore$ for the word *therefore*.

10. SUBTRACTION is the process of finding what remains when one number is taken away from another number of the same kind.

The number taken away is called the **subtrahend**.

The number from which it is taken is called the **minuend**.

The result is called the **difference** or **remainder**.

Subtraction is denoted by a short horizontal line (–) placed between the numbers. Thus $8 - 5$ means that 5 is to be taken from 8. In words, $8 - 5$ is read thus; 8 *minus* 5, or 8 *less* 5.

The line (–) is called the **sign of subtraction**, or **minus**.

If the minuend and the subtrahend are *equal* numbers, the difference is 0.

11. MULTIPLICATION is a short way of finding the sum of any number of given equal numbers.

The *number* of given equal numbers is called the **multiplier**.

The number to be multiplied, *i.e.* any one of the given equal numbers, is called the **multiplicand**.

The result of the multiplication is called the **product** of the multiplier and the multiplicand, and the multiplier and the multiplicand are called the **factors** of the product.

Multiplication is denoted by an *inclined* cross (×) called the **sign of multiplication**. It is placed between the numbers. For example, 3 multiplied by 4, which means $3 + 3 + 3 + 3$, is written 3×4. In words 3×4 is read 3 *multiplied by* 4.

12. DIVISION is the process of finding how often one number contains another. The former is said to be divided by the latter.

The number to be divided is called the **dividend**.

The number by which it is divided is called the **divisor.**

The number which tells how many times the divisor is contained in the dividend is called the **quotient.**

Division is denoted by the mark ÷, which is called the **sign of division.** It is placed between the numbers, *after* the dividend and *before* the divisor; thus $8 \div 2$ means 8 *divided by* 2.

If the dividend does not contain the divisor an exact number of times, and we take away from the dividend as many times the divisor as we can, what is left is called the **remainder.** When there is no remainder, the division is said to be **exact.**

Thus 8 contains 2 four times exactly, and there is no remainder. But 9 contains 2 four times, leaving a *remainder* 1.

13. In **exact** division the relation between Divisor, Dividend and Quotient may be written in either of the two forms

$$\text{Dividend} \div \text{Divisor} = \text{Quotient},$$

or

$$\text{Quotient} \times \text{Divisor} = \text{Dividend}.$$

In **inexact** division,

$$\text{Quotient} \times \text{Divisor} + \text{Remainder} = \text{Dividend}.$$

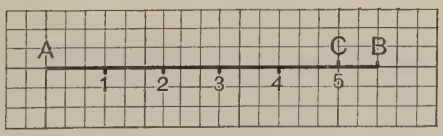
If, for example, a number 17 be divided by 3, the *quotient* is 5 and the remainder is 2. This may be expressed in the form,

$$17 = 3 \times 5 + 2.$$

If we represent numbers by letters, and if we suppose that, when a number represented by N is divided by a, the quotient is b and the remainder c, we may express this in the form,

$$N = a \times b + c.$$

14. The relation between Divisor, Dividend and Quotient, given above, may be illustrated by measuring, on squared paper,



a length AB, equal to 17 units, and then finding by actual trial how many times a length of 3 units is contained in this length. Marking off successive lengths of 3 units each, the ends of these lengths fall respectively at the points 1, 2, 3, 4, 5, in the figure. The end of 5 lengths of 3 units brings us to the point C. There then remains a length CB, which clearly contains 2 units.

It is evident from the figure that

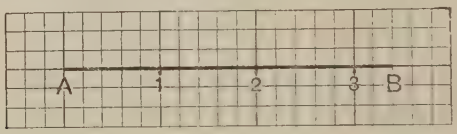
$$AB = AC + CB,$$

i.e. that 17 *units* $= 5 \times 3$ *units* $+ 2$ *units*,

or that Dividend = Quotient × Divisor + Remainder.

15. By taking different numbers, and working out for himself similar cases with the help of squared paper, the pupil will see that Division is really only repeated subtraction, and that the final Remainder is what is left when the Divisor has been subtracted from the Dividend as often as it can be. He will see too that the remainder relates to units *of the same name* as the Dividend.

16. We may also look at the matter in another way: We may enquire what number is contained 3 times in the 17 units. Using squared paper again, we see that the multiple of 3 next less than



17 is 15, or 3×5. The length which is contained 3 times in a length of 17 units, is therefore a length of 5 units, and there are 2 units over. In this case the Divisor is an abstract number, the *Quotient* is 5 *units*, and the Remainder is 2 units as before.

17. When the Divisor is a *concrete* number, the Quotient is an *abstract* number, and the process is called **Quotition.**

When the Divisor is an *abstract* number, and the Quotient is a *concrete* number, the process is called **Partition.**

II. HOW NUMBERS ARE EXPRESSED.

18. The art of expressing numbers by symbols is called **Notation**; and the art of expressing them by words is called **Numeration.**

The number which denotes *one* more than 9 is called **ten**, and plays a most important part both in notation and in numeration.

If, for example, we have a large number of similar coins, how can we count the coins and represent the number of them by means of the digits, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9?

Let us first take out ten coins and put them together in a pile. Then ten more in another pile; then ten more, and so on; and let us do this as often as we can, until there are less than ten coins left over. This remainder, being less than ten, may be denoted by one of the above digits. Suppose it is 3.

Let us next treat the piles of ten in a similar manner. Let us take ten piles and put them together in a group; then ten more in another group; then ten more in a third group, and so on; until, as before, we have less than ten piles left. If this remainder be 7 piles, we will place the digit 7 to the left of the 3, thus, 73. The number of coins in each group of ten piles, each pile containing ten pieces, is called a **hundred.**

We will now put together ten groups of a *hundred* each to form a larger group, the number of coins in which is called a **thousand**, and we will continue the grouping until the number of *hundreds* left over is less than ten. If 3 *hundreds* be left over, we place the digit 3 to the left of the 73, thus getting 373.

The process of putting together ten smaller groups to form one larger group must be carried on until at some step the number of larger groups that can be formed is itself less than ten. If this happens when we have formed the groups which contain a *thousand* each, that is, if the number of *thousand*-groups is only nine, we put the digit 9 to the left of the 373, thus, 9373, and the process is at an end. The expression 9373 will represent the number of coins.

19. For the number of coins in the successive groups, each of which is formed by putting together ten of the next smaller groups, we have the following names:

a unit,
ten,
one hundred,
one thousand,
ten thousand,
one hundred thousand,
one million,
ten million,
one hundred million,
one thousand million,
ten thousand million,
one hundred thousand million,
one billion.

Next we have *ten* billion, *one hundred* billion, and so on, until we come to *one million billion.* For this a new word is introduced, a million billion being called a **trillion.**

NOTE.—In the United States and in France, a *billion* means a thousand million; a *trillion*, a thousand billion; and so on.

20. The number of pieces in *one* group of ten is written 10. The number of pieces in one group of ten,

and 1 piece more, is called	*eleven*,	and is written	11,
... 2	*twelve*,	.....	12,
... 3	*thirteen*,	.....	13,
... 4	*fourteen*,	.....	14,
... 5	*fifteen*,	.....	15,
... 6	*sixteen*,	.....	16,
... 7	*seventeen*,	.....	17,
... 8	*eighteen*,	.....	18,
... 9	*nineteen*,	.....	19.

The number

in *two* groups of ten is called	*twenty*,	and is written	20,
... *three*	*thirty*,	.....	30,
... *four*	*forty*,	.....	40,
... *five*	*fifty*,	.....	50,
... *six*	*sixty*,	.....	60,
... *seven*	*seventy*,	.....	70,
... *eight*	*eighty*,	.....	80,
... *nine*	*ninety*,	.....	90,
... *ten*	*one hundred*,	.....	100.

Also 7 groups of ten and 4 additional units are spoken of as *seventy-four* units, and the number is written 74. In a similar manner other intermediate numbers are named and written.

21. Let us return now to the number 9373. Taking the digits one by one beginning at the right hand, we know that

the first digit 3 denotes 3 coins, or 3 *units*;
the next digit 7 ..... 7 *tens*, or *seventy units*;
the next digit 3 ..... 3 *hundreds*;
and the remaining digit 9 ..... 9 *thousands.*

Therefore we may express the number 9373 in the words

nine thousand, three hundred, and seventy-three.

NOTE.—To express a number in words, we begin with the left-hand digit, and we end with the units.

22. Similarly, the number 123765215 denotes

5 *units*,
1 *ten*,
2 *hundreds*,
5 *thousands*,
6 *ten thousands*, *or* 60 *thousands*,
7 *hundred thousands*,
3 *millions*,
2 *ten millions*, *or* 20 *millions*,
and 1 *hundred millions*.

This may be shortened by collecting together all the thousands, and all the millions; and expressing the five units and the one group of ten by the single word *fifteen*. The number therefore denotes

fifteen,
two hundred,
seven hundred and sixty-five thousand,
and *one hundred and twenty-three million*;

and we express it thus,

One hundred and twenty-three million, seven hundred and sixty-five thousand, two hundred and fifteen.

23. If in Art. 18, there had been no hundreds left over, we should have placed the digit 0 instead of 3 to the left of the 7; the number would then have been represented by the digits 9073, and in words by *nine thousand, and seventy-three.*

If moreover there had been no tens left over as well as no hundreds, the number would have been represented by the digits 9003, and in words by *nine thousand and three.*

24. To aid us in reading a large number it is convenient to place a comma in front of every third digit, counting from the right. For example, if a number be written thus,

2,510,763,254,318,

we can see at a glance that 318 represents the number of *hundreds*, *tens*, and *units*, 254 the number of *thousands*, 763 the number of *millions*, 510 the number of *thousand millions*, and 2 the number of *billions*. We can now read the number much more readily, thus,

Two billion,
five hundred and ten thousand, seven hundred and sixty-three millon,
two hundred and fifty-four thousand,
three hundred, and *eighteen.*

25. All numbers between 10 and 100 are represented by *two* digits. The highest, *ninety-nine*, is represented by 99, and means 9 *tens* $+$ 9 *units*.

If to this we add 1, we get

9 *tens* $+ 10$ *units*,
or 10 *tens* $+ 0$ *units*,
or 1 *hundred* $+ 0$ *tens* $+ 0$ *units*.

This is represented by 100, and requires *three* digits.

All numbers between 100 and 1000 need *three* digits. The highest is 999; if we add 1 to it we get 1000, which requires four digits.

The greatest number of *four* digits is 9999. If to this we add 1 we get 10000, for which we need *five* digits; and so on.

26. In the number 9373, the digit 3 appears twice; in one place it means 3 units, and in the other 3 hundreds. Hence, although a digit always has the same *absolute* value, yet in consequence of the difference of position the one 3 here denotes a hundred times as much as the other.

The principle of the **value** of a digit depending upon its **position** among the others is a fundamental one in our method of notation, and is called the PRINCIPLE OF LOCAL VALUE. The method was known in Europe to some extent about the 12th century, and it is pretty certain that *one* of its avenues of approach was through the Moors of Spain. It is called the **Arabic** system.

27. For the numbers 100, 1000, 10000, etc., it will be convenient to have a short form which will at the same time shew their relation to the number 10.

Since 100 denotes *ten tens*, or 10 multiplied by 10, we will denote it by 10^2. The little figure 2 placed above on the right hand is called an **index**, for it indicates the number of tens that are multiplied together. Similarly we will represent 1000 by 10^3, 10000 by 10^4, and so on.

The expression 10^4 is read as 10 *raised to the power* 4, or 10 *to the fourth;* and 10^8 as 10 *to the eighth.* The expressions 10^2 and 10^3, however, instead of being read as 10 *to the second* and 10 *to the third*, are more commonly spoken of as 10 *squared* and 10 *cubed* respectively.

Using this **index notation**, we see that

$$6507312 = 6 \times 10^6 + 5 \times 10^5 + 0 \times 10^4 + 7 \times 10^3 + 3 \times 10^2 + 1 \times 10 + 2.$$

The number 10 which plays so important a part in this mode of expressing numbers is called the **radix** of the **scale of notation**. The radix 10 is used by all civilized people, and was probably adopted originally because of the primitive custom, not uncommon even now, of counting with the ten fingers.

28. The *index notation* may be used also with any other number than 10. For example,

$$4 = 2 \times 2 = 2^2,$$
$$8 = 2 \times 2 \times 2 = 2^3,$$
$$25 = 5 \times 5 = 5^2,$$
$$625 = 5 \times 5 \times 5 \times 5 = 5^4,$$

and generally, if a represent a number, and n denote the number of a's which are multiplied together, the product may be represented by a^n; thus,

$$a \times a \times a \times a \times \ldots \text{ to } n \text{ factors} = a^n.$$

29. If we multiply $10 \times 10 \times 10$ by $10 \times 10 \times 10 \times 10$, we get $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$; *i.e.* if we multiply 10^3 by 10^4, we get 10^7; and, generally, if we multiply one power of 10 by another power of 10, the result is a power of 10, whose index is obtained by adding the indices of the powers multiplied. This property holds in the case of *any* number, and may be stated generally thus:

$$a^m \times a^n = a^{m+n},$$

where a, m and n represent any numbers.

Similarly, $$a^m \div a^n = a^{m-n}.$$

EXAMPLES. 1.

To be done first orally, then in writing.

State in figures:

1. Forty-five; fifty-four.
2. Fifty-one; fifteen.
3. Eighteen; eighty-one.
4. Eleven; one hundred and one; one hundred and eleven.
5. Seventeen; one hundred and seven; one hundred and seventy.
6. Two hundred and one; one hundred and two; one hundred and twenty; two hundred and ten.
7. One hundred and seventeen; seven hundred and eleven; one hundred and seventy-one; seventeen hundred and one.

State in figures:

8. One thousand one hundred and eleven; one thousand and eleven; one thousand and one.

9. Nine hundred and fourteen thousand, and forty-one.

10. One million, forty thousand, and fourteen.

11. One million and one; one million one thousand and one; one million one hundred and one; one million and sixty.

12. Sixty-four million, two hundred and six thousand.

13. One hundred and eleven million, two hundred and thirty-four thousand and five.

14. Nineteen million, nine hundred thousand and nineteen.

15. One hundred and sixty million and six.

16. Ten million, one thousand, one hundred and one.

17. Three hundred and four million, and forty thousand.

18. Ten million, three hundred thousand and three.

19. Eight million, eight thousand, and eighteen.

20. One billion, five million, and six.

21. Six hundred and six billion, three hundred and forty-one.

EXAMPLES. 2.

To be done first orally, then in writing.

Express in words:

1. 11, 101, 110.
2. 12, 21, 102, 201, 120, 210.
3. 23, 32, 203, 302, 230, 320.
4. 28, 82, 208, 802, 280, 820.
5. 45, 54, 504, 540, 405, 450.
6. 61, 16, 106, 601, 610, 160.
7. 111, 1101, 1011, 1110.
8. 1111, 10111, 11011, 11101, 11110.
9. 2153, 5213, 3125, 3521.
10. 20153, 52013, 50213, 30125, 35201, 21053, 25310, 32150.
11. 11111, 110111, 110011, 110001, 100001.
12. 76541, 760541, 706415, 765401, 765410.
13. 700541, 610045, 400006, 100400, 104005.
14. 1000001, 1001001, 1110001, 1000010, 1010101.
15. 1235132, 1035234, 3050204, 3000215, 5321000.

16. 11111111, 10000001, 10000100, 10101010, 11100001.

17. 90009200, 99200200, 90020090, 29000092.

18. 6007002005, 4219037613, 8010706010.

III. REMARKS ON THE FIRST FOUR RULES.

30. We will illustrate the ordinary rules for addition and subtraction by expressing each number with the index notation of Art. 27. As an example of addition we will consider 35689, 3216 and 28. In the index notation these are represented by

$$3\times10^4+5\times10^3+6\times10^2+8\times10+9,$$
$$3\times10^3+2\times10^2+1\times10+6,$$
and
$$2\times10+8.$$

By addition we find the sum to be

$$3\times10^4+8\times10^3+8\times10^2+11\times10+23.$$

But 23 units are the same as $2\times10+3$, hence the sum

$$=3\times10^4+8\times10^3+8\times10^2+13\times10+3,$$

the 2 tens being transferred to the next column, thereby increasing it from 11 tens to 13 tens. Again, since $13\times10=1\times10^2+3\times10$, we see finally that the sum

$$=3\times10^4+8\times10^3+9\times10^2+3\times10+3,$$
$$=38933.$$

From the above process we deduce the common rule.

31. As an example of subtraction we will take 386 from 1458. Writing the smaller number below the other, we have

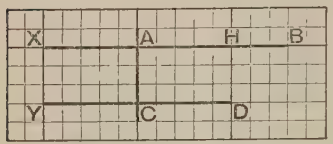
$$1\times10^3+4\times10^2+5\times10+8,$$
and
$$3\times10^2+8\times10+6.$$

If we subtract 6 units from 8 units, the remainder is 2 units. We cannot, however, subtract 8 tens from 5 tens; but if we add 10×10 to the upper line and 10^2 to the lower, the numbers then become

$$1\times10^3+4\times10^2+15\times10+8,$$
and
$$4\times10^2+8\times10+6.$$
The difference
$$=1\times10^3+0\times10^2+7\times10+2,$$
$$=1072.$$

Hence we can deduce the usual rule.

NOTE.—In the above example we have assumed that we may add equal numbers, 10×10 and 10^2, to the minuend and to the subtrahend respectively.



The accompanying figure on squared paper shows graphically that the difference between the new minuend and the new subtrahend is the same as the difference between the original minuend and the original subtrahend.

AB and CD represent the original minuend and subtrahend; XB and YD represent what AB and CD become when the equal lengths XA and YC respectively are added to them. The difference HB clearly remains unaffected by these changes.

If a, b and c be understood to represent any numbers, this property may be stated in a general form thus:

$$(a+b)-(c+b)=a-c.$$

32. Instead of the common process in subtraction the student will often find it convenient to determine how much must be *added* to the subtrahend to make up the minuend. For example, to find the difference between 32102 and 27568, the mental steps are as follows:

32102
27568
4534

(i) 8 and 4 make 12; put down **4**, and carry 1;
(ii) 1 to 6 makes 7, and 3 make 10; put down **3**, and carry 1;
(iii) 1 to 5 makes 6, and 5 make 11; put down **5**, and carry 1;
(iv) 1 to 7 makes 8, and 4 make 12; put down **4**, and carry 1;
(v) 1 to 2 makes 3.

Here the operation stops, and we find the difference to be 4534.

This method can be applied also when we wish to multiply a number by any of the numbers from 2 to 12, and *simultaneously* to subtract the product from a minuend; the final result being written down at once. As an example we will subtract 8 times 23478 from 784592 *in one operation*. Thus:

784592
23478 × 8
Difference = 596768

The mental steps by which we obtain the result at once are as follows:

(i) $8 \times 8 = 64$, and **8** make 72; put down **8**, and carry 7;
(ii) $8 \times 7 + 7 = 63$, and **6** make 69; put down **6**, and carry 6;
(iii) $8 \times 4 + 6 = 38$, and **7** make 45; put down **7**, and carry 4;
(iv) $8 \times 3 + 4 = 28$, and **6** make 34; put down **6**, and carry 3;
(v) $8 \times 2 + 3 = 19$, and **9** make 28; put down **9**, and carry 2;
(vi) 2 and **5** make 7; put down **5**.

Here the operation ends and we have the required difference 596768.

This process is employed in the Shortened form of Long Division known as the **Italian Method** (Art. 35). It may also be used to obtain, in one operation, the *sum* of one number and a multiple of another.

EXAMPLES. 3.

Find *in one operation* the value of

1.	$56892 - 7 \times 3456$.	2.	$311234 - 3 \times 9742$.
3.	$4978431 - 8 \times 476821$.	4.	$9090901 - 9 \times 672358$.
5.	$68743 - 11 \times 6078$.	6.	$19904690 - 12 \times 947842$.
7.	$74329 + 3 \times 7265$.	8.	$123567 + 8 \times 93742$.
9.	$476259 + 7 \times 84682$.	10.	$98765 + 9 \times 346721$.

33. We will next consider multiplication. The product of 5763 multiplied by 154 means that we take the first number 154 times over. But $154 = 100 + 50 + 4$. If therefore we multiply 5763 by 100, then by 50, and then by 4, and add the results together, we shall get the product required. We arrange the work as below:

		or	
5763			5763
154			154
576300	= 100 times 5763		5763
288150	= 50 times 5763		28815
23052	= 4 times 5763		23052
887502	= 154 times 5763		887502

It is customary to omit the final zeros of the 1st and 2nd lines; but the student must be very careful to place the three partial products, so that each right-hand digit may come under, or in the same column with, the multiplying figure which produces it; *e.g.*, the final figures 3, 5, 2 must be placed under the multiplying figures 1, 5, 4, respectively.

34. It may be noticed that the final zeros in the successive products are usually omitted in division also. For example, the division of 887502 by 154, if stated fully, would appear as in the margin. The quotient written in the usual way is 5763.

154) 887502 (	5000 + 700 + 60 + 3
770000	
117502	
107800	
9702	
9240	
462	
462	

35. When the pupil has had sufficient practice in long division, he may shorten the process by omitting the products and writing down only the successive differences (Art. 32). This is called the **Italian Method.**

By this method, the division of 13877136 by 3276 would appear as in the margin.

The figures of the quotient should be placed, as in this example, each immediately over the dividend figure which is used in obtaining it.

Quotient = 4236

3276) 13877136

7731

11793

19656

To obtain the first remainder 773 by the process of Art. 32 the mental steps would be as follows:

(i) $4 \times 6 = 24$; and **3** make 27; write down **3** and carry 2.

(ii) $4 \times 7 = 28$; and 2 carried make 30; and **7** make 37; write down **7** and carry **3.**

(iii) $4 \times 2 = 8$; and 3 carried make 11; and **7** make 18; write down **7** and carry **1.**

(iv) $4 \times 3 = 12$; and 1 carried makes 13.

The difference, as before, is 773. We now annex the digit 1 from the dividend, making 7731, and, for the second and following steps, we proceed again as described above.

36. (i) The correctness of a **sum** may be ascertained by subtracting one of the numbers from the sum of all; the difference should be the sum of the others.

(ii) The correctness of a **difference** may be ascertained by adding it to the subtrahend; the result should be the minuend.

(iii) The correctness of a **product** may be tested by using the old multiplier as multiplicand, and the old multiplicand as multiplier. The result should be the same as before.

For example, consider the dots in the margin. The number of dots altogether is the product of the number of dots in a row, 9, multiplied by the number of rows, 5. It is also the product of the number of dots in a column, 5, multiplied by the number of columns, 9.

Hence $9 \times 5 = 5 \times 9$.

If we use a and b to represent any whole numbers whatever, it may be shewn in this manner that $a \times b = b \times a$.

(iv) The correctness of a **product** may also be ascertained by dividing the product by one of the factors. The quotient should be the other factor, and there should be no remainder.

(v) The correctness of a **product** may be tested also by the process called **casting out the nines**, which is as follows:

Divide the sum of the digits of one factor by 9, *and find the remainder; divide the sum of the digits of the other factor by* 9, *and find the remainder; multiply the two remainders together, and divide the result by* 9. *The remainder after this last division should be the same as the remainder when we divide the sum of the digits of the product itself by* 9.

If these remainders are different, the product is certainly wrong; if they are the same, we cannot say that the product is certainly right; but it is very likely to be so. This test in fact will not disclose an error which is due to the misplacing of one of the partial products; nor an error, from any cause, which is a multiple of 9.

If, for instance, two of the digits in the product be interchanged, or if 9 be written by mistake for 0, or 0 for 9, the product will of course be wrong, but this test will not give any hint that it is so.

In applying the test it will be noticed that we do not require the *quotients*, but only the remainders. In adding the digits we may therefore disregard a digit 9, and also any digits whose *sum* is 9, because these will not affect a remainder. In the number 743589362, for example, we may disregard the digits 7 and 2, 4 and 5, 3 and 6, and 9. We are thus left with nothing more than the digits 8 and 3, whence we can see at once that the remainder will be 2.

If none of the digits may be disregarded for these reasons, we proceed to add the digits together beginning at the left hand, and when the sum becomes greater than 9, we subtract 9 from it and go on with the remainder only. Treating the number 743589362 in this way, the successive mental steps of the addition would be 7, 11, **2**, 5, 10, **1**, 9, **0**, 9, **0**, 3, 9, **0**, 2. The darker figures are those obtained after subtracting 9 from a sum greater than 9, or equal to 9. The last figure 2 is the final remainder.

We will now apply the test to the multiplication of 3274 by 5621.

In the number 3274 the digits 2 and 7 may be disregarded, and we see at once that the remainder is 7. In the number 5621 the digits 6, 2 and 1 may be disregarded, and we see that the remainder is 5.

The product of the remainders is 5×7, or 35, and this gives a remainder 8.

	3274
	5621
	16370
	19644
	6548
	3274
	18403154

In the number 18403154 we may disregard 1, 8, 5 and 4; and we are left with 4, 0, 3, and 1, which give a remainder 8.

The two remainders thus agree, and the test is satisfied.

If the remainders had been found to be different, there would certainly have been an error, either in the addition, or in one of the partial products 16370, 19644, 6548, 3274. We should then have tested each of these numbers in turn, until the error had been located.

(vi) The correctness of a **product** may be tested also by the process called **casting out the elevens**, which may be stated thus:

Find the remainders when the multiplicand and the multiplier are divided by 11. *Multiply these remainders together and divide the result by* 11. *The remainder then obtained should be the same as the remainder obtained from dividing the product itself by* 11.

The remainder after division by 11 may be obtained conveniently by the following process. We will consider the number $7\dot{4}3\dot{5}8\dot{9}3\dot{6}\dot{2}$, the digits in the *odd* places of which we will distinguish by dots placed *above* them, and those in the *even* places by dots placed *below* them. We will then add together in turn the digits in the *odd* places, taking them in the order 2, 3, 8, 3, 7; and from the result we will subtract separately and in turn the digits in the *even* places, taking these in the reverse order, 4, 5, 9, 6.

If, as we subtract, we find that at any step the minuend is less than the digit which should next be taken from it, we increase that minuend by 11.

Using the number 743589362, the sums obtained successively in the first part of the process, and the differences obtained successively in the second part, are as follows:

2, 5, 13, 16, 23, **19**, 14, 5, **16**, 10,

the increased minuend, obtained by adding 11 to 5, being shewn in heavier type. The last number, 10, represents the remainder obtained when 743589362 is divided by 11.

We will now, by this method, test again the accuracy of the number 18403154 which we found for the product 3274×5621. The successive sums and remainders are as follows:

for $3\dot{2}7\dot{4}$; 4, 6, 3, **14**, 7, remainder = 7;

for $5\dot{6}2\dot{1}$; 1, 7, 2, 0; remainder = 0;

for $1\dot{8}4\dot{0}3\dot{1}5\dot{4}$; 4, 5, 5, 13, 12, 8, 5, 0; remainder = 0

The product 7×0, when divided by 11, gives a remainder 0. The remainder when 18403154 is divided by 11 is also 0. The test is therefore satisfied.

This test would not detect an error which is a multiple of 11; but it will happen very seldom that an error remains undetected if both the **nine** test and the **eleven** test are applied.

(vii) It should be noticed that the *number of digits* in a product is *never greater* than the sum of the number of digits in the multiplicand and the number of digits in the multiplier, and it can only be *less* than that sum by 1. For instance, the product of a number of 4 figures and a number of 5 figures cannot be greater than 9999×99999, or 999890001; and it cannot be less than 1000×10000, or 10000000.

It may be noticed too that the product of two numbers such as 3274 and 56219 must lie between 3000×50000 and 4000×60000, *i.e.* between 150000000 and 240000000.

These are but rough tests, yet they will serve sometimes to prevent a pupil from overlooking results which are not merely wrong but are manifestly impossible.

(viii) The correctness of a **quotient** may be ascertained by multiplying the quotient by the divisor, and adding the remainder (if any) to the product. The result so obtained will be the same as the *dividend*, if the quotient be correct.

The correctness of a **quotient** may also be tested by *casting out the nines* and by *casting out the elevens.*

37. There are certain multipliers and divisors for which we may adopt special methods.

(i) *To multiply by* 9.

We know that $9 = 10 - 1$. Hence if we multiply the number by 10 we take it *once* too often; if therefore we subtract the number itself from this product, we clearly get the same as if we had multiplied the number at once by 9.

But the product of a number by 10 is obtained by merely placing a 0 at the end of the number. Hence multiplication by 9 is reduced to simple subtraction of one number from another.

For example, 10 times 6729403 = 67294030,
and once ..... = 6729403;
∴ by subtraction, 9 times ..... = 60564627.

(ii) *To multiply by* 99.

We know that $99 = 100 - 1$; therefore we multiply the number by 100, *i.e.* we place two 0's at the end, and subtract the number itself from the result. Thus:

100 times 6729403 = 672940300,
and once ..... = 6729403;
∴ 99 times ..... = 666210897.

(iii) *To multiply by* 999, 9999, 99999, *etc.*

These may all be treated in the same way as 9 and 99; for

$$999 = 1000 - 1,\quad 9999 = 10000 - 1,\quad 99999 = 100000 - 1,\text{ etc.}$$

Hence we place at the end of the number as many 0's as there are 9's in the multiplier, and from the result we subtract the number itself.

(iv) *To multiply by* 99996.

In this case we place 5 *noughts* at the end of the multiplicand, and from the result we subtract 4 *times* the multiplicand. Thus

$$\begin{aligned} &100000 \text{ times } 763021 = 76302100000,\\ \text{and}\quad &4 \text{ ..... } = \quad 3052084;\\ \therefore\ &99996 \text{ ..... } = 76299047916. \end{aligned}$$

This method should be adopted whenever the multiplier differs but little from 10, 100, 1000, 10000, etc., *i.e.* from a *power* of 10.

(v) *To multiply by* 5.

We know that 10 is twice 5, and that if we multiply by 10 the result is twice as much as it should be. If then we divide this product by 2, the quotient will be the product required. For example,

$$\begin{aligned} &10 \text{ times } 763021 = 7630210,\\ \therefore\ &5 \text{ ..... } 763021 = 3815105. \end{aligned}$$

One advantage of this process is that we arrive at the figures of the result in their proper order, beginning at the left hand.

(vi) *To multiply by* 25.

We know that 100 is 4 times 25, and that if we multiply the number by 100, we take it 4 times as often as we should. We therefore divide this result by 4. With the same multiplicand as before the operation is effected thus:

$$4 \,|\, \underline{76302100}$$
$$19075525$$

(vii) *To multiply by* 125.

In this case, for similar reasons, we place three 0's at the end of the multiplicand and divide by 8. With the same multiplicand we obtain the result as follows:

$$8 \,|\, \underline{763021000}$$
$$95377625$$

(viii) *To multiply by* 12111.

We notice that $12111 = 12100 + 11$,
and that $12100 = 1100$ times 11.

Hence, if 78932 be the multiplicand, we may arrange the work as in the margin.

The line (i) is 11 times 78932; the line (ii) is 1100 times (i), and therefore 12100 times 78932.

```
   78932
   12111
  868252 (i)
955077200 (ii)
955945452
```

(ix) *To multiply by* 5678109.

We notice that the multiplier $= 9 + 8100 + 5670000$. Consequently we may arrange the work in *three* lines. We will suppose the multiplicand to be 78932612.

The line (i) is 9 times the multiplicand; the line (ii) is 900 times (i) and therefore 8100 times the multiplicand; the line (iii) is 700 times (ii) and therefore 5670000 times the multiplicand. Consequently the result obtained by adding these three lines is the product required.

```
   78932612
    5678109
  710393508 (i)
639354157200 (ii)
447547910040000 (iii)
448187974590708
```

NOTE.—The pupil should keep a careful watch for such a relation between the digits of the multiplier as we have used here.

(x) *To multiply by any number from* 11 *to* 19.

We will suppose the multiplicand to be 57643, and the multiplier to be 17. The work would appear in full as in the margin.

```
 57643
    17
 57643
403501
979931
```

It is evident from this that we have to add 576430 to 7 times 57643. This can be effected in one operation by the following steps:

7 times 3 $= 21$; write down 1 and carry *2*;
7 times 4, and *2* $= 30$; add 3, making 33; write down 3 and carry *3*;
7 times 6, and *3* $= 45$; add 4, making 49; write down 9 and carry *4*;
7 times 7, and *4* $= 53$; add 6, making 59; write down 9 and carry *5*;
7 times 5, and *5* $= 40$; add 7, making 47; write down 7 and carry *4*;
to the *4* carried, add 5, making 9; write down 9.

Thus we obtain the result 979931. We may describe the process thus: *Multiply the figures of the multiplicand successively by 7, and add to each result that figure of the multiplicand which is to the right of the one multiplied.*

NOTE.—This process may be adopted also when the multiplier is a number of two digits, of which the *units'* digit is 1. In that case we add the figure to the *left* of the one multiplied.

(xi) *To divide by* 5, 25, 125.

We may multiply the number by 2, 4, 8 ..., and divide the result by 10, 100, 1000, ... respectively.

38. To divide by 35 by **short division**, we divide successively by 5 and 7. If, for example, we divide 3726589 by 5, the quotient is 745317, and the remainder is 4 *units*. If we now divide 745317 by 7, the quotient is 106473, and the remainder is 6 *sets of* 5 *units*.

Thus the complete remainder after division by 35

$$= 6 \textit{ sets of } 5 \textit{ units and } 4 \textit{ units}$$
$$= 6 \times 5 \textit{ units} + 4 \textit{ units}$$
$$= 34 \textit{ units}.$$

Hence, to find the complete remainder, we *Multiply the second remainder by the first divisor and add the first remainder.*

Example. *Divide* 3762905 *by* $3 \times 5 \times 7$ *and find the remainder.*

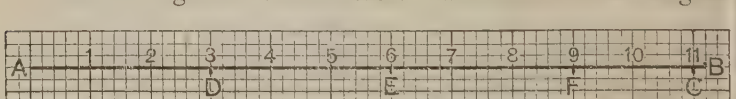
$$105 \left\{ \begin{array}{l} 15 \left\{ \begin{array}{l|l} 3 & 3762905 \\ 5 & \underline{1254301}\ldots 2 \\ 7 & \underline{250860}\ldots 1 \\ & 35837\ldots\ldots 1 \end{array} \right\} 5 \\ \qquad\qquad 20 \end{array} \right.$$

After division by 3 and 5 the complete remainder is 5. To find the complete remainder after the next division, we consider 15 as our first divisor and 5 as our first remainder. We may proceed in this way, whatever be the number of successive divisors.

39. The method of finding the complete remainder in short division may be illustrated by using squared paper. We will consider the division of 56 by 15, using the factors 3 and 5 in succession. In the usual way the operation would appear as in the margin.

$$15 \left\{ \begin{array}{l|l} 5 & 56 \\ 3 & \underline{11}\ldots 1 \\ & 3\ldots 2 \end{array} \right\} 11$$

We will take a line AB 56 units in length, and we will measure off from it lengths of 5 units each. The ends of these lengths



will be at the points marked 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11. The last will be at the point marked 11 or C, and there will be a remainder CB containing 1 unit.

We will next count these lengths 3 at a time, and we will mark the ends of these new sections with the letters D, E, F respectively. The remainder after this operation will be the length FB, and we see that

$$\begin{aligned} FB &= FC + CB \\ &= 2 \textit{ lengths of } 5 \textit{ units} + 1 \textit{ unit} \\ &= 2 \times 5 \textit{ units} + 1 \textit{ unit} \\ &= 11 \textit{ units.} \end{aligned}$$

The pupil should work out other examples in this way for himself.

EXAMPLES. 4.

Multiply:

1. 12302, 2. 67301, 3. 562143,
4. 487621, 5. 327412, 6. 7621985,

by 9, 99, 9999, 994, 9992, 99980, 5, 25, and 125.

Multiply in *two* lines:

7. 32162 by 427. 8. 3543 by 648.
9. 4016 by 637. 10. 72681 by 11011.
11. 31127 by 14412. 12. 61763 by 1089.

Multiply in *three* lines:

13. 92135 by 10813212. 14. 87623 by 105613212.
15. 45989 by 864729. 16. 57109 by 3125625125.

Divide 28937257 by

17. $3 \times 5 \times 7$, 18. $2 \times 3 \times 5$, 19. $3 \times 7 \times 11$,
20. $2 \times 3 \times 5 \times 11$, 21. $3 \times 5 \times 7 \times 11$, 22. $2 \times 3 \times 5 \times 7 \times 11$,

by short division, and find the complete remainder in each case.

Multiply 32745 in *one* line by

23. 16. 24. 17. 25. 18. 26. 19.
27. 41. 28. 81. 29. 101. 30. 121.

31. If 1374 divides 523689***, without remainder, find the missing figures which the stars (*) represent.

IV. THE ROMAN SYSTEM OF NOTATION.

40. Before the introduction of the Arabic system into Europe, the Roman system of notation was most generally used, and as it is still used for some purposes we will give a brief account of it.

41. The Roman symbols are

I, V, X, L, C, D, M.

Each by itself represents a number, but the numbers are not consecutive; thus:

I represents 1,	C represents 100,
V 5,	D 500,
X 10,	M ..... 1000.
L 50,	

42. To represent any other number we take those of the seven letters whose values amount to it by addition, but in order to lessen the number of letters used we take as many as possible of those of higher value. The letters may be arranged in any order, for in the Roman system we find no such principle as that of Local Value, except in a few cases to be noticed presently. It is customary, however, to write the letters of highest value at the left hand, the others following them in order. Thus:

8 = VIII,	276 = CCLXXVI,
23 = XXIII,	801 = DCCCI,
58 = LVIII,	1600 = MDC,
87 = LXXXVII,	1885 = MDCCCLXXXV.

43. The exceptional cases, in which the letters have a meaning dependent upon their position, appear in the notation for 4, 9, 40, 90, 400, 900.

To represent 4 by the method described we should have IIII; but instead of this it is customary to use IV; the position of the symbol of higher value, to the *right* of that of lower value, denoting that the lower symbol must be *subtracted from*, and not added to, the higher. Thus

$$VI = 5 + 1 = 6, \text{ but } IV = 5 - 1 = 4.$$

Again, we represent 9 by IX, instead of by VIIII; the relative positions of the two letters again denoting subtraction; thus

$$XI = 10 + 1 = 11, \text{ but } IX = 10 - 1 = 9.$$

On the same principle 40 is represented by XL, which denotes L − X; 90 by XC; 400 by CD; 900 by CM, and 1900 by MCM.

44. The Roman symbol for 500 is now generally written D, but is sometimes met with in the form Iↀ. The latter leads directly to the representation of still higher numbers, every ↀ annexed to Iↀ increasing the number tenfold.

Again, the Roman symbol for 1000, which is generally written M, is sometimes met with in the form CIↀ. In this case too if we place C in front and also ↀ at the end, we increase the value tenfold. Thus

Iↀ = 500,	CIↀ = 1000,
Iↀↀ = 5000,	CCIↀↀ = 10000,
Iↀↀↀ = 50000,	CCCIↀↀↀ = 100000.

45. Lastly, a bar placed over a letter multiplies its value a thousandfold; thus

$$\overline{\text{M}} = 1000, \text{ and } \overline{\text{M}} = 1000000.$$

46. The Arabic notation is not only adapted to the *representation* of numbers, but is such as to simplify the actual working out of calculations. The Romans, however, made their calculations with *counters* or *calculi*. From this practice the English word *calculation* is derived.

EXAMPLES. 5.

Express each of the following numbers in the Arabic notation; (i) *viva voce*, (ii) in writing.

1. IV, XIII, XXVII, XCVIII, XↀIX.
2. CLIX, CCLXIV, CCCXXXVIII, DIX, DXC.
3. DCXC, DCXXVIII, DCIX.
4. CDXCIX, DCCXXI, CDXXIX.
5. MCC, MDCLX, MDCCVIII.
6. CMXCIX, MCMXC, MMDCCCLVIII.
7. $\overline{\text{X}}$MCCLXXXVI, $\overline{\text{X}}$MMMDCXCIV.
8. $\overline{\text{LX}}$MCCI, $\overline{\text{LX}}$X, $\overline{\text{LXX}}$$\overline{\text{X}}$MMDCV.
9. $\overline{\text{LIX}}$CDLII, $\overline{\text{XCIX}}$CDXIX.
10. $\overline{\text{DLX}}$DCCXLIX. 11. $\overline{\text{MMDCLIC}}$IX.

EXAMPLES. 6.

Express the following numbers in the Roman notation; (i) *viva voce*, (ii) in writing.

1. 4, 14, 24, 34, 44, 54, 64, 74, 84, 94, 104, 114.
2. 9, 19, 29, 39, 49, 59, 69, 79, 89, 99, 109, 119.
3. 149, 144, 194, 199.
4. 400, 440, 444, 449, 490, 494, 499.
5. 900, 940, 944, 949, 990, 994, 999.
6. 1584, 1499, 1602, 1351, 1885.
7. 3050, 3149, 2139, 3273, 2768.
8. 5670, 8953, 7320, 6666, 8888.
9. 67241, 40069, 98762, 45798, 99999.
10. 1567840, 3476521, 3005763.

V. FACTORS AND PRIME NUMBERS.

47. If two or more numbers be multiplied together the result is called the **product**, and the numbers multiplied are called **factors** of the product.

Thus 6, being the result of multiplying 2 by 3, is called the *product* of 2 and 3; and 2 and 3 are called the *factors* of 6.

All numbers are multiples of unity, and each contains itself once. Any number can therefore be divided by itself and by unity, but if it cannot be divided without remainder by *any other* number, it is said to be **prime**, and is called **a prime number**. We may therefore define a prime number as *a number which has no factor or divisor but itself and unity.*

Numbers which are not prime are called **composite**.

Taking the numbers in order it will be seen that 2, 3, 5, 7, 11, 13, 17, etc., are prime numbers, and that 4, 6, 8, 9, 12, 14, 15, 16, etc., are composite.

48. A series of numbers in which each is greater by 1 than that which precedes it, such as 5, 6, 7, 8, are called **consecutive** numbers.

Numbers which are exactly divisible by 2, such as 2, 4, 6, 8, etc., are called **even** numbers. Hence no *even* number can be *prime* except 2.

Numbers which are *not* exactly divisible by 2, such as 1, 3, 5, 7, etc., are called **odd** numbers. Hence when an odd number is divided by 2, the *remainder* is always 1.

It follows from these definitions that every even number is of the form $2n$, and that every odd number is of the form $2n+1$, or $2n-1$.

49. Such a number as 12 is the product of 6 and 2, also of 4 and 3, and also of 2, 2 and 3, but the last of these sets, namely, 2, 2 and 3, has a property which the others have not, for 2, 2 and 3 are all *prime*.

Many numbers can be separated into factors in more than one way, just as 12 can be; but *no* number can be separated in more than one way into *prime* factors. When therefore we speak of separating a number into factors, we shall as a rule mean *prime* factors. In the case of 12 we have

$$12 = 2 \times 2 \times 3 \quad = 2^2 \times 3,$$

and, similarly,

$$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5.$$

50. The prime factors of a number can be found by trying the prime numbers in succession, beginning with 2. When we have found one factor, we divide the number by it, and then treat the quotient in a similar way; and we continue this process until we arrive at a divisor which gives a quotient less than itself.

For example, in the case of the number 150, we try divisors 2, 3, 5, 7, 11 in succession; but when we try 13, we get a quotient 11 and a remainder 7. The quotient 11 being less than the divisor 13, we need try no further. All prime factors of 150 lie therefore between 2 and 11 inclusive.

What numbers to try will readily be seen after a little practice, but it will be well, however, to remember the following **rules of divisibility**:

(i) **2** is a factor of all numbers whose last digit can be divided by 2.

(ii) **4** is a factor if the number composed of the last 2 digits can be divided by 4.

(iii) **8** is a factor if the number composed of the last 3 digits can be divided by 8.

(iv) **5** is a factor if the last digit be either 0 or 5.

(v) 10 is a factor if the last digit is 0.

(vi) 3 is a factor if the sum of the digits can be divided by 3.

(vii) 9 is a factor if the sum of the digits can be divided by 9.

(viii) 6 is a factor if both 2 and 3 are factors.

(ix) 12 is a factor if both 3 and 4 are factors.

(x) 11 is a factor if, when the 1st, 3rd, 5th, etc., digits are added together, and also the 2nd, 4th, 6th, etc., the difference between these sums is 0, or can be divided by 11.

51. We will apply these rules to the number 133056.

(i) 2 is a factor, because the last digit 6 can be divided by 2; hence

$$133056 = 2 \times 66528.$$

(ii) 3 is a factor of 66528, because the sum of the digits, viz. 27, can be divided by 3; thus

$$133056 = 2 \times 3 \times 22176.$$

(iii) 11 is a factor of 22176, because the sum of the 1st, 3rd, and 5th digits is 9, and the sum of the 2nd and 4th digits is also 9; the difference of these sums being 0, it follows that 22176 is divisible by 11. Hence

$$133056 = 2 \times 3 \times 11 \times 2016.$$

(iv) 9 is a factor of 2016, because the sum of the digits, viz. 9, is divisible by 9. Thus we have

$$133056 = 2 \times 3 \times 11 \times 9 \times 224.$$

(v) 4 is a factor of 224, because the number formed by the last two digits, viz. 24, can be divided by 4. Thus we have

$$\begin{aligned} 133056 &= 2 \times 3 \times 11 \times 9 \times 4 \times 56 \\ &= 2 \times 3 \times 11 \times 3^2 \times 2^2 \times 2^3 \times 7 \\ &= 2^6 \times 3^3 \times 7 \times 11. \end{aligned}$$

We have worked out this example as above, in order to shew the application of the rules, but the separation of a number into factors may be effected briefly, thus:

$$\begin{array}{r|l} 8 & 133056 \\ 8 & 16632 \\ 9 & 2079 \\ 3 & 231 \\ 7 & 77 \\ & 11 \end{array}$$

$$\begin{aligned} \therefore\ 133056 &= 8 \times 8 \times 9 \times 3 \times 7 \times 11 \\ &= 2^3 \times 2^3 \times 3^2 \times 3 \times 7 \times 11 \\ &= 2^6 \times 3^3 \times 7 \times 11. \end{aligned}$$

52. The following list of prime numbers between 1 and 1009 may be useful:

PRIME NUMBERS.

1	59	139	233	337	439	557	653	769	883
2	61	149	239	347	443	563	659	773	887
3	67	151	241	349	449	569	661	787	907
5	71	157	251	353	457	571	673	797	911
7	73	163	257	359	461	577	677	809	919
11	79	167	263	367	463	587	683	811	929
13	83	173	269	373	467	593	691	821	937
17	89	179	271	379	479	599	701	823	941
19	97	181	277	383	487	601	709	827	947
23	101	191	281	389	491	607	719	829	953
29	103	193	283	397	499	613	727	839	967
31	107	197	293	401	503	617	733	853	971
37	109	199	307	409	509	619	739	857	977
41	113	211	311	419	521	631	743	859	983
43	127	223	313	421	523	641	751	863	991
47	131	227	317	431	541	643	757	877	997
53	137	229	331	433	547	647	761	881	1009

EXAMPLES 7. *Oral.*

1. What are the prime factors of
   6, 9, 10, 14, 21, 33, 35, 38, 49?
2. What are the prime factors of
   8, 12, 18, 20, 27, 28, 30, 42, 45?
3. What are the prime factors of
   16, 24, 48, 56, 60, 64, 72, 80, 81, 96?
4. Which of the following numbers contain the factor (i) 2, (ii) 3, (iii) 4, (iv) 5, (v) 6, (vi) 8, (vii) 9, (viii) 11, (ix) 12?
   512, 795, 11172, 1235, 2859, 6467, 11599, 14916, 38187, 53729, 601830.

EXAMPLES. 8.

Separate each of the following numbers into its prime factors:

1.	60.	2.	75.	3.	78.	4.	98.
5.	102.	6.	105.	7.	210.	8.	231.
9.	252.	10.	168.	11.	315.	12.	429.
13.	525.	14.	594.	15.	612.	16.	693.
17.	715.	18.	840.	19.	854.	20.	1089.

Separate each of the following numbers into its prime factors:

21.	1331.	22.	1430.	23.	1456.	24.	3465.
25.	3675.	26.	4536.	27.	4620.	28.	4851.
29.	5250.	30.	7623.	31.	11025.	32.	14157.
33.	28413.	34.	48510.	35.	51425.	36.	89712.
37.	405769.	38.	750684.	39.	750750.	40.	561924000.

53. If a and b represent any two numbers, the following general principles are frequently useful:

(i) $(a+b)\times(a-b)=a^2-b^2$,

and conversely, $a^2-b^2=(a+b)\times(a-b)$.

(ii) $(a+b)^2=a^2+2\times a\times b+b^2$,

and (iii) $(a-b)^2=a^2-2\times a\times b+b^2$.

They may be proved as follows:

(i) $$\begin{aligned}(a+b)\times(a-b)&=a\times(a-b)+b\times(a-b)\\&=a\times a-a\times b+b\times a-b\times b\\&=a^2-b^2.\end{aligned}$$

(ii) $$\begin{aligned}(a+b)^2&=(a+b)\times(a+b)\\&=a\times(a+b)+b\times(a+b)\\&=a\times a+a\times b+b\times a+b\times b\\&=a^2+2\times a\times b+b^2.\end{aligned}$$

(iii) $$\begin{aligned}(a-b)^2&=(a-b)\times(a-b)\\&=a\times(a-b)-b\times(a-b)\\&=a\times a-a\times b-b\times a+b\times b\\&=a^2-2\times a\times b+b^2.\end{aligned}$$

As an illustration of the practical use of (i) we will consider the expression $100003^2-99997^2$. If we were to multiply 100003 by itself, and 99997 also by itself, and to subtract the latter result from the former, we should get what is required, but the calculation would be somewhat laborious. If, however, we use (i), there will be an obvious saving of labour. For

$$\begin{aligned}100003^2-99997^2&=(100003+99997)\times(100003-99997)\\&=200000\times 6\\&=1200000.\end{aligned}$$

Again, if we require the square of 81, the labour may be lessened by using (ii); thus

$$\begin{aligned} 81^2 &= (80+1)^2 \\ &= 80^2 + 2\times 80 + 1 \\ &= 6400 + 160 + 1 \\ &= 6561. \end{aligned}$$

We might find the value of 99997^2 by using (iii). Thus:

$$\begin{aligned} 99997^2 &= (100000-3)^2 \\ &= 10000000000 - 2\times 3\times 100000 + 3^2 \\ &= 10000000009 - 600000 \\ &= 9999400009. \end{aligned}$$

EXAMPLES. 9.

Find the value of

1.	$35^2 - 29^2$.	2.	$47^2 - 42^2$.	3.	$98^2 - 48^2$.
4.	$116^2 - 109^2$.	5.	$247^2 - 147^2$.	6.	$324^2 - 319^2$.
7.	$560^2 - 540^2$.	8.	$8947^2 - 947^2$.	9.	$6598^2 - 3402^2$.
10.	19×21.	11.	95×105.	12.	87×93.
13.	389×411.	14.	625×575.	15.	891×909.
16.	1203×1197.	17.	1075×1125.	18.	1288×1312.
19.	105^2.	20.	112^2	21.	119^2.
22.	692^2.	23.	820^2.	24.	650^2.
25.	996^2.	26.	9998^2.	27.	10003^2.

54. In Art. 50 we gave rules which enable us to ascertain, almost by inspection, whether a number is divisible, or not, by certain divisors. We will now consider the general reasons upon which these rules are based.

We will suppose the digits of the number to be represented by the letters a, b, c, d, ... taken in order, a being the right-hand digit, *i.e.* the digit in the *units'* place, b that in the *tens'* place, c that in the *hundreds'* place, and so on; and we will represent the number itself by N. It follows that

$$\mathrm{N} = a + b\times 10 + c\times 100 + d\times 1000 + e\times 10000 + \ldots,$$

or

$$\mathrm{N} = a + 10b + 100c + 1000d + 10000e + \ldots.$$

It is of course understood that each of the letters a, b, c, d, e,... stands for one of the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9; and also that the *divisibility* of one number by another implies that the quotient is an integer, and the remainder 0.

(i) Divisibility by 2. We have

$$\mathrm{N} = a + 10b + 100c + 1000d + 10000e + \ldots,$$

$$\therefore \frac{\mathrm{N}}{2} = \frac{a}{2} + \text{an integer}.$$

Hence N is divisible by 2, *i.e.* $\frac{N}{2}$ is an integer, if $\frac{a}{2}$ is an integer, *i.e.* if a is divisible by 2. Consequently a number is divisible by 2, if the digit in the *units'* place is divisible by 2, *i.e.* if the digit in the units' place is 0, 2, 4, 6 or 8.

(ii) **Divisibility by 4.** We have

$$N = a + 10b + 100c + 1000d + 10000e + \ldots,$$

$$\therefore \frac{N}{4} = \frac{a + 10b}{4} + \text{an integer.}$$

Hence N is divisible by 4, if $a + 10b$ is divisible by 4, *i.e.* if the number formed by the *two* digits at the right hand consists of 0's only, or is divisible by 4.

(iii) **Divisibility by 8.** We have

$$N = a + 10b + 100c + 1000d + 10000e + \ldots,$$

$$\therefore \frac{N}{8} = \frac{a + 10b + 100c}{8} + \text{an integer.}$$

Hence N is divisible by 8, if $a + 10b + 100c$ is divisible by 8, *i.e.* if the number formed by the *three* digits at the right hand consists of 0's only, or is divisible by 8.

NOTE.—Similar reasoning will enable us to see that a number is divisible by 16, 32, 64, ..., if the number formed by the last 4, 5, 6, ..., digits at the right hand consists of 0's only, or is divisible by 16, 32, 64, ..., respectively.

(iv) **Divisibility by 5.** We have

$$N = a + 10b + 100c + 1000d + 10000e + \ldots,$$

$$\therefore \frac{N}{5} = \frac{a}{5} + \text{an integer.}$$

Hence N is divisible by 5, if a is divisible by 5, *i.e.* if the right-hand digit is 0 or 5.

NOTE.—It may be shown in a similar way that a number is divisible by 25, if the *two* right-hand digits are 00, 25, 50, or 75; and by 125, if the *three* right-hand digits are 000, 125, 250, 375, 500, 625, 750, or 875.

(v) **Divisibility by 10.** In this case the necessary condition is that the right-hand digit is 0.

(vi) **Divisibility by 3.** We have

$$N = a + 10b + 100c + 1000d + 10000e + \ldots,$$

$$\therefore \frac{N}{3} = \frac{a}{3} + 3b + \frac{b}{3} + 33c + \frac{c}{3} + 333d + \frac{d}{3} + 3333e + \ldots$$

$$= \frac{a + b + c + d + e + \ldots}{3} + \text{an integer.}$$

Hence N is divisible by 3, if $a + b + c + d + e + \ldots$ is divisible by 3, *i.e.* if the *sum of its digits* is divisible by 3.

Example. In the case of 13762084 the sum of the digits is 31, and in the case of 13762884 the sum of the digits is 39; hence we may conclude that the latter number *is*, but the former is *not* divisible by 3.

(vii) **Divisibility by 9.** We have

$$N = a + 10b + 100c + 1000d + 10000e + \ldots,$$

$$\therefore \frac{N}{9} = \frac{a}{9} + b + \frac{b}{9} + 11c + \frac{c}{9} + 111d + \frac{d}{9} + 1111e + \frac{e}{9} + \ldots$$

$$= \frac{a+b+c+d+e+\ldots}{9} + \text{an integer.}$$

Hence N is divisible by 9, if $a+b+c+d+e+\ldots$ is divisible by 9, *i.e.* if the *sum of its digits* is divisible by 9.

(viii) **Divisibility by 6.** A number is divisible by 6, if it is divisible by 2 and also by 3; *i.e.* (a) if the right-hand digit is either 0, 2, 4, 6 or 8; and (b) if the sum of all its digits is divisible by 3.

(ix) **Divisibility by 12.** A number is divisible by 12, if it is divisible by 4 and also by 3; *i.e.* (a) if the number formed by the last two digits at the right hand consists of 0's only, or is divisible by 4; and (b) if the sum of all its digits is divisible by 3.

(x) **Divisibility by 11.** We have

$$N = a + 10b + 100c + 1000d + 10000e + \ldots$$

$$= a + (11b - b) + (99c + c) + (1001d - d) + (9999e + e) + \ldots$$

$$= a - b + c - d + e - \ldots + 11b + 99c + 1001d + 9999e + \ldots.$$

Hence N is divisible by 11, if $a - b + c - d + e - \ldots$ is divisible by 11,

i.e. if $a + c + e + \ldots - b - d - f - \ldots$ is divisible by 11,

i.e. if $(a + c + e + \ldots) - (b + d + f + \ldots)$ 11,

i.e. if the difference between the sum of the digits in the *even* places and the sum of the digits in the *odd* places is either 0 or a multiple of 11.

55. Casting out the nines. The process explained in Art. 36 may be used to test the correctness of a result obtained by any one of the four simple rules. We found that

$$\frac{N}{9} = \frac{\text{the sum of the digits}}{9} + \text{a whole number;}$$

consequently, the remainder (if any) when a number is divided by 9 must be equal to the remainder (if any) when *the sum of the digits* is divided by 9. For example, if we add

$$(\textit{one multiple of } 9) + a$$

to

$$(\textit{another multiple of } 9) + b,$$

the result will be

$$(\textit{a multiple of } 9) + (a + b),$$

and the remainder when this is divided by 9 will be the same as the remainder when $(a+b)$ is divided by 9. A similar result will clearly be obtained in subtraction.

Again, if we multiply

(*one multiple of* 9) + a by (*another multiple of* 9) + b,

the result will be (*a multiple of* 9) + $a \times b$,

and the remainder when this is divided by 9 must be the same as the remainder when $a \times b$ is divided by 9.

We may state the property in general terms, thus: If a and b be the remainders when two numbers A and B respectively are divided by 9, then the remainders when A + B, A − B and A × B are divided by 9 will be the same as the remainders when $a+b$, $a-b$ and $a \times b$ respectively are divided by 9.

In the case of exact division, if Q be the quotient when A is divided by B, it follows that $A = Q \times B$; and if a, b and q be the respective remainders when A, B and Q are divided by 9, it will be seen that the remainder, when $b \times q$ is divided by 9, must be a.

In the case of inexact division, $A = Q \times B + R$; and if r be the remainder when R is divided by 9, the remainder when $b \times q + r$ is divided by 9 must be a.

EXAMPLES. 10.

(*This set of examples will be found most useful when used as the foundation of a lecture.*)

1. Supply the missing figures so that

(i) 34*7 may be divisible by 9.
(ii) 1*567 ..... 11.
(iii) 159* ..... 6.
(iv) 7**217 ..... 99.
(v) 635** ..... 75.

2. Prove that the product of any three consecutive integers is divisible by 6.

3. Prove that the product of any four consecutive integers is divisible by 24.

4. Prove that the difference between the squares of any two consecutive integers is equal to the sum of the numbers.

5. Prove that the difference between the squares of two consecutive odd integers is divisible by 8.

6. Prove that the difference between two numbers denoted by the same digits in different orders is divisible by 9.

7. In how many ways can 112 be resolved into factors, excluding unity and the number itself?

8. The four integers next lower than 81, and the four next higher than 81, are written down and added together; shew that the sum is divisible by 9, and explain why.

9. Prove that the square of a number cannot be divisible by 9, if the number itself is not divisible by 3.

10. The difference of two numbers is divisible by 9, and their product is divisible by 9; prove that *both* of the numbers must be divisible by 3.

11. For what values of n will 11 divide $10^n + 1$ exactly?

12. Shew that 9 will always divide $10^n - 1$ exactly. When will 99 divide it?

13. For what values of n will 111 divide $10^n - 1$ exactly, and what will be the quotient when $10^{12} - 1$ is divided by 111?

14. Write down all the prime numbers between 50 and 100. Explain why there cannot be *three* consecutive odd numbers among them.

15. In multiplying a certain number by a number consisting of 3 digits, a boy places the partial product due to the units' digit in the position where that due to the hundreds' digit ought to be, and the product due to the hundreds' digit where that due to the units' digit ought to be. He tests the result by casting out nines and elevens, and both tests are satisfied. Explain this.

VI. GREATEST COMMON MEASURE.

56. A number which is a factor of two or more numbers is said to be a **common factor**, or **common measure**, of the numbers. We exclude unity, which is a common measure of all numbers.

The *greatest* number which will divide each of two or more numbers is called their **Highest Common Factor**, or **Greatest Common Measure**, and is denoted by the letters **H.C.F.** or **G.C.M.**

57. The G.C.M. of two or more numbers may often be readily determined by separating each number into its prime factors, and this should always be done when possible. Consider, for example, the numbers 1008 and 2064.

We find that $1008 = 2^4 \times 3^2 \times 7$,
and $2064 = 2^4 \times 3 \times 43$;
hence their G.C.M. $= 2^4 \times 3 = 48$,

for this number is the product of all the prime factors that are common to both.

58. *To find the* G.C.M. *of two numbers whose prime factors cannot readily be found.*

The method will be shewn first by an example. We will find the G.C.M. of 82407 and 7241.

(i) Divide the greater number 82407 by the smaller 7241

$$\begin{array}{r|l} 7241 & 82407\,(11 \\ & \underline{7241} \\ & 9997 \\ & \underline{7241} \\ & 2756 \end{array}$$

(ii) Divide the first divisor 7241 by the remainder 2756.

$$\begin{array}{r|l} 2756 & 7241\,(2 \\ & \underline{5512} \\ & 1729 \end{array}$$

(iii) Divide the second divisor 2756 by the remainder 1729.

$$\begin{array}{r|l} 1729 & 2756\,(1 \\ & \underline{1729} \\ & 1027 \end{array}$$

(iv) Divide the third divisor 1729 by the remainder 1027.

$$\begin{array}{r|l} 1027 & 1729\,(1 \\ & \underline{1027} \\ & 702 \end{array}$$

(v) Divide the fourth divisor 1027 by the remainder 702.

$$\begin{array}{r|l} 702 & 1027\,(1 \\ & \underline{702} \\ & 325 \end{array}$$

(vi) Divide the fifth divisor 702 by the remainder 325.

$$\begin{array}{r|l} 325 & 702\,(2 \\ & \underline{650} \\ & 52 \end{array}$$

(vii) Divide the sixth divisor 325 by the remainder 52.

$$\begin{array}{r|l} 52 & 325\,(6 \\ & \underline{312} \\ & 13 \end{array}$$

(viii) Divide the seventh divisor 52 by the remainder 13.

$$\begin{array}{r|l} 13 & 52\,(4 \\ & \underline{52} \end{array}$$

There is now no remainder, so the operation terminates. The **last divisor**, 13, is the G.C.M. required.

The work may be arranged more concisely thus:

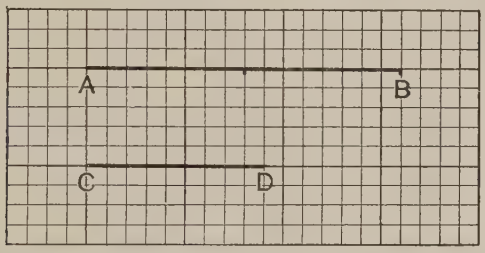
$$\begin{array}{rl|l} 2) & 7241 & 82407 \\ & & \underline{7241} \\ & & 9997 \\ & 5512 & 7241 \\ \hline 1) & 1729 & 2756\,(1 \\ & 1027 & 1729 \\ \hline 2) & 702 & 1027\,(1 \\ & 650 & 702 \\ \hline 4) & 52 & 325\,(6 \\ & 52 & 312 \\ \hline & & \underline{13} = \text{G.C.M.} \end{array}$$

or thus (Art. 35)

$$\begin{array}{rl|l} & 7241 & 82407 \\ & & \underline{9997} \\ & 1729 & 2756 \\ & \underline{702} & 1027 \\ & 52 & 325 \\ & & \text{G.C.M.} = \underline{13} \end{array}$$

59. The process is based upon the principle that any number which divides each of two numbers divides also (i) their sum, (ii) their difference, (iii) any multiples of either, and (iv) the sum and the difference of any multiples of the numbers.

The pupil can at once convince himself of the truth of these statements by measuring off lines of various lengths on squared paper. For example, the lines AB and CD in the figure contain 16 units and 9 units respectively. If CD be placed in the same



line and continuous with AB, it will be seen that the sum of the two lines will contain 25 of the same units; similarly, their difference will contain 7 of the units; *i.e.* the unit is contained exactly both in AB + CD and in AB − CD.

It may be seen in a similar way that the unit is contained exactly in any *multiples* of AB and CD, and in the *sum* and in the *difference* of any *multiples*.

It may also be seen that the converse of these principles is *not* true; a length of 5 units is contained exactly in AB + CD, but is *not* contained exactly in either AB or CD by itself.

60. It follows from the principles stated that all factors which divide 7241 and 82407

will also divide 7241 and 82407 − 11 × 7241, *i.e.* 7241 and 2756;
and will also divide 2756 and 7241 − 2 × 2756, *i.e.* 2756 and 1729;
and will also divide 1729 and 2756 − 1729, *i.e.* 1729 and 1027;
and will also divide 1027 and 1729 − 1027, *i.e.* 1027 and 702;
and will also divide 702 and 1027 − 702, *i.e.* 702 and 325;
and will also divide 325 and 702 − 2 × 325, *i.e.* 325 and 52;
and will also divide 52 and 325 − 6 × 52, *i.e.* 52 and 13.

But 13 is the greatest number which will divide both 13 and 52. It is therefore the greatest number which will divide each number in each pair of numbers given above, *i.e.* each divisor and each dividend throughout the operation. Therefore 13 is the greatest number which will divide both 7241 and 82407 exactly.

NOTE I.—Our object in the above process is to find the greatest *common* factor. If then the divisor at any stage of the process has a *prime* factor which is not a factor also of the dividend, or if the dividend has a *prime* factor which is not a factor also of the divisor, that factor may be removed from the divisor or from the dividend, as the case may be, and may be disregarded. The divisor, or the dividend, is thus reduced and the process is simplified. The factor may be removed, as we have stated, because we have no interest in numbers which are not factors of both.

We may illustrate this matter by referring to the example worked out in Art. 58. The second divisor, 2756, is divisible by 2 and again by 2, and 2 is not a factor of 7241. We may therefore use 689 as divisor instead of 2756; the dividend remaining 7241 as before.

Again, the fifth divisor 702 is divisible by 3 and the dividend 1027 is not. We may therefore replace 702 by 234.

By promptly removing such factors much labour may be saved, but the pupil must be careful on two points, (i) the factor removed must be a *prime* number, and (ii) it must *not* be a factor of *both* numbers.

NOTE II.—If the divisor and the dividend at any stage of the process can be easily factorized, the operation may be completed in the manner of Art 57.

And if the divisor and the dividend at any stage *have* a common factor, that factor may be removed from both, but the G.C.M. obtained at the end of the operation must be *multiplied by any common factor thus treated*, in order that we may get the G.C.M. required. The reason for doing this will be clear if the principles already explained are properly understood, but to illustrate what we have said we will consider the two numbers

$$130338 \text{ and } 741663,$$

$$\text{i.e. } 9\times 14482 \text{ and } 9\times 82407.$$

The G.C.M. $= 9 \times$ the G.C.M. of 14482 and 82407

$= 9 \times$ the G.C.M. of 7241 and 82407 (removing the factor 2)

$= 9 \times 13$ (Art. 58)

$= 117.$

61. As another example we will find the G.C.M. of 695 and 544. In (ii) differences only are written down as in Art. 35.

(i)

544	695
453	544
91	151
60	91
31	60
29	31
2	29
2	28
	1

Hence the G.C.M. is 1.

(ii)

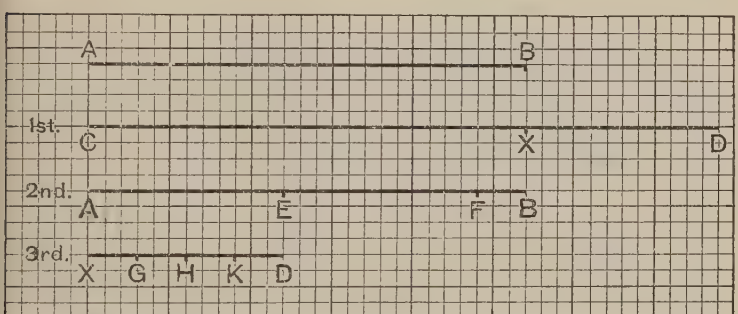
544	695
91	151
31	60
	29

The last two numbers, 29 and 31, being primes, the operation stops here, and we see that the only common measure is unity.

In defining the term Common Measure in Art. 56 we excluded unity. Hence 695 and 544 have no common measure in the usual meaning of the term. Numbers such as these are said to be **prime to one another.**

62. *Geometrical illustration of the process of finding the* G.C.M. *of two numbers.*

We will use squared paper, and we will take as our unit the side of one of the small squares. We will suppose that we wish to find the G.C.M. of AB which contains 27 units, and CD which contains 39 units.



(i) From CD we mark off as many lengths equal to AB as we can. It happens that we can mark off only one, namely CX. The remainder is XD (12 units).

(ii) From AB we now mark off as many lengths equal to XD as we can. We can mark off two such lengths, namely AE and EF. The remainder is FB (3 units).

(iii) From XD we now mark off as many lengths equal to FB as we can, and we find that we can mark off exactly four. There is no remainder.

The length FB is the G.C.M. of AB and CD, *i.e.* it is the greatest length which is contained exactly both in AB and in CD. Thus 3 is the G.C.M. of 39 and 27.

NOTE.—It is clear that FB is the greatest length which is contained exactly in XD and FB; and therefore in XD and 2XD + FB, *i.e.* in XD and AB; and therefore in AB and AB + XD, *i.e.* in AB and CD.

If again we suppose each division on the squared paper to represent 4 units, AB will represent 108 units, CD will represent 156 units, and FB will represent 12 units. The G.C.M. of 108 and 156 is therefore 12. Hence,

if a factor be removed from *both* numbers, and the G.C.M. of the quotients be found, it follows that to obtain the required G.C.M. we must multiply the G.C.M. of the quotients by the factor which was removed.

Again if the length CD be doubled while AB remains of the same length, the G.C.M. is still equal to FB, or 3 units. But it would not be so if 2 were a factor of the number of units in AB. The pupil should prove this for himself by drawing the actual lengths on squared paper.

63. *To find the* G.C.M. *of three or more numbers.*

(i) When the numbers can easily be separated into prime factors, this should be done (Art. 57).

Example. *Find the* G.C.M. *of* 196, 350, 728, *and* 924.

We have $196 = 4 \times 49 = 2^2 \times 7^2$;

$350 = 2 \times 175 = 2 \times 7 \times 25 = 2 \times 5^2 \times 7$;

$728 = 7 \times 104 = 7 \times 8 \times 13 = 2^3 \times 7 \times 13$;

and $924 = 4 \times 231 = 2^2 \times 11 \times 21 = 2^2 \times 3 \times 7 \times 11$.

The highest power of 2 which will divide each of these numbers is 2 itself. The highest power of 7 that will do so is 7 itself. Of the other factors 3, 5, 11, 13, there are none common to all. Hence

$$\text{G.C.M.} = 2 \times 7 = 14.$$

NOTE.—After each number has been separated into prime factors, the rules to be observed are that (a) the G.C.M. must contain *all* the factors which are *common* to *all* the numbers, and (b) the index of any factor in the G.C.M. must be that of the *lowest* power to which it is raised in any of the numbers.

(ii) When the numbers cannot be easily separated, the method of Art. 58 should be used.

Suppose, for example, that A, B and C be three numbers whose G.C.M. is wanted. We first find the G.C.M. of A and B. Call this D. Then since D and its factors are the only numbers which will divide both A and B, it is plain that we have now only to find the product of those prime factors of D which will also divide C. Hence the G.C.M. of A, B and C is the G.C.M. of D and C; D being the G.C.M. of A and B.

Example. *To find the* G.C.M. *of* 65442, 43446, *and* 9022.

It will be found that the G.C.M. of 65442 and 43446 is 78, which

$$= 2 \times 3 \times 13.$$

We have now to find the greatest divisor of $2 \times 3 \times 13$ which will also divide 9022. By trial we find that 2 and 13 will divide 9022, and that 3 will not. Hence G.C.M. required $= 2 \times 13 = 26$.

NOTE.—If D, the G.C.M. of 65442 and 43446, could not readily have been separated into factors, we should have found the G.C.M. of D and 9022 by the method of Art. 58.

64. In Art. 59 we stated that the common method of finding the G.C.M. of two numbers is based upon the principle that any number which exactly divides *each* of two numbers divides also (i) their sum, (ii) their difference, (iii) any multiples of either number, and (iv) the sum and the difference of any multiples. We will now consider these principles more generally.

Suppose that two numbers be represented by A and B, and suppose that each is divisible, without remainder, by a number represented by x. If p and q represent the respective quotients, then

$$\mathsf{A}=p\times x \text{ and } \mathsf{B}=q\times x;$$

whence it follows that

(i) $\mathsf{A}+\mathsf{B}=p\times x+q\times x=(p+q)\times x,$

(ii) $\mathsf{A}-\mathsf{B}=p\times x-q\times x=(p-q)\times x,$

(iii) $\left.\begin{aligned} m\times\mathsf{A}&=m\times p\times x\\ n\times\mathsf{B}&=n\times q\times x\end{aligned}\right\},$

(iv) $\left.\begin{aligned} m\times\mathsf{A}+n\times\mathsf{B}&=m\times p\times x+n\times q\times x=(m\times p+n\times q)\times x\\ m\times\mathsf{A}-n\times\mathsf{B}&=m\times p\times x-n\times q\times x=(m\times p-n\times q)\times x\end{aligned}\right\}.$

We see that x is a factor of each of the expressions on the right-hand sides of the equalities, so that x will divide

$$\mathsf{A}+\mathsf{B},\quad m\times\mathsf{A},\qquad m\times\mathsf{A}+n\times\mathsf{B},$$
$$\mathsf{A}-\mathsf{B},\quad n\times\mathsf{B},\quad \text{and}\quad m\times\mathsf{A}-n\times\mathsf{B},$$

without remainder in any case.

EXAMPLES. 11. *Oral.*

State the G.C.M. of

1.	4 and 6.	**2.**	6 and 8.	**3.**	6 and 9.
4.	8 and 12.	**5.**	9 and 12.	**6.**	6 and 14.
7.	8 and 14.	**8.**	10 and 15.	**9.**	8 and 18.
10.	12 and 18.	**11.**	10 and 24.	**12.**	10 and 25.
13.	15 and 25.	**14.**	18 and 27.	**15.**	21 and 28.
16.	21, 28 and 35.	**17.**	18, 27 and 36.	**18.**	15, 25 and 50.

EXAMPLES. 12.

Find the G.C.M. of

1.	45 and 72.	**2.**	45 and 81.	**3.**	72 and 162.
4.	132 and 165.	**5.**	54 and 258.	**6.**	72 and 480.
7.	45 and 162.	**8.**	216 and 258.	**9.**	646 and 950.
10.	616 and 1518.	**11.**	720 and 960.	**12.**	936 and 2925.
13.	9756 and 8496.			**14.**	429 and 6942.
15.	3252 and 4248.			**16.**	42336 and 53088.
17.	10353 and 14877.			**18.**	2145 and 3471.

Find the G.C.M. of

19. 4929 and 43672.
20. 94248 and 105336.
21. 61655 and 70092.
22. 66429 and 169037.
23. 110319 and 127308.
24. 33495 and 106260.
25. 228684 and 364089.
26. 45, 72 and 162.
27. 45, 72 and 480.
28. 162, 729 and 4374.
29. 372, 994 and 3132.
30. 218, 484 and 73170.
31. 492, 744 and 1044.
32. 975, 3150 and 146250.
33. 1326, 3094 and 4420.
34. 1288, 1736 and 104.
35. 30, 110, 140 and 680.
36. 12558, 20769, 47403 and 12581.
37. 128, 136, 256, 442 and 940.
38. 102, 612, 476, 816 and 428.
39. Find the greatest number which, when divided into 3912 and 92832, leaves remainders 40 and 25 respectively.
40. Find the greatest number such that, if 111300 and 91400 be divided by it, the remainders are respectively 33 and 23.
41. Write down all the numbers between 400 and 1400 which have 211 as their G.C.M.
42. Three rods, measuring respectively 14 feet, 42 feet and 119 feet, are to be divided into portions all of the same length. If the lengths of the portions are as great as possible, into how many portions will each rod be divided?
43. If 1475 kilograms = 3245 pounds, what is the least number of kilograms equal to an exact number of pounds?
44. If 1272 francs = 1007 shillings, what is the least number of francs which can be expressed as an exact number of shillings?

VII. LEAST COMMON MULTIPLE.

65. A number which can be divided without remainder by two others, or more, is a **common multiple** of them. Thus 12 is a common multiple of 2, 3, 4 and 6.

The *least* number which can be so divided is called their **Least Common Multiple**, and is denoted by the letters **L.C.M.**

66. The L.C.M. may be determined very readily when the numbers can be separated into prime factors.

Let us again consider the numbers 1008 and 2064. We found that $1008 = 2^4 \times 3^2 \times 7$, and $2064 = 2^4 \times 3 \times 43$.

Now the L.C.M. must contain all the prime factors in 1008 and in 2064, and each of these factors must occur at least as often in the L.C.M. as it occurs in either of the numbers themselves. For example, 2 occurs *four* times in both, and must therefore occur *four* times in the L.C.M. Also, as 3 occurs *once* in 2064 and *twice* in 1008, it must occur *twice* in the L.C.M.;

$$\therefore \text{ L.C.M.} = 2^4 \times 3^2 \times 7 \times 43 = 43344.$$

67. To find the L.C.M. of three or more numbers we proceed in exactly the same way.

Example. *Find the* L.C.M. *of* 196, 350, 728, 924.

We have from the Example of G.C.M. in Art. 63,

$$196 = 2^2 \times 7^2, \qquad 728 = 2^3 \times 7 \times 13,$$
$$350 = 2 \times 5^2 \times 7, \qquad 924 = 2^2 \times 3 \times 7 \times 11;$$
$$\therefore \text{ L.C.M.} = 2^3 \times 3 \times 5^2 \times 7^2 \times 11 \times 13$$
$$= 8 \times 3 \times 25 \times 49 \times 143$$
$$= 600 \times 49 \times 143$$
$$= 85800 \times 49$$
$$= 600600 \times 7$$
$$= 4204200.$$

NOTE.—When the prime factors have been found, it will be seen that the L.C.M. must have for its factors all the different factors which occur in any of the numbers, and the *index* of each factor must be that of the *highest* power to which it is raised in any of them.

68. We will now consider the form in which the work can be best arranged when the numbers are numerous and small.

Suppose we have to find the L.C.M. of

1, 3, 5, 9, 12, 14, 16, 96, 128.

We see that 3, 12 and 16 are factors of 96; hence all numbers divisible by 96 must also be divisible by 3, 12 and 16. We may therefore disregard 3, 12 and 16 altogether.

Consequently we cross out 1, 3, 12 and 16 thus:

~~1~~, ~~3~~, 5, 9, ~~12~~, 14, ~~16~~, 96, 128,

and we state the work as follows:

2	~~1~~,	~~3~~,	5,	9,	~~12~~,	14,	~~16~~,	96,	128
	3	5,	9,		7,		48,	64	
		5,	3,		7,		~~16~~,	64	

The divisors 2 and 3 are prime factors which are common to two or more of the numbers. In all cases we divide by such **prime** factors as will divide two or more of the numbers. We begin with the least, and we take them in order. We write down the quotients below such numbers as can be divided, and we bring down unaltered the numbers which cannot be divided.

In the second line, for example, 5 and 9 are brought down unaltered, because they are not divisible by 2, but 14, 96 and 128 are reduced to 7, 48 and 64.

In the third line we strike out 16, because it is a factor of 64 and may therefore be neglected; there are then four numbers, no two of which have a common factor. Hence all the factors which must occur in the L.C.M. are 2, 3, 5, 3, 7, and 64;

$$\therefore \text{ L.C.M.} = 2 \times 3 \times 5 \times 3 \times 7 \times 64 = 40320.$$

Example ii. *Find the* L.C.M. *of* 7, 12, 15, 27, 35, 40, *and* 45.

$$\begin{array}{r|l} 2 & 7,\ 12,\ 15,\ 27,\ 35,\ 40,\ 45 \\ 2 & 6,\ 27,\ 35,\ 20,\ 45 \\ 3 & 3,\ 27,\ 35,\ 10,\ 45 \\ 3 & 9,\ 35,\ 10,\ 15 \\ 5 & 3,\ 35,\ 10,\ 5 \\ & 3,\ 7,\ 2 \end{array}$$

$$\therefore \text{ L.C.M.} = 2 \times 2 \times 3 \times 3 \times 5 \times 3 \times 7 \times 2 = 7560.$$

69. When two numbers cannot readily be separated into prime factors, we may adopt another method.

The G.C.M. of the numbers is the product of all their common factors; hence if we divide the numbers by their G.C.M., the quotients will be the products of all the factors that are *not* common.

The L.C.M. of the two numbers is therefore the product of the G.C.M. and the two quotients; in other words, it is the product of either number and the quotient obtained on dividing the other number by the G.C.M.; or, again, it is the quotient obtained on dividing the product of the numbers by their G.C.M

As an example, we will consider 43709 and 109981. Their G.C.M. = 109, and if we divide 43709 by 109, the quotient is 401; hence the L.C.M. = $401 \times 109981 = 44102381$.

70. To find the L.C.M. of three or more numbers which can not readily be separated into factors, we find the L.C.M. of two of them as in Art. 69; then the L.C.M. of this and another, and

so on until we have used all. The last L.C.M. is the L.C.M. required.

71. In Art. 69 we explained the general method of determining the L.C.M. of two numbers. We will now suppose the letters A and B to represent the two numbers, and G to represent their G.C.M.; and when the numbers are divided by G, we will suppose the quotients to be a and b respectively. Consequently

$$A = a \times G \text{ and } B = b \times G;$$

and it follows from the meaning of G that there is no factor common to a and b, *i.e.* that a and b are *prime* to one another.

The L.C.M. of A and B must therefore be $a \times b \times G$, which may be written in any of the forms

$$a \times b \times G,\ a \times B,\ b \times A \text{ and } A \times B \div G;$$

and we see also that L.C.M. $\times$ G.C.M. $= A \times B$.

EXAMPLES. 13. *Oral.*

Find the L.C.M. of

1.	2 and 4.	2.	4 and 8.	3.	2 and 6.
4.	2, 3 and 6.	5.	2, 3 and 4.	6.	2, 3 and 5.
7.	3, 4 and 5.	8.	3, 5 and 7.	9.	3, 5 and 6.
10.	2, 5 and 7.	11.	2, 6 and 5.	12.	2, 6 and 8.
13.	3, 4 and 10.	14.	5, 8 and 10.	15.	5, 12 and 15.
16.	6, 12 and 24.	17.	7, 10 and 14.	18.	6, 10 and 18.
19.	8, 16 and 24.	20.	10, 40 and 60.		

EXAMPLES. 14.

Find the L.C.M. of

1.	8, 12 and 20.	2.	6, 14 and 49.
3.	9, 12 and 21.	4.	18, 20 and 30.
5.	4, 18, 21 and 20.	6.	18, 36, 60 and 72.
7.	15, 13, 39 and 65.	8.	15, 16, 56 and 70.
9.	25, 120, 15 and 84.	10.	18, 36, 45 and 81.
11.	18, 56, 50 and 72.	12.	12, 36, 72 and 120.
13.	13, 14, 39 and 49.	14.	16, 28, 35 and 63.
15.	18, 24, 32 and 48.	16.	26, 33, 39 and 44.
17.	44, 48, 52 and 96.	18.	32, 48, 40 and 14.
19.	12, 22, 55 and 40.	20.	16, 21, 49 and 72.
21.	15, 25, 39 and 63.	22.	7, 17, 68 and 238.
23.	35, 105, 210 and 750.	24.	8, 12, 20, 30 and 80.
25.	24, 16, 6, 4 and 20.	26.	3, 5, 12, 36 and 45.

Find the L.C.M. of

27. 9, 12, 16, 18 and 28.
28. 28, 56, 100, 180 and 125.
29. 21, 45, 60, 84 and 90.
30. 12, 18, 27, 30 and 40.
31. 12, 32, 52, 72 and 162.
32. 15, 35, 55, 75 and 165.
33. 16, 90, 91, 280 and 455.
34. 8, 12, 18, 24, 28 and 30.
35. 4, 11, 18, 20, 48 and 72.
36. 5, 7, 16, 28, 48 and 76.
37. 8, 16, 24, 64, 12 and 100.
38. 20, 27, 28, 35, 45 and 63.
39. 508 and 889.
40. 936 and 2925.
41. 1729 and 5850.
42. 5757 and 40380.
43. 84, 672 and 472.
44. 153, 204 and 221.
45. 110, 1155 and 1470.
46. 165, 855 and 9500.
47. 3696, 286 and 19656.
48. 1836, 1482, 1938 and 8398.

49. Find the smallest weight which can be divided into an exact number of parcels weighing 54 pounds, 42 pounds, 28 pounds, 22 pounds, or 15 pounds.

50. Find the least number by which 825 must be multiplied in order to produce a multiple of 715.

51. Find the least number which, when divided by 6179 and by 8851, leaves a remainder 151 in each case.

52. Three persons start together to walk along a road at the same rate. The lengths of their strides are respectively 28 inches, 30 inches and 32 inches. How far will they go before they will be 'in step' again? How many steps will each have taken at that moment?

53. Four bells begin tolling at the same time, and continue at intervals of 25, 30, 36 and 48 seconds respectively. How often will each bell have tolled when all again sound at the same instant?

54. If 1 mile = 5280 feet, 1 knot = 6080 feet and 1 kilometre = 3280 feet, find the least distance which is an exact number of miles, knots, or kilometres. Express the distance in feet, in miles, in knots, and in kilometres.

55. Find the least number which when divided by 5 leaves a remainder 3; when divided by 7 leaves a remainder 5; and when divided by 13 leaves a remainder 11.

56. The G.C.M. of two numbers is 113, and their L.C.M. is 228825. One of the numbers is 2825; find the other.

57. The L.C.M. of three numbers is 23100, and their G.C.M. is 5: if two of the numbers are 700 and 770, find the possible values of the third number.

MISCELLANEOUS EXAMPLES. 15.

1. Write in words 15006021 : and state how many times a number is multiplied by annexing 3 zeros to it.

2. Write down in figures, seven million thirteen thousand five hundred and one.

3. Add together 6543, 807, 54091, 9999 and 38.

4. Subtract 4726085 from 123408963.

5. Multiply 74562801 by 25, and by 463.

6. Divide 173913712 by 63728.

7. Find the G.C.M. of 10353 and 14877.

8. Find the L.C.M. of 4, 11, 18, 20, 36 and 48.

9. Find the least number which will leave a remainder 3 when divided either by 5, 7, 9, or 11.

---

10. Write in words 9010201 ; and in figures, one hundred and four million seven thousand and eighty-two.

11. Add together 9873, 361, 10458, 94 and 7290.

12. From 9768452 take 839204.

13. Multiply 59873 by 125 ; and by 4907 in 2 lines.

14. If seven hundred and eighty-four thousand one hundred and thirty-five be divided by four hundred and twenty-five, what will be the remainder?

15. Find the prime factors of 27720.

16. Find the G.C.M. of 27720 and 51744.

17. Find the L.C.M. of 32, 48, 40 and 14.

18. Four bells toll at intervals of 3, 4, 5 and 8 seconds, beginning together ; after how many seconds will they again toll together?

---

19. Multiply one hundred million one hundred thousand and ten by five thousand and five.

20. Divide five hundred and one million five hundred by three thousand eight hundred and fifty.

21. What number is as much less than 57321 as it is greater than 32899?

22. Divide 7807 into three parts, such that the first is 193 greater, and the third is 207 less, than the second.

23. Find the G.C.M. of 23760 and 26136.

24. Find the L.C.M. of 125, 136, 180, 24 and 27.

25. How many times is the G.C.M. of 48, 36, 72 and 24 contained in their L.C.M.?

26. Three men journey 14, 35 and 38 miles a day respectively. At what distance from the starting point is the nearest place at which they all put up?

27. Find the missing figures in the following addition sums:

(i)
$$\begin{array}{r} 3456*8 \\ *324* \\ *6*123 \\ \hline 959045 \end{array}$$

(ii)
$$\begin{array}{r} 45*7* \\ 23*1 \\ *2931 \\ *246 \\ \hline 104196 \end{array}$$

---

28. 1542 counters are divided into 3 heaps. The number in two of the heaps is 905, and the number in another two of the heaps is 1021. How many are there in each heap?

29. Why is the addition of a 0 to the right of a whole number equivalent to the multiplication of that number by ten?

30. In a cricket match A, B and C together scored 280 runs. If A and B together made 192, and C made 11 more than B, how many did each make?

31. 32736 is the product of three consecutive integers; find them.

32. Find the L.C.M. of 48, 27, 81, 165 and 110.

33. Find the least number which is divisible by all the numbers from 1 to 15 inclusive.

34. Find the sum of all the primes between 70 and 100.

35. The G.C.M. of two numbers is 7, and their L.C.M. is 140. Find the possible values of the numbers.

36. Find the least number which, when divided by 33, 171 and 1900, will always leave the same remainder 21.

---

37. Three years ago A's age was double B's. Seven years hence the sum of their united ages will be 83 years. Find the ages of A and B.

38. How many figures are required in numbering the pages of a book of 365 pages?

39. The product of two numbers is 15580656, and one of them is 6552; find the other.

40. What is the greatest factor common to 689 and 1573?

41. What is the difference between the G.C.M. and the L.C.M. of 160, 352 and 992?

42. 9405 is the product of two integers which differ by 4; find them.

43. The divisor is 48, the quotient 596, and the remainder 10. What is the dividend?

44. The multiplicand is 407, and the product 10989. What is the multiplier?

45. Find the greatest number which will divide 1625, 2281 and 4217, and leave remainders 8, 4 and 4 respectively.

---

46. Subtract six million eighty thousand six hundred and seventy-two from eight million nine hundred and sixty-seven thousand and twenty-five.

47. A is 75 miles distant from D; C is 40 miles distant from A, and B is 50 miles distant from D. If A, B, C, D are situated in a straight line in the order named, find their distances apart.

48. In a division sum the Dividend is 21 times, and the Quotient is 4 times, the Remainder; find the Divisor.

49. If 7855568 be divided by 4173, find the remainder.

50. If 4832796 is the product, and 1208199 the multiplicand, what is the multiplier?

51. What is the G.C.M. of 40457 and 420325?

52. Resolve 18, 16, 36, 44, 45, 48, 63, 121 into their prime factors, and thence find their L.C.M.

53. Find the greatest number which will divide 5053, 23947, 28367 and 925933, and leave remainders 13, 7, 17 and 1 respectively.

54. Supply the missing figures in the following subtractions:

(i)	4***4	(ii)	*84562
	947		8*1*3
	26205		6*7*7*

---

55. A box contains 785 balls whose colours are red, white and blue. The red and white together number 605; and the white and blue number 471. How many are there of each?

56. How often does the sum of 81459 and 54306 contain their difference?

57. A certain number, divided by 243, gives a quotient 1306 and a remainder 92. Find the number.

58. 256000***** is exactly divisible by 159601; find the digits in the dividend which are missing.

59. Find the G.C.M. of 237655 and 367285.

60. Find the sum of all the primes between 700 and 900.

61. Resolve 460992 into its prime factors.

62. 20391 gallons of beer and 49287 gallons of stout have to be put into an exact number of barrels, all of the same size and as big as possible, without mixing the beer and stout together. Find how much each barrel must hold.

63. Supply the missing figures in the following multiplications:

(i)
```
  3**7*
     9
 -------
 **12*2
```

(ii)
```
  ***6*
     7
 -------
 3*29*6
```

---

64. The difference of two numbers is 3 times the smaller number, and the sum of the numbers is 37805; find the numbers.

65. Find the sum, difference, product and quotient of 537152 and 763.

66. 867 is the product of two factors, one of which is 3 times the other; find the factors.

67. Divide 1437981 by 144 by short division.

68. Find the least number which, when divided by 12, 15, 20, leaves remainders 8, 11, 16 respectively.

69. Divide 2330790 by 2310 by short division.

70. Is 7303 prime or composite?

71. When a certain number is divided by 11, the remainder is 1; shew that, if it be divided by 22, the remainder must be 1 or 12.

72. Supply the missing figures in the following multiplications:

(i)
```
   823
    **
 -------
  ***
 ***5
 -------
 *****
```

(ii)
```
   97*
    *8
 -------
  ***
 ****
 -------
 ****0
```

given that the multiplicand is divisible by 3.

---

73. Prove that $35^2 = 4 \times 300 + 25$; and prove similar statements for 15^2, 45^2, 65^2, 95^2. Hence deduce a rule for writing down at sight the square of a number of two digits ending in 5.

74. Find the number which subtracted from 80000 leaves 57735, and divide it by 365.

75. Find the G.C.M. of 929181 and 1012891.

76. Find the greatest and least numbers of six digits which are exactly divisible by 789.

77. Resolve 132288 into its prime factors.

78. 43680 is the product of four consecutive integers; find them.

79. Divide 2141055 by 2205, by short division.

80. Find the least number which when divided by 21, 24, 56, 168 leaves remainders 9, 12, 44, 156 respectively.

81. Supply the missing figures in these multiplication sums:

(i)
```
  *52*
    3*
 ------
 4**2
  ***8
 ------
 4****
```

(ii)
```
  ****
    9*
 ------
 ****7
  ****1
 ------
 3310*1
```

---

82. What is meant by the digits 6, 5, 2, 9 respectively in the number 615209? Express 4167306002015 in words

83. Five years ago A was 3 times as old as B. If A is 30 years older than B, find the present ages of A and B.

84. Divide 7654321 by 126, (i) by long division, (ii) by short division.

85. What would be the dividend if 902341 were the divisor, 978 the quotient, and 1857 the remainder?

86. Find the greatest number which will divide 13956 and 14565, and leave a remainder 7 in each case.

87. In how many ways can 663 be resolved into two factors?

88. Divide 24182235 by 24255, by short division.

89. Four men can walk 27, 28, 35 and 45 miles per day respectively; what is the least distance that could be walked over by each in an exact number of days?

90. Supply the missing figures in the following division sums:

(i) $8 \overline{)\,3*2*}$ $\quad *2*\ldots 1$

(ii) $7 \overline{)\,3*5*8}$ $\quad *93*\ldots 2$

VIII. (a). MEASURES OF MONEY.

72. We have hitherto considered only *abstract* numbers; that is, we have considered numbers merely as multiples of a unit, without specifying, or considering in any way, what sort of thing the unit might be.

The next step is from abstract to concrete quantities, the nature of the unit now being specified. We will begin with one of the most important of such quantities, namely, MONEY.

In primitive times, and even in these days among certain primitive people, buying and selling is carried on by what is called **barter**. This in its simplest form means that a man *exchanges* what he has, and does not want, for what he has not but does want. When a hunter exchanges his skins and furs for tea, spirits, guns or gunpowder, or a schoolboy 'swops' a penknife for foreign stamps, we have simple cases of *barter*.

But we can easily see that difficulties would often arise in transactions of this kind, and to avoid as many of these difficulties as possible, it is customary to estimate the value of all articles of commerce in terms of *one* substance only. The substance chosen is **Gold**, which for this reason is called the *common medium of exchange*.

Whoever has anything to sell accepts for it a certain quantity of gold, and with this gold he buys what he needs.

73. The gold is issued in little flat and circular pieces called *coins*, each of which is stamped with certain marks and contains a definite quantity of gold. In our own country we have gold coins of two sizes in general circulation. They are called a **sovereign** and a **half-sovereign**; *two half-sovereigns* together being, for purposes of exchange, of exactly the same value as *one sovereign*.

74. With only the gold coins we could of course buy only such things or quantities as are worth an exact number of sovereigns or half-sovereigns. To represent values of less than a half-sovereign we have therefore coins made of silver, and for still smaller values coins made of bronze.

The principal *silver* coin is called a **shilling**, and is such that 20 of them represent the same value as *one* sovereign, and 10 the same value as *one* half-sovereign.

The principal *bronze* coin is called a **penny**, and is such that 12 represent the same value as one shilling.

The other silver coins in circulation are called

(i) a crown, which is worth 5 shillings,
(ii) a double florin, ... 4 shillings,
(iii) a half-crown, ... 2 shillings and 6 pence,
(iv) a florin, ... 2 shillings,
(v) a sixpenny piece, ... 6 pence,
(vi) a threepenny piece, ... 3 pence.

The other bronze coins are (i) a half-penny, 2 of which make one penny, and (ii) a farthing, 4 of which make one penny.

75. It should be remembered that values are properly estimated in gold only, and that strictly all our coins ought therefore to be of gold only: but a gold shilling, and still more a gold penny, would be so very small that for the sake of convenience we use larger coins of the less valuable metals, silver and bronze. Now these coins of silver and bronze are not really of the value they represent; the silver in 20 shillings is not really worth as much as the gold in one sovereign, nor is the bronze in 12 pennies worth as much as the silver in one shilling. Payments have to be made therefore as far as possible in gold, the silver and bronze being used only to make up the odd shillings and pence. In fact a person to whom payment is being made is allowed by law to refuse to receive more than the amount of 40 shillings in silver, 12 pence in pence or halfpence, or 6 pence in farthings.

76. The value of an article is estimated in English money as a certain number of pounds, shillings and pence; the shillings and pence representing what remains over and above an exact number of pounds. A sum of money such as 7 pounds, 12 shillings and 5 pence, is written £7. 12*s*. 5*d*.

The symbol £ stands for the Latin word *libra*, the letter *s* for *solidus*, and the letter *d* for *denarius*; these words were the names of certain Roman coins or sums of money.

One farthing is represented by placing $\frac{1}{4}$ immediately after the number which denotes pence, two farthings or one half-penny by the symbol $\frac{1}{2}$, and three farthings by $\frac{3}{4}$. Thus 7 pounds, 12 shillings, 5 pence and 3 farthings, is written £7. 12*s*. $5\frac{3}{4}d$. The reason for using the symbols $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ will be explained when we consider *fractions*.

77. The relations between the pound, the shilling, the penny and the farthing are stated in the following

ENGLISH MONEY TABLE.

4 **farthings** make 1 **penny.**
12 **pence** ..... 1 **shilling.**
20 **shillings** ..... 1 **pound.**

78. It should also be remembered that

4 crowns	make £1,		80 threepences	make £1,	
5 double florins	... £1,		240 pennies	... £1,	
8 half-crowns	... £1,		480 half-pennies	... £1,	
40 sixpences	... £1,		960 farthings	... £1.	
60 fourpences	... £1,				

79. Of coins used formerly but not now in use in the British Isles, the most important was the **guinea**, which was equivalent to 21 shillings; the word *guinea* is still commonly used in speaking of this amount. Of other obsolete coins, we need only mention the *moidore* 27*s.*, the *angel* 10*s.*, the *mark* 13*s.* 4*d.*, and the *noble* 6*s.* 8*d.* The value of the *angel* was the same as that of the present *half-sovereign*. It derived its name from the figure which it bore of the archangel Michael killing the dragon.

80. In France, Belgium and Switzerland, the principal coin is called a **franc**, and this amount is divided into 100 equal parts, each of which is called a **centime**. A *franc* is equivalent to about $9\frac{1}{2}d.$

In Germany the principal coin is called a **mark**, and this amount is divided into 100 equal parts, each of which is called a **pfennig**. A *mark* is nearly equal to 1*s.*

In the United States the principal coin is called a **dollar**, and this amount is divided into 100 equal parts, each of which is called a **cent**. A *dollar* is equivalent to about 4*s.* 2*d.*

VIII. (b). REDUCTION OF MONEY.

81. Sums of money expressed in terms of the same unit, for example, 15 shillings and 12 shillings, are said to be of the same **denomination**; and a sum of money expressed in terms of a larger unit is of **higher** denomination than when expressed in terms of a smaller unit. For example, 2 pounds is of a higher denomination than 40 shillings.

When we find how many farthings there are in a given number of pounds, shillings and pence, or the number of pounds, shillings and pence equivalent to a given number of farthings; or, generally, when we convert a quantity from one denomination to another, we are said to **reduce** it, and the process is called **reduction**. Thus 20 shillings become 1 pound when *reduced* to pounds, and 240 pence when *reduced* to pence.

82. We know that in a pound there are 20 shillings, hence to find the number of shillings in a given number of pounds we multiply the number of pounds by 20.

Similarly, to find the number of pence in a given number of shillings we multiply the number of shillings by 12.

And, again, to reduce pence to farthings we multiply the number of pence by 4.

Example i. *Reduce* £23 *to shillings.*

£23
20
460 shillings.

∴ no. of shillings = 460.

Example ii. *Reduce* £14 *to pence.*

£14
20
280 = no. of shillings,
12
3360 = no. of pence.

∴ no. of pence = 3360.

83. When we have to reduce to farthings a sum of money which is not a whole number of pounds or shillings, we multiply the number of *pounds* by 20, and add to the product the number of odd *shillings*; we then multiply this result by 12, and add the number of odd *pence*; lastly, we multiply the number of pence by 4, and add the odd farthings.

Example i.

Reduce £17. 14*s.* 5*d.* *to pence.*

£17. 14*s.* 5*d.*
20
340
14
354 = no. of shillings,
12
4248
5
4253 = no. of pence.

Example ii.

Reduce £21. 19*s.* 6¾*d.* *to farthings.*

After a little practice we can add the odd shillings, pence and farthings without using separate lines.

£21, 19*s.* 6¾*d.*
20
439 = no. of shillings,
12
5274 = no. of pence,
4
21099 = no. of farthings.

84. To reduce foreign money from the higher denomination to the lower, francs to centimes, marks to pfennige, dollars to cents, we multiply by 100. For example,

23 francs 15 centimes = 2300 c. + 15 c. = 2315 c.,

and 5 francs 4 centimes = 500 c. + 4 c. = 504 c.

The pupil will see from these examples that, in the case of foreign money, reduction from the higher denomination to the

lower is effected simply by writing down the figures in the order in which they occur. But it must be carefully noticed that *two* figures are always necessary to express the number of the smaller denomination. When the number of the smaller denomination is *less than* 10, as in the second example, we must prefix 0. Thus, for purposes of reduction, 4 c. is written 04 c., and for 5 francs 4 centimes we write 504 c., **not** 54 c.

85. If the *place-values* of the digits be considered the pupil will easily understand that reduction in the case of foreign money is nothing more than a change of notation. In the case of 23 f. 15 c., for example, if we take 1 c. as the unit, it is evident that

the digit 5 represents 5 units,
the digit 1 ..... 1 ten,
the digit 3 ..... 3 hundreds (since 1 f. = 100 c.),
and the digit 2 ..... 2 thousands (since 10 f. = 1000 c.)

Thus, with the ordinary notation of numbers explained in Chapter II., we see that 23 f. 15 c. = 2315 units = 2315 centimes.

Similarly, 55 marks 8 pfennige = 5508 pfennige, and 613 dollars 81 cents = 61381 cents.

EXAMPLES. **16.** *Oral.*

1. In the following amounts how many pence are there?

1*s.* 2*d.*, 1*s.* 6*d.*, 1*s.* 8*d.*, 2*s.*, 2*s.* 6*d.*, 2*s.* 8*d.*, 3*s.*, 3*s.* 4*d.*, 3*s.* 6*d.*, 4*s.*, 4*s.* 2*d.*, 4*s.* 6*d.*, 5*s.*, 5*s.* 6*d.*, 6*s.*, 6*s.* 6*d.*, 6*s.* 8*d.*, 7*s.*, 7*s.* 6*d.*, 8*s.*, 8*s.* 4*d.*, 8*s.* 6*d.*, 9*s.*, 9*s.* 6*d*, 10*s.*, 11*s.*, 11*s.* 6*d.*, 12*s.*, 12*s.* 6*d.*, 13*s.*, 13*s.* 4*d.*, 13*s.* 6*d.*, 14*s.*, 14*s.* 6*d.*, 15*s.*, 16*s.*, 16*s.* 8*d.*, 17*s.*, 17*s.* 6*d.*, 18*s.*, 19*s.*, 20*s.*, 21*s.*

2. How many shillings are there in

£2, £5, £10, £3. 4*s.*, £6. 17*s.*, £43. 12*s.*, £82. 9*s.*, £129. 5*s.*, £150. 10*s.*, £175. 15*s.*?

3. How many pence are there in

5 crowns, 8 half-crowns, 30*s.*, 1 guinea, 2 guineas?

4. How many half-crowns are there in

7*s.* 6*d.*, 5*s.*, 10*s.*, 12*s.* 6*d.*, 15*s.*, 17*s.* 6*d.*, 22*s.* 6*d.*, £5, £50, £100, £60. 17*s.* 6*d.*?

5. Express in centimes

3 f. 13 c., 110 f. 50 c., 14 f. 4 c., 51 f. 1 c., 9 f. 21 c., 10 f. 10 c., 200 f. 8 c., 130 f. 30 c., 46 f. 7 c., 150 f. 5 c.

6. Express in cents

7 dol. 10 c., 80 dol. 75 c., 10 dol. 7 c., 55 dol. 5 c.,
14 dol. 15 c., 100 dol. 15 c., 30 dol. 9 c., 500 dol. 50 c.

7. Express in pfennige

17 m. 5 pf., 8 m. 12 pf., 120 m. 9 pf., 20 m. 16 pf.,
12 m. 3 pf., 18 m. 21 pf., 19 m. 80 pf., 50 m. 75 pf.

EXAMPLES. 17.

Reduce

1. £41, £53, £105, £368, £7284 to shillings.

2. £51. 7*s.*, £119. 12*s.*, £233. 2*s.*, £5612. 19*s.* to shillings.

3. £45, £67, £195, £6125, £47890 to pence.

4. £5. 6*s.*, £125. 14*s.*, £798. 5*s.*, £3429. 11*s.* to pence.

5. £3. 6*s.* 8*d.*, £15. 4*s.* 2*d.*, £234. 17*s.* 6*d.*, £1329. 18*s.* 4*d.*, £4798. 3*s.* 6*d.* to pence.

6. £17. 4*s.* 6*d.*, £48. 6*s.* 9*d.*, £149. 16*s.* 8*d.*, £367. 2*s.* 9*d.*, £5621. 12*s.* 10*d.* to farthings.

7. £49. 6*s.* $2\frac{1}{4}d.$, £176. 15*s.* $3\frac{3}{4}d.$, £798. 12*s.* $7\frac{1}{2}d.$, £4513. 4*s.* $11\frac{1}{2}d.$, £98729. 19*s.* $5\frac{1}{4}d.$ to farthings.

8. £15, £128, £6725, £48936, £208471, (i) to crowns, (ii) to sixpences.

9. £8. 15*s.*, £14. 10*s.*, £327. 5*s.*, £5189. 15*s.*, £17129. 5*s.*, (i) to crowns, (ii) to fourpences.

10. £7. 2*s.* 6*d.*, £87. 5*s.*, £415. 7*s.* 6*d.*, £816. 17*s.* 6*d.*, £1920. 15*s.*, (i) to half-crowns, (ii) to sixpences.

11. £10. 12*s.* 6*d.*, £34. 7*s.* 6*d.*, £513. 10*s.*, £3201. 12*s.* 6*d.*, £47912. 15*s.*, (i) to half-crowns, (ii) to threepences.

12. 7652 half-crowns, 347 guineas, 650 half-sovereigns, to half-pence.

86. In the *converse* operation, the reduction of farthings to pounds, shillings and pence, we divide successively by 4, 12 and 20; the successive remainders give the number of odd farthings, pence and shillings respectively.

Example 1. *Reduce* 576331 *farthings to pounds.*

Dividing 576331 by 4, the quotient is 144082, and the remainder is 3; which means that 576331 farthings are equal to 144082 pence with 3 farthings over.

Dividing 144082 by 12, the quotient is 12006, and the remainder is 10; which means that 144082 pence are equal to 12006 shillings with 10 pence over.

Lastly, dividing 12006 by 20, the quotient is 600, and the remainder is 6; which means that 12006 shillings are equal to 600 pounds with 6 shillings over. The complete operation is shewn thus:

4 | 576331
12 | 144082 pence, and 3 farthings over,
20 | 12006 shillings, and 10 pence over,
600 pounds, and 6 shillings over.

∴ 576331 farthings = £600. 6*s*. 10¾*d*.

Example ii. *Reduce* 3720945 *farthings to pounds, shillings and pence.*

4 | 3720945
12 | 930236*d*. . ¼*d*.
20 | 77519*s*. . 8¼*d*.
£3875. 19*s*. 8¼*d*.

∴ 3720945 farthings = £3875. 19*s*. 8¼*d*.

87. To change the unit in terms of which a quantity is expressed, we find the G.C.M. of the two units, the old and the new. To express in half-guineas, for example, a sum of money already expressed in half-crowns, we first find the G.C.M. of a half-guinea and a half-crown. This G.C.M. is 6*d*. We then multiply the number, of half-crowns by 5 to reduce to sixpences, and we divide the number of sixpences by 21 to reduce to half-guineas.

Example i. *How many half-guineas are there in* 525 *half-crowns?*

525 half-crowns,
5
21 { 3 | 2625 sixpences,
7 | 875
525 half crowns = 125 half-guineas.

Example ii. *How many half-guineas are there in* £302. 8*s*. *?*

The largest sum in terms of which each can be expressed is 6*d*. We therefore reduce £302. 8*s*. to sixpences and then divide by 21, which is the number of sixpences in a half-guinea.

£302. 8*s*. 0*d*.
40
21 { 3 | 12096 sixpences,
7 | 4032
576 half-guineas.

88. To reduce foreign money from the smaller denomination to the higher, *e.g.* from centimes to francs, from pfennige to marks, from cents to dollars, we divide by 100. This can be done at once by inspection. The last two figures of the dividend

form the remainder, and the others the quotient. If, for example, 37556 be divided by 100, the quotient is 375 and the remainder is 56. Hence 37556 centimes = 375 f. 56 c.

89. By considering the place-values of the digits the pupil will see as before that this change is merely a change of notation, in which we have replaced 100 c., *i.e.* 100 units in the old notation by 1 unit in the new, *i.e.* by 1 f. For, taking the digits of the number 37556 centimes in turn it is clear that

the digit 6 denotes 6 units, or 6 c.,
the digit 5 ..... 5 tens, or 50 c.,
the digit 5 ..... 5 hundreds, or 500 c., or 5 f.,
the digit 7 ..... 7 thousands, or 7000 c., or 70 f.,
the digit 3 ..... 3 ten-thousands, or 300 f.

Therefore 37556 c. = 375 f. 56 c.

EXAMPLES. 18. *Oral.*

1. How many shillings, and pence over, are there in the following numbers of pence?

15, 18, 20, 21, 22, 24, 26, 27, 30, 32, 36, 38, 40, 41, 43, 45, 48, 50, 54, 60, 65, 70, 72, 76, 80, 84, 87, 90, 93, 96, 99, 100, 110, 112, 115, 120, 125, 130, 132, 140, 144, 150, 156, 160, 168, 170, 175, 180, 188, 192, 200, 204, 210, 216, 220, 228, 230, 240.

2. Express in francs and centimes:

175 c., 2105 c., 31556 c., 56708 c., 3215 c., 176321 c., 74625 c., 1005 c., 5006 c., 97165 c., 100007 c., 80009 c.

3. Express in dollars and cents:

967 c., 875 c., 906 c., 8115 c., 10065 c., 5015 c., 805 c., 68401 c., 73216 c., 30004 c., 1089 c., 15007 c.

EXAMPLES. 19.

Reduce

1. 671, 8429, 56123, 612347, 872963 farthings to pence.
2. 472, 3261, 56203, 142891, 3428072 pence to shillings.
3. 342, 2349, 84725, 432109, 8572018 shillings to pounds.
4. 472, 8932, 4789, 62084, 3720895 pence to pounds.
5. 743, 2089, 36201, 847298, 63478251 farthings to shillings.

Reduce

6. 5621, 842089, 3628471, 48562987 half-pence to pounds.

7. 6234, 58106, 68472097, 561842091 farthings to pounds.

8. 84714, 623469, 3712086, 65432178 shillings to guineas.

9. 1575 half-crowns to half-guineas.

10. How many articles at 3*d.* can be bought for 57 guineas?

11. Each of three bags contains £10. 5*s.*; one in half-crowns, the second in sixpences, and the third in threepenny pieces. How many coins are there in each bag?

12. How many $2\frac{1}{2}$*d.* stamps can I buy for £1. 10*s.*?

13. A bag contains 575 francs in 5-franc pieces. How many coins are there in it? How many 10-centime pieces would there be, if it contained only coins of this value?

14. How many 10-centime pieces are equivalent to thirty-five 20-franc pieces?

15. How many sous, each worth 5 centimes, are there in a Napoleon worth 20 francs?

IX. COMPOUND ADDITION.—BRITISH MONEY.

90. A **compound** quantity is a quantity in the expression of which there are terms of different denominations. For example, £5. 6*s.* 10*d.* is a compound quantity, for in it there are terms of three different denominations: pounds, shillings and pence.

Compound Addition is the addition of compound quantities of the same kind, and is the same in principle as the addition of abstract numbers.

91. In adding abstract numbers we place the numbers so that the digits which denote *units* are all one under another in a column, the digits which denote *tens* also one under another in a column, and so on.

Similarly, when we add together several sums of money, we arrange them so that the numbers which denote *pence* are all one under another in a column, those which denote *shillings* one under another in a column, and so also those which denote *pounds*.

Again, in simple addition we add together the unit digits, carry forward multiples of 10, and set down the remainder. So in adding sums of money, we add together the numbers of pence, carry forward shillings or multiples of 12 pence, and set down the remainder. And we treat the other columns in a similar way.

Example i. *Add together* £37. 6*s.* 10½*d.*, £42. 12*s.* 6¼*d.* and £94. 15*s.* 4¾*d.*

We arrange the expressions one under another.

£.	*s.*	*d.*
37 .	6 .	10½
42 .	12 .	6¼
94 .	15 .	4¾
174 .	14 .	9½

We then begin at the right hand by adding together the quantities of lowest denomination, just as in simple addition we begin by adding the units. Adding together ¾ which represents 3 farthings, ¼ which represents one farthing, and ½ which represents 2 farthings, we get 6 farthings, which is the same as 1 penny and 2 farthings.

We write down ½, the symbol for 2 farthings, under the other symbols for farthings; and we carry the penny to increase the pence-column.

Adding now the pence-column, we find that it amounts to 21 pence. But 21 pence are equivalent to 1 shilling and 9 pence. We therefore write down the 9 pence at the foot of the pence-column, and transfer the 1 shilling to increase the shillings-column.

The shillings-column now amounts to 34 shillings. Hence we write down 14 shillings, and transfer 1 pound to increase the pounds-column.

Lastly, adding the pounds-column, we get £174.

The result of addition is therefore £174. 14*s.* 9½*d.*

Example ii. *Add together* £257. 2*s.* 9¾*d.*, £61. 14*s.* 5½*d.*, £1351. 17*s.* 2¼*d.*, £1090. 1*s.* 11¾*d. and* £2. 15*s.* 4¼*d.*

£.	*s.*	*d.*
257 .	2 .	9¾
61 .	14 .	5½
1351 .	17 .	2¼
1090 .	1 .	11¾
2 .	15 .	4¼
2763 .	11 .	9½

EXAMPLES. 20. *Oral.*

Add together

1. 2*d.*, 3*d.* and 4*d.*
2. 8*d.* and 7*d.*
3. 9*d.*, 10*d.* and 11*d.*
4. 4*d.*, 5*d.*, 8*d.* and 9*d.*
5. 6*d.*, 7*d.*, 8*d.* and 9*d.*
6. 8*d.*, 9*d.*, 10*d.* and 11*d.*
7. 7*d.*, 1*s.* and 1*s.* 1*d.*
8. 8*d.*, 10*d.*, 3*d.* and 1*s.*
9. 2*d.*, 1*s.* 2*d.*, 1*s.* 1*d.* and 1*s.*
10. 3*d.*, 1*s.* 8*d.*, 1*s.* and 2*s.*
11. 1*s.* 1*d.*, 1*s.* 3*d.*, 1*s.* and 2*s.* 5*d.*
12. 1*s.* 2*d.*, 1*s.* 1*d.*, 2*s.* 8*d.* and 2*s.*
13. 1*d.*, 1*s.*, 2*s.* 8*d.* and 3*s.* 2*d.*
14. 3*d.*, 2*s.* 5*d.*, 3*s.* 4*d.* and 6*s.*
15. 1*s.* 9*d.*, 2*s.* 3*d.*, 1*s.* 8*d.* and 3*s.* 4*d.*
16. 2*s.* 6*d.*, 12*s.* 6*d.*, 15*s.* and 17*s.* 6*d.*
17. 3*s.* 4*d.*, 6*s.* 8*d.*, 17*s.* 8*d.* and 21*s.*
18. 6*s.* 8*d.*, 13*s.* 4*d.*, 5*s.* 9*d.* and 14*s.* 3*d.*

Add together

19. 6*d.*, 13*s.* 6*d.*, 12*s.* 3*d.* and 14*s.* 9*d.*

20. 3*s.* 4*d.*, 2*s.* 8*d.*, 5*s.* 6*d.* and 14*s.* 9*d.*

EXAMPLES. 21.

1.
d.
$2\frac{1}{4}$
$5\frac{1}{4}$
$7\frac{1}{4}$
$9\frac{1}{4}$

2.
d.
$3\frac{1}{2}$
$2\frac{1}{4}$
4
$10\frac{1}{2}$

3.
d.
$1\frac{3}{4}$
$2\frac{1}{2}$
$3\frac{1}{2}$
$5\frac{1}{4}$

4.
d.
$6\frac{3}{4}$
$7\frac{3}{4}$
$5\frac{1}{2}$
$9\frac{1}{4}$

5.
d.
$9\frac{1}{2}$
$10\frac{1}{4}$
$11\frac{3}{4}$
$10\frac{1}{2}$

6.
s.	*d.*
1	7
9	4
3	2
4	9

7.
s.	*d.*
7	2
3	11
5	6
6	3

8.
s.	*d.*
9	10
6	4
12	6
15	1

9.
s.	*d.*
5	6
7	8
9	6
5	4

10.
s.	*d.*
3	$2\frac{1}{4}$
2	$1\frac{1}{2}$
9	$6\frac{3}{4}$
3	$5\frac{1}{2}$
2	$7\frac{3}{4}$

11.
s.	*d.*
11	$1\frac{1}{4}$
3	$7\frac{1}{2}$
7	$9\frac{3}{4}$
10	$8\frac{1}{2}$
4	$6\frac{3}{4}$

12.
s.	*d.*
9	$11\frac{1}{4}$
10	$5\frac{1}{2}$
7	$10\frac{3}{4}$
4	$7\frac{1}{4}$
7	6

13.
s.	*d.*
5	$6\frac{1}{2}$
3	$4\frac{3}{4}$
2	$8\frac{1}{4}$
14	$9\frac{3}{4}$
18	$7\frac{3}{4}$

14.
£.	*s.*	*d.*
1	5	4
1	3	6
3	2	1
1	0	5

15.
£.	*s.*	*d.*
3	3	2
2	1	3
4	10	1
7	2	5

16.
£.	*s.*	*d.*
2	13	5
7	2	3
8	1	8
3	2	10

17.
£.	*s.*	*d.*
7	14	$10\frac{1}{4}$
5	6	$8\frac{3}{4}$
17	4	11
3	15	$9\frac{1}{2}$
5	6	$8\frac{3}{4}$

18.
£.	*s.*	*d.*
3	2	$1\frac{1}{4}$
5	6	$10\frac{1}{2}$
15	11	$4\frac{3}{4}$
13	5	$8\frac{1}{2}$
6	8	4

19.
£.	*s.*	*d.*
17	5	$3\frac{3}{4}$
2	3	$7\frac{1}{2}$
5	6	$4\frac{3}{4}$
27	14	8
16	15	$3\frac{3}{4}$

20.
£.	*s.*	*d.*
7	8	$2\frac{1}{2}$
8	2	$3\frac{3}{4}$
9	3	$7\frac{1}{4}$
31	14	$5\frac{3}{4}$
10	13	4
5	12	$6\frac{1}{4}$

21.
£.	*s.*	*d.*
11	10	$4\frac{1}{2}$
14	8	2
5	6	$8\frac{1}{2}$
13	17	$4\frac{1}{4}$
7	3	9
2	4	$11\frac{1}{2}$

22.
£.	*s.*	*d.*
8	10	0
7	2	$0\frac{1}{2}$
24	5	$7\frac{1}{4}$
18	3	$5\frac{3}{4}$
5	0	$1\frac{1}{2}$
32	16	$7\frac{1}{4}$

23.

£	s.	d.
17 .	3 .	$1\frac{1}{2}$
5 .	13 .	$0\frac{3}{4}$
35 .	4 .	$5\frac{1}{2}$
41 .	0 .	6
4 .	7 .	$2\frac{1}{4}$
5 .	17 .	$6\frac{1}{2}$

24.

£.	s.	d.
4 .	3 .	$0\frac{1}{2}$
17 .	6 .	$8\frac{3}{4}$
55 .	2 .	$6\frac{1}{2}$
40 .	17 .	6
15 .	0 .	$7\frac{3}{4}$
31 .	14 .	$6\frac{1}{2}$

25.

£.	s.	d.
5 .	6 .	8
7 .	0 .	$10\frac{1}{4}$
62 .	14 .	$9\frac{3}{4}$
70 .	10 .	$2\frac{1}{2}$
89 .	3 .	$6\frac{3}{4}$
5 .	0 .	$2\frac{1}{4}$

26.

£.	s.	d.
51 .	10 .	$6\frac{1}{2}$
10 .	19 .	$8\frac{1}{4}$
15 .	5 .	$6\frac{3}{4}$
33 .	10 .	$4\frac{1}{2}$
47 .	12 .	$5\frac{1}{4}$
56 .	10 .	9
40 .	5 .	$6\frac{1}{2}$

27.

£.	s.	d.
16 .	9 .	0
4 .	10 .	$6\frac{3}{4}$
92 .	3 .	$8\frac{1}{4}$
50 .	12 .	$11\frac{1}{2}$
43 .	8 .	2
3 .	6 .	$8\frac{3}{4}$
19 .	10 .	$4\frac{1}{4}$

28.

£.	s.	d.
9 .	10 .	$2\frac{1}{2}$
13 .	12 .	$9\frac{1}{4}$
89 .	2 .	1
65 .	17 .	$11\frac{3}{4}$
39 .	18 .	$9\frac{1}{2}$
54 .	16 .	$10\frac{1}{2}$
76 .	11 .	9

29.

£.	s.	d.
7 .	19 .	$10\frac{1}{2}$
91 .	3 .	$7\frac{1}{4}$
98 .	17 .	11
87 .	16 .	$10\frac{1}{2}$
74 .	11 .	$5\frac{3}{4}$
69 .	3 .	$7\frac{1}{2}$
21 .	14 .	$3\frac{3}{4}$

30.

£.	s.	d.
84 .	13 .	$6\frac{1}{2}$
92 .	12 .	$4\frac{3}{4}$
107 .	5 .	$9\frac{3}{4}$
87 .	16 .	4
4 .	7 .	$8\frac{1}{2}$
121 .	8 .	$6\frac{1}{4}$
	17 .	$8\frac{1}{2}$

31.

£.	s.	d.
131 .	16 .	4
	18 .	$6\frac{1}{2}$
84 .	3 .	$11\frac{3}{4}$
105 .	8 .	$4\frac{3}{4}$
3 .	6 .	8
98 .	17 .	$7\frac{1}{2}$
5 .	6 .	$8\frac{3}{4}$

32.

£.	s.	d.
37 .	19 .	$8\frac{3}{4}$
152 .	16 .	$10\frac{3}{4}$
99 .	4 .	$2\frac{1}{2}$
105 .	12 .	$9\frac{1}{2}$
6 .	3 .	$2\frac{3}{4}$
16 .	15 .	$10\frac{1}{2}$
9 .	19 .	6
25 .	16 .	$7\frac{1}{4}$

33.

£.	s.	d.
44 .	5 .	$7\frac{1}{2}$
139 .	13 .	$2\frac{1}{2}$
6 .	2 .	6
	14 .	$5\frac{1}{4}$
	3 .	$2\frac{3}{4}$
150 .	0 .	0
69 .	18 .	$3\frac{1}{2}$
43 .	14 .	$7\frac{1}{4}$

34.

£.	s.	d.
189 .	16 .	$7\frac{1}{4}$
203 .	7 .	$3\frac{3}{4}$
	17 .	$4\frac{3}{4}$
393 .	14 .	5
32 .	10 .	0
280 .	13 .	$4\frac{3}{4}$
93 .	15 .	7
1 .	6 .	8

35.

£.	s.	d.
560 .	12 .	$4\frac{1}{2}$
712 .	8 .	$9\frac{1}{4}$
62 .	3 .	$4\frac{1}{2}$
		$9\frac{1}{2}$
	13 .	$4\frac{3}{4}$
	3 .	$2\frac{1}{2}$
16 .	5 .	6
	13 .	4

36.

£.	s.	d.
3 .	17 .	$6\frac{1}{2}$
		$7\frac{3}{4}$
108 .	0 .	$11\frac{1}{2}$
	3 .	6
924 .	18 .	$7\frac{3}{4}$
876 .	15 .	$2\frac{1}{4}$
31 .	0 .	5
2 .	3 .	$6\frac{1}{2}$

37.

£.	s.	d.
	13 .	4
6 .	2 .	$8\frac{1}{2}$
26 .	17 .	$2\frac{1}{2}$
		$9\frac{3}{4}$
	7 .	$8\frac{1}{2}$
1071 .	12 .	3
895 .	3 .	$7\frac{1}{4}$
3 .	7 .	6

	£.	s.	d.
38.	8032	3	6
	29	14	$6\frac{1}{2}$
	3	7	$4\frac{2}{4}$
	41	19	$6\frac{3}{4}$
	689	3	$7\frac{1}{2}$
	7602	18	$10\frac{3}{4}$
	3421	3	4
	6308	1	$3\frac{1}{2}$
	2531	5	6
	21	17	4

	£.	s.	d.
39.	3	17	6
	291	6	$2\frac{1}{4}$
	85	7	4
	9827	19	$11\frac{3}{4}$
	8231	0	10
	6073	15	6
	10329	8	$7\frac{3}{4}$
	7609	2	6
	5730	15	$7\frac{3}{4}$
	3	0	$4\frac{1}{2}$

	£.	s.	d.
40.	20376	13	4
	542	1	$8\frac{3}{4}$
	29	15	10
	70483	9	$6\frac{3}{4}$
	89763	12	$0\frac{1}{4}$
	8532	4	5
	75431	19	$11\frac{3}{4}$
	65948	2	6
	82103	0	$10\frac{1}{2}$
	6975	12	$8\frac{1}{2}$

X. COMPOUND SUBTRACTION.—BRITISH MONEY.

92. **Compound** subtraction is the subtraction of one compound quantity from another of the same kind.

93. To subtract one sum of money from another we place the sum to be subtracted below the one from which it is to be taken, pence below pence, shillings below shillings, and pounds below pounds.

Example. *Subtract* £3. 4*s.* 6*d.* *from* £5. 15*s.* 10*d.*

(a) The lower figure in each column is less than the upper one. If therefore we take 6 from 10, 4 from 15, and 3 from 5, the remainders are 4, 11 and 2 respectively, and the result is £2. 11*s.* 4*d.*

£.	s.	d.
5	15	10
3	4	6
2	11	4

(b) Otherwise, by the method of Art. 32, thus:

6*d.* and 4*d.* make 10*d.*; write down 4*d.*,
4*s.* ... 11*s.* ..... 15*s.*; 11*s.*,
£3 ... £2 ..... £5; ..... £2.

Hence the result is £2. 11*s.* 4*d.*

94. If any term in the lower line be greater than the term of the same denomination in the upper line, we again proceed as in simple subtraction.

Example i. *Subtract* £6. 17*s.* 9*d.* *from* £9. 10*s.* 8*d.*

(a) We cannot take 9 pence from 8 pence, so we increase the 8 pence to 20 pence, and at the same time we increase the 17*s.* in the lower line to 18*s.* The remainder from the pence column is then 11 pence.

£.	s.	d.
9	10	8
6	17	9
2	12	11

Again, we cannot take 18*s.* from 10*s.*, so we increase 10*s.* to 30*s.*, and we increase the £6 in the lower line to £7.

The difference is thus found to be £2. 12*s.* 11*d.*

(b) By the process of Art. 32, the mental steps are as follows:

(i) 9*d.* and 11*d.* make 1*s.* 8*d.*; write down 11*d.* and carry 1*s.*

(ii) 1*s.* and 17*s.* make 18*s.*; and 12*s.* make 30*s.*; write down 12*s.* and carry £1.

(iii) £1 and £6 make £7; and £2 make £9; write down £2.

Example ii. *In one operation take the sum of* £17. 16*s.* $4\frac{1}{2}d.$ *and* £15. 8*s.* $3\frac{3}{4}d.$ *from* £35. 16*s.* 7*d.*

We add each column of the subtrahends, and subtract each sum from the corresponding term in the top line by the process of Art. 32. Thus:

£.	*s.*	*d.*
35	16	7
17	16	$4\frac{1}{2}$
15	8	$3\frac{3}{4}$
2	11	$10\frac{3}{4}$

(i) $3f.$ and $2f.$ make $1\frac{1}{4}d.$; and $3f.$ make $2d.$; write down $3f.$ and carry $2d.$

(ii) $2d.$, $3d.$ and $4d.$ make $9d.$; and **10***d.* make 1*s.* $7d.$; write down **10***d.* and carry 1*s.*

(iii) 1*s.*, 8*s.* and 16*s.* make 25*s.*; and **11***s.* make £1. 16*s.*; write down **11***s.* and carry £1.

(iv) £1, £15, and £17 make £33; and £2 make £35; write down £2.

Hence the result is £2. 11*s.* $10\frac{3}{4}d.$

EXAMPLES. 22. *Oral.*

Subtract

1. $\frac{1}{4}d.$ from $\frac{1}{2}d.$
2. $\frac{1}{4}d.$ from $\frac{3}{4}d.$
3. $\frac{1}{2}d.$ from $\frac{3}{4}d.$
4. $1\frac{1}{4}d.$ from $6\frac{1}{2}d.$
5. $2\frac{1}{2}d.$ from $10\frac{3}{4}d.$
6. $5\frac{1}{4}d.$ from $8\frac{3}{4}d.$
7. $7\frac{1}{4}d.$ from $10\frac{3}{4}d.$
8. $6\frac{1}{2}d.$ from $9\frac{3}{4}d.$
9. $10\frac{1}{4}d.$ from $11\frac{1}{2}d.$
10. 1*s.* 7*d.* from 12*s.* 8*d.*
11. 3*s.* 6*d.* from 17*s.* 10*d.*
12. 4*s.* $5\frac{1}{4}d.$ from 18*s.* $6\frac{3}{4}d.$
13. 10*s.* $9\frac{3}{4}d.$ from 15*s.* $10\frac{3}{4}d.$
14. 14*s.* $9\frac{1}{2}d.$ from 18*s.* $11\frac{3}{4}d.$
15. 8*s.* $7\frac{3}{4}d.$ from 14*s.* $9\frac{3}{4}d.$
16. 16*s.* $4\frac{1}{2}d.$ from 17*s.* $7\frac{3}{4}d.$
17. 10*s.* $7\frac{1}{4}d.$ from 15*s.* $10\frac{1}{2}d.$
18. 8*s.* $10\frac{1}{2}d.$ from 19*s.* $2\frac{3}{4}d.$
19. 14*s.* $6\frac{3}{4}d.$ from 15*s.*
20. 15*s.* $11\frac{3}{4}d.$ from 20*s.*

EXAMPLES. 23.

Subtract

1. £1. 2*s.* 6*d.* from £1. 10*s.* 9*d.*
2. £3. 14*s.* 3*d.* from £5. 17*s.* 6*d.*
3. £13. 5*s.* 6*d.* from £25. 11*s.* 10*d.*
4. £14. 10*s.* 2*d.* from £36. 14*s.* 1*d.*
5. £17. 3*s.* 10*d.* from £19. 6*s.* 6*d.*
6. £21. 7*s.* 6*d.* from £42. 6*s.* 10*d.*
7. £45. 12*s.* 9*d.* from £57. 8*s.* 6*d.*
8. £28. 13*s.* $6\frac{1}{2}d.$ from £45. 8*s.* $2\frac{1}{4}d.$
9. £59. 14*s.* $6\frac{3}{4}d.$ from £145. 10*s.* $4\frac{1}{2}d.$

Subtract

10. £98. 5*s.* 11¼*d.* from £142. 1*s.* 8*d.*
11. £192. 19*s.* 7*d.* from £301. 17*s.* 6¾*d.*
12. £348. 17*s.* 10¾*d.* from £456. 4*s.* 3½*d.*
13. £562. 15*s.* 3¾*d.* from £618. 12*s.* 1¼*d.*
14. £781. 9*s.* 7½*d.* from £820. 7*s.* 4¼*d.*
15. £659. 17*s.* 8¼*d.* from £782. 10*s.* 7¼*d.*
16. £896. 11*s.* 5¾*d.* from £1072. 8*s.* 2¼*d.*
17. £1076. 9*s.* 11½*d.* from £3761. 5*s.* 6*d.*
18. £5209. 5*s.* 7½*d.* from £6987. 0*s.* 1½*d.*
19. £8260. 13*s.* 9¼*d.* from £10687. 7*s.*
20. £16987. 0*s.* 10*d.* from £37659. 0*s.* 6*d.*
21. £24329. 6*s.* 4*d.* from £34284. 3*s.* 2*d.*
22. £56234. 17*s.* 10*d.* from £78941. 12*s.* 6*d.*
23. £72841. 3*s.* 6*d.* from £189432. 2*s.* 4*d.*
24. £132982. 12*s.* 8*d.* from £156241. 9*s.* 6*d.*
25. £218032. 0*s.* 5*d.* from £451805. 13*s.* 4*d.*

In one operation take the sum of

26. £32. 5*s.* 6*d.* and £40. 3*s.* 7*d.* from £93. 7*s.* 8*d.*
27. £13. 4*s.* 7½*d.*, £22. 19*s.* 4¼*d.* and £37. 13*s.* 2*d.* from £101. 7*s.* 6*d.*
28. £35. 2*s.* 1*d.*, £10. 9*s.* 6½*d.* and £52. 1*s.* 10*d.* from £212. 15*s.* 4*d.*
29. £21. 6*s.* 10*d.*, £15. 2*s.* 3¼*d.* and £47. 18*s.* 9*d.* from £115. 6*s.* 7½*d.*
30. £5. 6*s.* 4½*d.*, £31. 15*s.* 10¼*d.*, £43. 18*s.* 5¾*d.* and £25. 16*s.* 4¼*d.* from £371. 14*s.* 6⅝*d.*

XI. COMPOUND MULTIPLICATION.—BRITISH MONEY.

95. **Compound Multiplication** is the multiplication of compound quantities.

In principle it is similar to simple multiplication or the multiplication of abstract numbers; that is to say, it is equivalent to repeated addition simplified by the recollection of certain known facts and results which are stored up in the *multiplication table*, in the *money table*, and in the *tables of weights and measures*.

96. The multiplier must always be an *abstract* number, for it denotes the number of equal quantities added together.

For example, we can multiply £5. 6*s.* 8*d.* by 3, but we cannot multiply it by £3, or 3*s.*, or 3 pence.

97. In multiplying a sum of money, we write the multiplier immediately underneath the term of lowest denomination. We then multiply, and carry forward multiples of 12 pence and multiples of 20 shillings in the same way as in addition.

Example i. *Multiply* £65. 13*s.* 7¾*d.* *by* 7.

The work is stated as in the margin.

We know that 7 times 3 farthings = 21 farthings = 5¼d., therefore we write down ¼*d.*, and carry forward 5 pence.

Again, 7 times 7 pence = 49 pence, which become 54 pence when we add the 5 pence brought forward. Also 54 pence are equivalent to 4 shillings and 6 pence. We therefore write down 6 pence and carry forward 4 shillings.

Next, 7 times 13 shillings = 91 shillings, which become 95 shillings when we add the 4 shillings brought forward. And we know that 95 shillings are the same as 4 pounds and 15 shillings. Therefore we write down 15 shillings and carry forward 4 pounds.

Lastly, 7 times £65 = £455, which become £459 when we add the £4.

Hence the result is £459. 15*s.* 6¼*d.*

£.	*s.*	*d.*
65 .	13 .	7¾
		7
459 .	15 .	6¼

Example ii. *Multiply* £26. 7*s.* 3½*d.* *by* 11.

The 3 shillings carried forward from the pence, and the 77 shillings obtained by the multiplication of 7 shillings by 11, make 80 shillings altogether: and 80 shillings are exactly the same as £4. Hence we write down 0 in the shillings-place, and carry forward £4.

NOTE.—It should be noticed that 11 times £26 is £286, and that 11 times £27 is £297, so that the answer to the question must lie between £286 and £297. A **rough approximation** of this kind can be obtained very easily and quickly, and this should always be done. The pupil should learn to satisfy himself that the answer which he gives is at least reasonable. It too often happens that an answer is given which is absurdly large or absurdly small.

£.	*s.*	*d.*
26 .	7 .	3½
		11
290 .	0 .	2½

Example iii. *Multiply* £84. 17*s.* 6¼*d.* *by* 24.

The multiplier 24 is the same as 4×6; therefore we multiply £84. 17*s.* 6¼*d.* by 4, and the product by 6.

NOTE.—A rough approximation gives an answer between £84 × 24 and £85 × 24, *i.e.* between £2016 and £2040. Therefore the answer must lie between £2016 and £2040.

£	*s.*	*d.*
84 .	17 .	6¼
		4
339 .	10 .	1
		6
2037 .	0 .	6

In the following set of examples the multipliers are either not greater than 12, or can easily be separated into *factors* which are not greater than 12.

EXAMPLES. 24.

Multiply

1. £17. 6*s*. 8*d*. by 5, 7, 9, 11.
2. £51. 17*s*. 6*d*. by 4, 6, 8, 10.
3. £83. 13*s*. $5\frac{1}{2}d$. by 3, 7, 8, 11.
4. £149. 10*s*. $6\frac{3}{4}d$. by 2, 7, 8, 12.
5. £542. 9*s*. $3\frac{1}{4}d$. by 5, 6, 10, 12.
6. £5. 7*s*. 6*d*. by 18, 24, 36.
7. £10. 9*s*. 8*d*. by 16, 22, 28.
8. £15. 10*s*. $4\frac{1}{2}d$. by 25, 32, 35.
9. £25. 16*s*. $3\frac{1}{4}d$. by 28, 56, 63.
10. £41. 5*s*. $10\frac{3}{4}d$. by 35, 45, 55.
11. £115. 18*s*. $7\frac{3}{4}d$. by 36, 42, 54.
12. £450. 4*s*. $10\frac{1}{4}d$. by 40, 56, 110.
13. £963. 19*s*. $5\frac{1}{2}d$. by 81, 121, 75.
14. £4012. 8*s*. $2\frac{1}{4}d$. by 120, 112, 128.
15. £3056. 0*s*. $5\frac{1}{2}d$. by 112, 270, 750.

Multiplication by Large Numbers.

98. When the multiplier is a large number which cannot be easily separated into simple factors, or when the simple factors are numerous, we may proceed as in the following examples.

Example 1. *Multiply* £43. 14*s*. $5\frac{3}{4}d$. *by* 7694.

The multiplier is 7694, or $7000+600+90+4$; we therefore multiply £43. 14*s*. $5\frac{3}{4}d$. by 7000, 600, 90 and 4, and add the products together.

To multiply by 7000, we multiply by the factors 10, 10, 10, and 7 successively. To multiply by 600, we multiply by 10, 10 and 6. To multiply by 90, we multiply by 10 and 9. The work can be arranged concisely as follows:—

£.	*s*.	*d*.		
43 .	14 .	$5\frac{3}{4}$	$\times 4$	(i) = the amount
		10		
437 .	4 .	$9\frac{1}{2}$	$\times 9$	(ii) = 10 times
		10		
4372 .	7 .	11	$\times 6$	(iii) = 100 times
		10		
43723 .	19 .	2	.....	(iv) = 1000 times
		7		
306067 .	14 .	2	.....	(v) = 7000 times
26234 .	7 .	6	.....	(vi) = 600 times
3935 .	3 .	$1\frac{1}{2}$	.....	(vii) = 90 times
174 .	17 .	11	.....	(viii) = 4 times
336412 .	2 .	$8\frac{1}{2}$	.....	(ix) = 7694 times

To get (vi) we multiply (iii) by 6.
To get (vii) ..... (ii) by 9.
To get (viii) ..... (i) by 4.
To get (ix) we add (v), (vi), (vii) and (viii).

NOTE.—When the multiplier is large, as in this case, it is specially important that the method of **Rough Approximation** should be applied, as, in the constant change of multiplier, the pupil is very liable to lose sight of the original question before him. In this case the answer must be between £43 × 7694 and £44 × 7694, *i.e.* between £330842 and £338536. This test would still leave a possible error of nearly £7694, but it would prevent the error being still greater. If, too, the test be applied also to the separate products which make up the final result, the limits of error may be reduced considerably.

The following process may also be used.

Example. ii. *Multiply* £47. 18*s.* 7¾*d.* *by* 253.

Multiply 253 by 3, the number of farthings; the result is 759 farthings. Dividing this by 4, we get 189¾ pence.

Next multiply 253 by 7, the number of pence; the result is 1771 pence, which, added to the 189 pence, makes 1960 pence altogether. Dividing this by 12, we get 163 shillings and 4 pence.

Next, multiplying 253 by 18, the number of shillings, and adding the result to the 163 shillings, we get 4717 shillings, which reduces to £235. 17*s.*

Lastly, multiplying 253 by 47, the number of pounds, and adding the result to the £235, we get £12126.

The complete result is therefore £12126. 17*s.* 4¾*d.*

253
3
4 | 759 farthings
189¾ pence
1771
12 | 1960¾ pence
163*s.* 4¾*d.*
253
2024
20 | 4717*s.*
235£. 17*s.*
1012
1771
£12126. 17*s.* 4¾*d.*

Rough Approximation.—The answer must lie between £47 × 253 and £48 × 253, *i.e.* between £11891 and £12144.

EXAMPLES. 25.

Multiply

1. £5. 16*s.* 7¼*d.* by 59, 61, 89.
2. £23. 4*s.* 6½*d.* by 37, 97, 83.
3. £56. 16*s.* 9¾*d.* by 79, 71, 87.
4. £15. 13*s.* 2¾*d.* by 101, 103, 107.
5. £17. 12*s.* 10½*d.* by 109, 113, 119.
6. £23. 7*s.* 5¼*d.* by 89, 109, 127.
7. £43. 13*s.* 7¾*d.* by 141, 163, 196.

Multiply

8. £41. 7*s.* 8¾*d.* by 726, 841, 963.
9. £6. 4*s.* 5½*d.* by 1036, 5041, 3724.
10. £7. 14*s.* 10*d.* by 7142, 3653, 2003.
11. £7. 12*s.* 7¼*d.* by 6073, 5326, 7932.
12. £4. 5*s.* 6*d.* by 4321, 1576, 3798.
13. £1. 13*s.* 9½*d.* by 3908, 2136, 6800.
14. £8. 13*s.* 4*d.* by 5303, 4008, 7065.
15. £5. 12*s.* 9¾*d.* by 3691, 5086, 2030.
16. £1. 15*s.* 7¾*d.* by 5036, 8921, 7145.
17. £2. 5*s.* 2½*d.* by 7934, 6512, 6074.
18. £3. 18*s.* 4¾*d.* by 1562, 1078, 6324.
19. £2. 3*s.* 2¼*d.* by 8008, 6003, 5401.
20. £5. 0*s.* 10*d.* by 5063, 8320, 11562.

XII. COMPOUND DIVISION.—BRITISH MONEY.

99. **Compound Division**, or the division of compound quantities, is of *two* kinds, which are called **Partition** and **Quotition** respectively.

The *first* is the converse of Compound Multiplication. In *multiplication* we add together a number of equal compound quantities, and find the sum; in *division* we separate a compound quantity into a given number of equal parts, and find the value of each part. The *second* is the determination of how many times one compound quantity is contained in another of the same kind.

In the first case the quotient is a *compound quantity* of the same kind; in the second it is an *abstract number*.

100. We will consider examples of the first kind.

Example. *Divide* £164. 16*s.* 10½*d.* *by* 3.

In simple division we begin at the left hand with the digits of highest order: so in compound division we begin at the left hand with the term of highest denomination. If we divide 164 by 3, the quotient is 54 and the remainder is 2. Hence, if we divide £164 by 3, the quotient is £54 and the remainder is £2.

We convert the remainder £2 into 40 shillings, and add to it the 16*s.* in the dividend; we thus get 56*s.* altogether. If now we divide 56*s.* by 3, the quotient is 18*s.* and the remainder is 2*s.*

Converting the 2*s.* into 24 pence, and adding the 10 pence from the dividend, we get 34 pence altogether. And if we divide 34 pence by 3, we get 11 pence for the quotient and 1 penny over.

Finally, converting the 1 penny into farthings, and adding the 2 farthings from the dividend, we get 6 farthings. Dividing this by 3, the quotient is 2 farthings, and there is no remainder. The result is £54. 18*s.* 11½*d.*

The operation may be stated briefly thus:

	£.	*s.*	*d.*
3	164 .	16 .	10½
	54 .	18 .	11½

101. If we multiply £54. 18*s.* 11½*d.* by 3, we shall find that the product is £164. 16*s.* 10½*d.* In this way a quotient may be verified.

102. The division of £164. 16*s.* 10½*d.* should **not** be performed by reducing it to farthings, dividing the number of farthings by 3, and bringing the quotient back to pounds, shillings and pence, for this process would be absurdly long.

103. The method of Art. 100 can be adopted conveniently only when the divisor is a number for which we should adopt the 'short' process in *simple* division. We cannot lay down a limit beyond which this 'short' method cannot be adopted, for the more practice the student has, and the more skilful he becomes in the use of figures, the farther he may go in the use of the method; but the method should certainly be adopted when the divisor is not greater than 12, or when it can be separated into *factors* which are not greater than 12.

Thus, if we have 56 for a divisor, knowing that $56 = 7 \times 8$, we first divide the given quantity by 7, and then the quotient by 8; for if we divide a quantity into 7 equal parts and each of these parts again into 8 equal parts, the latter parts must be such that 56 of them will make up the original quantity.

Example i. *Divide* £178. 3*s.* 0*d.* *by* 56.

	£.	*s.*	*d.*
7	178 .	3 .	0
8	25 .	9 .	0
	3 .	3 .	7½

The result is £3. 3*s.* 7½*d.* That this is correct may be ascertained by multiplying it by 56.

Rough Approximation gives an answer about £178 ÷ 56, *i.e.* between £3 and £4.

Example ii. *Divide* £12903. 3*s.* 9*d.* by 756.

	£.	*s.*	*d.*
12	12903 .	3 .	9
7	1075 .	5 .	3¾
9	153 .	12 .	2¼
	17 .	1 .	4½

We find that $756 = 12 \times 7 \times 9$.

Rough Approximation gives an answer about £12903 ÷ 756, *i.e.* between £17 and £18.

104. The process by which we obtain the **complete remainder** when we divide a *number* by "short division," may be extended also to the division of a *compound* quantity. As an example we will divide £18. 17*s*. 5*d*. by 15, using the factors 3 and 5 in succession.

	£.	*s*.	*d*.	
3	18	17	5	
5	6	5	9	...2*d*.
	1	5	1	...4*d*.

15 = 3 × 5; 14 pence.

Thus £6. 5*s*. 9*d*. = £1. 5*s*. 1*d*. × 5 + 4*d*.,

and £18. 17*s*. 5*d*. = £6. 5*s*. 9*d*. × 3 + 2*d*.;

∴ £18. 17*s*. 5*d*. = £1. 5*s*. 1*d*. × 5 × 3 + 4*d*. × 3 + 2*d*.

Hence the remainder = (4 × 3 + 2) pence = 14*d*.

EXAMPLES. 26. *Oral.*

Divide

1. £3. 17*s*. 6½*d*. by 2.
2. £479. 5*s*. 3½*d*. by 2.
3. £16. 3*s*. 4½*d*. by 3.
4. £563. 15*s*. 7½*d*. by 3.
5. £78. 4*s*. 5*d*. by 4.
6. £732. 11*s*. 3*d*. by 4.
7. £46. 16*s*. 4¼*d*. by 5.
8. £1032. 7*s*. 8½*d*. by 5.
9. £531. 3*s*. 7½*d*. by 6.
10. £3217. 10*s*. 1½*d*. by 6.
11. £58. 17*s*. 10¾*d*. by 7.
12. £982. 6*s*. 4½*d*. by 7.
13. £741. 11*s*. 10*d*. by 8.
14. £5720. 2*s*. 8*d*. by 8.
15. £835. 1*s*. 4½*d*. by 9.
16. £3794. 19*s*. 9¾*d*. by 9.
17. £4031. 12*s*. 3½*d*. by 10.
18. £8379. 8*s*. 11½*d*. by 10.
19. £5284. 18*s*. 1*d*. by 11.
20. £9305. 17*s*. 3*d*. by 11.
21. £7349. 2*s*. 6*d*. by 12.
22. £18372. 13*s*. 3*d*. by 12.

EXAMPLES. 27.

Divide, by short division,

1. £213. 15*s*. 0*d*. by 15, 18.
2. £1253. 0*s*. 0*d*. by 16, 21.
3. £1207. 15*s*. 0*d*. by 24, 30.
4. £1639. 9*s*. 0¾*d*. by 25, 75.
5. £12056. 5*s*. by 144 and 125.
6. £144281. 3*s*. 1½*d*. by 121 and 210.
7. £829. 12*s*. 7½*d*. by 441 and 343.
8. £98049. 17*s*. 6*d*. by 660 and 231.
9. £12855. 3*s*. by 1008 and 1584.
10. £48131. 0*s*. 3¾*d*. by 1575 and 1925.

11. £55436. 18*s.* 1½*d.* by 5445 and 3267.
12. £530866. 17*s.* 6*d.* by 2772 and 1925.
13. £687044. 10*s.* 7½*d.* by 6237 and 2430.

105. When the divisors are large and cannot easily be separated into small factors, the work may be stated in full as in the margin: the *only difference* is that reductions which have hitherto been done mentally are here written down.

Example.

Divide £130264. 9*s.* 6*d.* *by* 9416.

Rough Approximation gives an answer about £130264 ÷ 9416, *i.e.* between £13 and £14.

The quotient = £13. 16*s.* 8¼*d.*

	£.	*s.*	*d.*
Quotient =	13	16	8¼
9416) 130264	.	9	. 6
	36 04		
	7 56		
	20		
	157129 shillings		
	62969		
	6473		
	12		
	77682 pence		
	2354		
	4		
	9416 farthings		

EXAMPLES. 28.

Divide

1. £58. 1*s.* by 43.
2. £98. 15*s.* by 79.
3. £117. 16*s.* 8*d.* by 101.
4. £169. 6*s.* 8*d.* by 127.
5. £177. 7*s.* 11*d.* by 131.
6. £152. 13*s.* 11½*d.* by 137.
7. £184. 17*s.* 11¾*d.* by 139.
8. £146. 19*s.* 7¾*d.* by 149.
9. £190. 19*s.* 10¼*d.* by 181.
10. £302. 4*s.* 3¼*d.* by 211.
11. £617. 8*s.* 7½*d.* by 223.
12. £538. 17*s.* 9¼*d.* by 227.
13. £946. 8*s.* 6½*d.* by 241.
14. £3262. 4*s.* 3¾*d.* by 251.
15. £6303. 11*s.* 5½*d.* by 271.
16. £13960. 12*s.* 11½*d.* by 401.
17. £9810. 9*s.* 2¾*d.* by 409.
18. £7306. 11*s.* 11¾*d.* by 503.
19. £20766. 0*s.* 3¼*d.* by 613.
20. £46680. 6*s.* 6¼*d.* by 719.
21. £121701. 5*s.* 6½*d.* by 821.
22. £144438. 1*s.* 7½*d.* by 919.
23. £283812. 15*s.* 7½*d.* by 1009.
24. £4420895. 0*s.* 3¾*d.* by 3001.

106. The second kind of division is the determination of how many times one compound quantity is contained in another of the same kind. In this case the quotient will always be an *abstract number*. We first reduce both compound quantities to simple quantities of the same denomination.

For example, to divide £3. 2*s.* 6*d.* by 4*s.* 2*d.*, we reduce both to pence; and we divide the number of pence in £3. 2*s.* 6*d.* by the number of pence in 4*s.* 2*d.*

Example i. *Divide* £3. 2*s.* 6*d.* *by* 4*s.* 2*d.*

s.	*d.*
4	2
12	
50 pence	

£.	*s.*	*d.*
3	2	6
20		
62		
12		
750 pence		

Hence the quotient = 15.

50	750 pence
	15

Example ii. *Divide* £100. 12*s.* 6*d.* *by* £4. 7*s.* 6*d.*

It is sufficient to reduce each of these expressions to *half-crowns.*

£	*s.*	*d.*
4	7	6
8		
35 half-crowns		

£.	*s.*	*d.*
100	12	6
8		
805 half-crowns		

35	5 | 805 half-crowns
	7 | 161
	23

Hence the quotient = 23.

NOTE.—The quantities should not be reduced to a lower denomination than is necessary. In Example (ii) it is not necessary to reduce them to *pence*; half-crowns are low enough.

107. When we wish to find how often one compound quantity is contained in another, we may proceed also as follows. We begin by multiplying the divisor by 10; we then multiply the product obtained by 10, and so on until we reach a stage at which it is clear that the next multiplication by 10 will produce a quantity *greater than* the dividend. At that stage we stop, and we use the products obtained in the manner shewn below.

Example. *How often is* £2. 17*s.* 4½*d.* *contained in* £2194. 11*s.* 10½*d.*?

	£.	*s.*	*d.*
Divisor =	2	17	4½
			10
Divisor × 10 =	28	13	9
			10
Divisor × 100 =	286	17	6

£.	*s.*	*d.*	
2194	11	10½	(765 = Quotient.
2008	2	6	= 700 × Divisor
186	9	4½	
172	2	6	= 60 × Divisor
14	6	10½	
14	6	10½	= 5 × Divisor

On the left we multiply the Divisor by 10, and the result obtained by 10, but we do not multiply by 10 a third time because it is clear that the result would be greater than the Dividend £2194. 11*s.* 10½*d.*

We next ascertain how often £286. 17*s.* 6*d.* will go into the Dividend, and we find that it will go 7 times. The first figure of the Quotient is therefore 7, which denotes 700. Subtracting 7 times £286. 17*s.* 6*d.* from £2194. 11*s.* $10\frac{1}{2}d.$, we get a remainder £186. 9*s.* $4\frac{1}{2}d.$ Ten times the divisor, or £28. 13*s.* 9*d.*, will go into £186. 9*s.* $4\frac{1}{2}d.$ 6 times, and the remainder is £14. 6*s.* $10\frac{1}{2}d.$ This gives the second figure of the quotient as 6, which denotes 60, making so far 760. The Divisor itself will go 5 times exactly into £14. 6*s.* $10\frac{1}{2}d.$, making the complete quotient 765.

EXAMPLES. 29.

Divide

1. £94. 5*s.* by £3. 5*s.*
2. £111. 6*s.* by £2. 2*s.*
3. £195. 15*s.* by £2. 5*s.*
4. £46. 11*s.* 8*d.* by £1. 1*s.* 8*d.*
5. £86. 3*s.* 4*d.* by £1. 16*s.* 8*d.*
6. £131. 16*s.* 3*d.* by £2. 6*s.* 3*d.*
7. £51. 13*s.* $8\frac{1}{4}d.$ by 9*s.* $3\frac{3}{4}d.$
8. £70. 6*s.* 2*d.* by £1. 7*s.* $0\frac{1}{2}d.$
9. £155. 13*s.* $3\frac{1}{4}d.$ by £5. 7*s.* $4\frac{1}{4}d.$
10. £367. 14*s.* $6\frac{1}{4}d.$ by £12. 13*s.* $7\frac{1}{4}d.$
11. £351. 15*s.* $10\frac{1}{2}d.$ by £8. 7*s.* $6\frac{1}{4}d.$
12. £447. 13*s.* $1\frac{3}{4}d.$ by £6. 2*s.* $7\frac{3}{4}d.$
13. £698. 3*s.* 7*d.* by £15. 17*s.* $4\frac{1}{4}d.$
14. £785. 15*s.* 9*d.* by £5. 6*s.* $2\frac{1}{4}d.$
15. £945. 19*s.* $1\frac{1}{2}d.$ by £3. 15*s.* $4\frac{1}{2}d.$
16. £1231. 18*s.* $0\frac{1}{2}d.$ by £2. 5*s.* $6\frac{1}{2}d.$
17. £2760. 8*s.* $7\frac{1}{2}d.$ by £2. 13*s.* $2\frac{1}{4}d.$
18. £2954. 0*s.* $3\frac{3}{4}d.$ by £1. 15*s.* $1\frac{1}{4}d.$
19. £4571. 19*s.* by £3. 2*s.* $5\frac{1}{2}d.$
20. £6030. 6*s.* $11\frac{1}{4}d.$ by £2. 4*s.* $6\frac{1}{4}d.$

XIII. ADDITION, ETC., OF FOREIGN MONEY.

108. In dealing with foreign money systems, in which there are only two denominations, one being 100 times the other, the special processes known as **Compound Rules** are no longer necessary. In addition and subtraction the number carried from one column to another is always the same as that which would be carried in **simple** addition or subtraction. The following example will illustrate this.

Example i. *Add together* 35 *f.* 35 *c.*, 215 *f.* 24 *c.*, 85 *f.* 7 *c. and* 396 *f.* 55 *c.*

We may either express each amount in centimes (*a*), or we may arrange the amounts in columns (*b*).

(*a*)		(*b*)	f.	c.
	3535 c.		35	. 35
	21524 c.		215	. 24
	8507 c.		85	. 7
	39655 c.		396	. 55
Sum =	73221 c.			
=	732 f. 21 c.	Sum =	732	. 21

The student should understand that (*a*) and (*b*) differ only in arrangement: the process is the same in both. Beginning at the right hand, the figure carried from the second column to the third is 1; it is quite immaterial whether we look upon it as 1 *franc* or as 100 *centimes*.

Example ii. *Subtract* 75 *dol.* 15 *c. from* 85 *dol.* 7 *c.*

(*a*)		(*b*)	dol.	c.
	8507 c.		85	. 7
	7515 c.		75	. 15
Diff. =	992 c.			
=	9 dol. 92 c.	Diff. =	9	. 92

109. In multiplication and division it is always best to express the multiplicand and the dividend in terms of the lower denomination, and then to multiply or divide as in 'simple' multiplication or division. The result obtained can easily be expressed in the ordinary form.

Example i. *Multiply* 23 *m.* 7 *pf. by* 51.

23 m. 7 pf. = 2307 pf.

51

Product = 117657 pf.

= 1176 m. 57 pf.

We have multiplied by 51 in one operation as explained in Art. **37**.

Example ii. *Divide* 457 *f.* 70 *c. by* 115.

Quotient = 398 c. = 3 f. 98 c.

115) 45770 c.

345

1127

1035

920

920

EXAMPLES. 30.

Add together

1. 23 f. 5 c.; 115 f. 25 c.; 78 f. 84 c.; 235 f. 76 c.
2. 117 f.; 235 f. 8 c.; 12 f. 54 c.; 567 f. 31 c.
3. 58 m. 20 pf.; 126 m. 9 pf.; 93 m.; 15 pf.; 187 m. 46 pf.
4. 96 dol. 25 c.; 12 dol.; 92 dol. 75 c.; 40 dol. 24 c.; 3 dol. 6 c.

Subtract

5. 127 f. 15 c. from 250 f. 12 c.
6. 4567 f. 75 c. from 5784 f. 14 c.
7. 419 dol. 50 c. from 500 dol. 25 c.
8. 742 m. 18 pf. from 8461 m. 78 pf.

Multiply

9. 18 f. 17 c. by 96.
10. 145 f. 30 c. by 100.
11. 456 f. 75 c. by 213.
12. 55 dol. 85 c. by 57.
13. 978 dol. 8 c. by 23.
14. 847 m. 9 pf. by 347.

Divide

15. 219 dol. 38 c. by 7.
16. 68 f. 40 c. by 15.
17. 191 f. 62 c. by 13.
18. 567 m. 12 pf. by 51.
19. 13603 f. 59 c. by 151.
20. 5424 dol. by 64.

MISCELLANEOUS EXAMPLES. 31.

1. Add together £413. 18*s.* 9*d.*, £129. 5*s.* 2½*d.*, £93. 17*s.* 4¾*d.*, £203. 1*s.* 11¼*d.*, £68. 11*s.* 10¼*d.* and £121. 0*s.* 1*d.*

2. Subtract £7658. 13*s.* 9½*d.* from £93714. 10*s.* 6*d.*

3. Multiply £35746. 13*s.* 6½*d.* by 316.

4. A has 2468 florins and B has 1975 half-crowns. Which has the most money, and by how much?

5. How many times does £43. 6*s.* 8*d.* contain £3. 6*s.* 8*d.*?

6. Three wheels, making 60, 36 and 24 revolutions in a minute, start with certain points in their circumferences downwards. When will they first come all together again in the same position?

7. If I pay £3. 7*s.* 6*d.*, £1. 5*s.* 9*d.* and £4. 3*s.* 4*d.* out of a £10 note, find *in one operation* how much will be left.

---

8. Add together £7296. 14*s.* 8¾*d.*, £417. 9*s.* 7½*d.*, £3246. 17*s.* 4¼*d.*, £29. 8*s.* 9½*d.*, £1877. 3*s.* 11*d.* and £824. 19*s.* 3¾*d.*

9. Reduce £201. 15*s.* 4½*d.* to halfpence.

10. A debt of £3. 17*s.* 6*d.* is paid in farthings. How many are required?

11. How often is £1. 17*s.* 6*d.* contained in £28. 2*s.* 6*d.*?

12. A man's wages for 28 days are £4. 0*s.* 6*d.* What does he earn per day?

13. A man pays into the bank 7 sovereigns, 7 half-sovereigns, 15 half-crowns, 11 florins, 8 shillings and 5 sixpences; how much does he pay in altogether?

14. If I pay £1. 17*s.* 2*d.*, £3. 16*s.* 2¼*d.* and £3. 18*s.* 9½*d.* out of a £10 note, find *in one operation* how much will be left.

---

15. Divide £153. 1*s.* 3¾*d.* by 9, and by 29.

16. Reduce £819. 14*s.* 3½*d.* to farthings.

17. Find the greatest number of which 33495 and 106260 are multiples.

18. The profits of a business, amounting to £904. 11*s.* 4*d.*, are equally divided among the partners. If each receives £56. 10*s.* 8½*d.*, what is the number of partners?

19. Goods were bought for £560. 15*s.* 6*d.*; packing cost £1. 10*s.*; carriage, £12. 6*s.* 8½*d.*; and import duty, £3. 8*s.* 6¾*d.* The goods were then sold for £655. 8*s.* 5*d.* What was the profit?

20. A man collects the following sums which are owing to him, viz., £5. 17*s.* 6*d.*, £2. 3*s.* 4*d.* and £4. 3*s.* 2*d.*, and he uses the money to pay the debts which he owes. He has 9*s.* left when he has paid all; how much did he owe?

21. To pay 15 workmen and a foreman a clerk brings £26. Each man is to draw £1. 10*s.* 4*d.* and the foreman £2. 10*s.* How much money will be left over?

---

22. Find the least number of which 26, 33, 39 and 44 are divisors.

23. After paying out of £100, £20. 13*s.* 4¾*d.* to the butcher, £30. 15*s.* 6½*d.* to the tailor, and £46. 13*s.* 11¼*d.* to the grocer, the remainder was divided among 27 persons. Find the share of each person.

24. A postman, whose pay for a week is 15*s.*, is fined 1*s.* 6*d.* if he comes in late, and at the end of 13 weeks he receives £8. 15*s.* 6*d.* How often was he late?

25. Find the cost of 504 oranges at 7 for 6*d.*

26. A man sells a house for £550 and thus gains £25. 10*s.*; what should he have sold it for so as to gain £75. 10*s.*?

27. £150 is to be divided between two men, one of whom is to receive £25 more than the other; what will each get?

28. The cost of a return ticket from A to B and back is £2. 3*s.* 6*d.*; how many times can a man make the double journey for £91. 7*s.*?

---

29. If William had £3. 17*s.* 6*d.* more than he actually has, he could spend £5 upon a bicycle and would have £1. 2*s.* 6*d.* left; how much has he?

30. If a sheep is bought for £1. 12*s.* 6*d.* and sold for £2. 3*s.* 0*d.*, what will be the profit on a flock of 300?

31. Find the cost of 51 kilograms of silver at 188 fr. 25 c. per kilogram.

32. A man pays 14 f. 75 c. for pamphlets which cost 25 c. each; how many does he buy?

33. A man has in his pocket 100 f. 75 c. and buys 3 articles at 5 f. 25 c. each, 5 at 8 f. 15 c. each and 2 at 12 f. 50 c. each; how much money has he left?

34. Find the cost of 24 metres of silk at 4 f. 25 c. per metre.

35. A merchant spends 820 f. in buying wine at 2 f. 5 c. per litre; how many litres does he buy?

---

36. A man changes a certain sum of English money in New York, and gets for it 70 dol. 20 c., at the rate of 1 dollar for 4*s.* 2*d.*; how many francs would he have got for it in Paris at the rate of 1 franc for 10*d.*?

37. Certain men dined together 247 times, and each man paid 2*s.* 3*d.* for each dinner. The whole sum paid by them was £9142. 1*s.* 9*d.* What was the number of men?

38. To stock a farm I had £2000. I bought 24 cows, 50 oxen, 250 sheep, 142 lambs, 5 farm horses and one hack for my own use. Taking the price of a cow at £15, an ox at £10. 10*s.*, a sheep at £2. 15*s.*, a lamb at 18*s.*, a farm horse at £24 and my own horse at £45, how much money had I left?

39. How many yards of cloth at 2*s.* 1*d.* a yard are equal in value to 75 yards at 5*s.* 7*d.* per yard?

40. The united fares of 17 persons travelling together were £9. 2*s.* 3*d.*, but 7 of them paid only half-fare; what was the full fare?

41. What is the greatest number of articles, each worth £2. 14*s.* 11¼*d.*, which can be purchased with £84? What sum will be left over?

42. A number of articles at 5*s.* 3d. each, and three times that number at 5*s.* 9*d.* each, cost altogether £486. What sum will be gained by selling them all at 6*s.* 6*d.* each?

---

43. If the total revenue of a country whose population is 34,908,560 be £107,780,179, how much does each person pay if they pay equal amounts?

44. I spent £291. 5*s.* on equal quantities of three different articles, costing respectively 13*s.* 3*d.*, 18*s.* 9*d.* and £1. 6*s.* 3*d.* each. How many of each did I buy?

45. A man mixes 400 litres of wine at 7 f. 25 c. per litre, and 154 litres at 9 f. 45 c. per litre, with a certain quantity of water. He sells the mixture at 6 f. 75 c. per litre, and gains in all 585 f. 70 c.; how much water does he add?

46. A sum of 115 f. is divided among four men, so that B receives 3 f. 20 c. more than A, C 2 f. 10 c. more than B, and D 6 f. 50 c. more than A; how much does each man receive?

47. A purse contains a certain number of sovereigns, four times as many shillings, and eight times as many threepenny pieces: the whole sum is £13; how many coins of each kind are there?

48. A tradesman laid out £19. 15*s.* 10*d.* in silk at 3*s.* 2*d.* per yard. Some yards got soiled and were unsaleable; the rest he sold for £21. 10*s.* 10*d.* at 3*s.* 11*d.* per yard. How many yards were soiled?

49. A sum of £50. 8*s.* is made up of sovereigns, half-sovereigns, half-crowns and shillings: if there are twice as many half-sovereigns as sovereigns, 4 times as many half-crowns and 13 times as many shillings, find the numbers of the coins.

XIV. MEASURES OF WEIGHT—BRITISH AND METRIC.

110. The value of a substance depends upon the *nature* of the substance, and also upon the *quantity* of it. We cannot here consider how it depends upon the nature, but we may estimate the quantity in two ways, (i) by **weight**, (ii) by **volume**.

There were formerly two systems of weights used in England, **Avoirdupois Weight** and **Troy Weight**. The former is still in general use; the latter is used only for weighing gold, silver and platinum. In 1898 the Imperial Parliament made lawful the use of the **Metric System** also.

TROY WEIGHT.

24 **grains** (grs.) make 1 **pennyweight** (dwt.),

20 **pennyweights** ..... 1 **ounce** (oz. tr.).

(The Troy pound of 12 ounces is not used now.)

AVOIRDUPOIS WEIGHT.

16 **drams** (drs.)	make 1 **ounce** (oz.),
16 **ounces**	..... 1 **pound** (lb.), or 7000 **grs.**
14 **pounds**	..... 1 **stone** (st.),
28 **pounds**	..... 1 **quarter** (qr.),
4 **quarters**, or 112 **pounds**	..... 1 **hundredweight** (cwt.),
20 **hundredweights**, or 2240 **pounds**	..... 1 **ton.**

111. A grain was originally the weight of a grain of wheat taken from the middle of the ear and well dried.

The word pennyweight is denoted by the symbol *dwt.*, in which the *d* denotes the Latin word *denarius*. The *d* used in denoting pence, and the *d* in the symbol for pennyweight have therefore the same origin.

The symbol *lb.*, which stands for the word *pound*, is a contracted form of the Latin word *libra*; so that the two symbols £ and lb. are contracted forms of the same word, the former being reserved to denote pounds *in money*, and the latter to denote pounds *in weight*.

The *c* in the symbol *cwt.* stands for the Latin word *centum*—a hundred. The English hundredweight originally contained 100 pounds, but was increased to 112 pounds by Edward III. in order that it might be subdivided into 8 stones of 14 lb. By this change the old ton of 20 hundredweights was increased from 2000 pounds to 2240 pounds. The use of the old 100 pound weight has been revived in Canada and the United States; it is called a **Cental**, and is in legal use in our corn trade with those countries.

In compounding medicines a weight of 20 grains is called 1 scruple (scr.), and a weight of 60 grains is called 1 drachm (dr.).

NOTE.—It is sometimes convenient to know that 1 half-crown weighs half an ounce, 3 pennies weigh one ounce, 5 halfpennies weigh one ounce, and 10 farthings weigh one ounce.

112. The **Ounce Troy** must be carefully distinguished from the **Ounce Avoirdupois**;

1 **oz. Troy** = 480 **grains,**

1 **oz. Avoirdupois** = $437\frac{1}{2}$ **grains.**

113. To reduce a weight expressed in tons, cwt., qrs., etc., to pounds or ounces, we proceed on exactly the same principle as in the case of money, (Art. 83). To reduce tons to cwt. we multiply by 20: to reduce cwt. to qrs. we multiply by 4: to reduce quarters to lb. we multiply by 28, and so on. The odd cwt., qrs., lb., etc., in the given weight must be added to the successive products respectively.

Example. *Reduce* 5 *tons* 13 *cwt.* 2 *qrs.* 9 *lb.* 13. *oz. to ounces.*

5 tons 13 cwt. 2 qrs. 9 lb. 13 oz.
20
113 cwt.
4
454 qrs.
28
908
3641
12721 lb.
16
203549 oz. ∴ no. of ounces = 203549.

EXAMPLES. 32. *Oral.*

1. In 5 oz., 8 oz., 9 oz., 11 oz., how many dwt.?
2. In 40 dwt., 60 dwt., 80 dwt., 180 dwt., how many ounces?
3. How many grains are there in 2 dwt., 3 dwt., 4 dwt., 5 dwt., 6 dwt., 10 dwt.?
4. How many dwt. are there in 72 grs., 120 grs., 216 grs., 192 grs.?
5. How many oz. Avoirdupois are there in 3 lb., 5 lb., 6 lb., 8 lb., 20 lb.?
6. Reduce 56 oz., 65 oz., 27 oz. to pounds.
7. How many lb. are there in 5 cwt., 7 cwt., 10 cwt.?
8. How many cwt. are there in 4 tons, 6 tons, 8 tons, 12 tons?
9. How many grains are there in 12 pounds?
10. By how much is 1 oz. of silver heavier than 1 oz. of lead?

EXAMPLES. 33. *Troy.*

Reduce to grains

1. 6 oz. 13 dwt. and 8 oz. 19 dwt.
2. 7 oz. 9 dwt. and 9 oz. 17 dwt.
3. 7 oz. 15 dwt. 16 grs. and 8 oz. 15 dwt. 18 grs.
4. 11 oz. 13 dwt. 14 grs. and 11 oz. 9 dwt. 21 grs.
5. 75 oz. 5 dwt. and 149 oz. 16 dwt.
6. 187 oz. 15 dwt. 23 grs.
7. 716 oz. 19 dwt. 14 grs.
8. 18481 oz. 12 grs.
9. 8976 oz. 1 gr.
10. 12036 oz. 2 dwt.

EXAMPLES. 34. *Avoirdupois.*

Reduce

1. 3 lb. to grains.
2. 271 lb. to grains.
3. 5725 lb. to grains.
4. 8 lb. 4 oz. to ounces.
5. 29 lb. 6 oz. to ounces.
6. 18 lb. 3 oz. to drams.
7. 10 lb. 11 oz. 10 drs. to drs.
8. 1 qr. 3 lb. to ounces.
9. 2 qrs. 13 lb. to ounces.
10. 1 qr. 12 lb. 3 oz. to drs.
11. 3 qrs. 25 lb. 11 oz. 7 drs. to drs.
12. 4 cwt. 1 qr. to lb.
13. 7 cwt. 20 lb. to oz.
14. 15 cwt. 2 qrs. 27 lb. to lb.
15. 5 tons to lb.
16. 3 tons 1 qr. 21 lb. to lb.
17. 113 tons 3 cwt. 15 lb. to lb.
18. 761 tons 16 cwt. 3 qrs. to lb.
19. 7902 tons 3 qrs. 19 lb. to lb.
20. 13 tons 3 qrs. 5 lb. to oz.

114. In the converse operation,—the reduction of drams to tons,—we divide successively by 16, 16, 28, 4 and 20. For if we divide a number of drams by 16, the quotient will represent the number of whole ounces which they make up. If, again, we divide a number of ounces by 16, the quotient will represent the number of whole pounds which they make up; and so on.

Example i. *Reduce* 3256794 *drams to tons.*

$$\begin{array}{rl|l|l} 16 & 4 & 3256794 & \\ & 4 & 814198 & .2 \\ 16 & 4 & 203549 \text{ ounces} & .2 \\ & 4 & 50887 & .1 \\ 28 & 4 & 12721 \text{ pounds} & .3 \\ & 7 & 3180 & .1 \\ & 4 & 454 \text{ quarters} & .2 \\ & 20 & 113 \text{ cwt.} & .2 \text{ qrs.} \end{array} \quad \begin{array}{l} \\ \\ 10 \text{ drams} \\ \\ 13 \text{ ounces} \\ \\ 9 \text{ pounds} \\ \\ \end{array}$$

5 tons 13 cwt. 2 qrs. 9 lb. 13 oz. 10 drs.

Example ii. *Reduce* 1537602 *grains to ounces Troy.*

We divide 1537602 *grains* by 24, and the quotient will denote *dwt.*; then the no. of *dwt.* by 20, and the quotient will denote *ounces*; the remainders, if any, must be written down after each division.

$$\begin{array}{rl|l|l} 24 & 4 & 1537602 & \\ & 6 & 384400 & .2 \\ & 20 & 64066 \text{ dwt.} & .4 \end{array} \quad \begin{array}{l} \\ \\ 18 \text{ grains} \end{array}$$

3203 oz. 6 dwt. 18 grs.

EXAMPLES. 35. *Troy.*

Reduce

1. 176 and 479 grs. to dwt.
2. 1451 and 2378 grs. to dwt.
3. 987 and 760 dwt. to oz.
4. 3715 and 5621 grs. to oz.
5. 8720 and 23476 grs. to oz.
6. 415 and 4721 dwt. to oz.
7. 5763 and 6307 dwt. to oz.
8. 14762 and 47285 grs. to oz.
9. 357601 grs. to oz.
10. 634792 grs. to oz.
11. 1475103 grs. to oz.
12. 1562489 grs. to oz.
13. 84320571 grs. to oz.
14. 93745291 grs. to oz.

EXAMPLES. 36. *Avoirdupois.*

Reduce

1. 150 oz. to lb.
2. 762 drams to lb.
3. 76043 drams to lb.
4. 76320 lb. to cwt.
5. 4576201 lb. to cwt.
6. 14030101 lb. to cwt.
7. 98746580 lb. to cwt.
8. 12713 oz. to cwt.
9. 100000 oz. to cwt.
10. 376210 oz. to cwt.
11. 1432014 oz. to cwt.
12. 153479 oz. to cwt.
13. 5430114 oz. to cwt.
14. 8760328 qrs. to tons.
15. 43705 lb. to tons.
16. 54031 lb. to tons.
17. 5462780 lb. to tons.
18. 14763051 lb. to tons.
19. 137620172 oz. to tons.
20. 7603254301 oz. to tons.

115. In the **Metric System** weights are expressed in **grams**, multiples of a gram, and parts of a gram. The relations between them are given in the following table:

METRIC MEASURES OF WEIGHT.

10 **milligrams** (mg.)	= 1	**centigram** (cg.).
10 **centigrams**	= 1	*decigram* (dg.).
10 *decigrams*	= 1	**gram** (g.).
10 **grams**	= 1	*decagram* (Dg.).
10 *decagrams*	= 1	*hectogram* (Hg.).
10 *hectograms*	= 1	**kilogram** (Kg.).

Each weight is 10 times the next smaller weight.

The prefixes *milli-*, *centi-*, *deci-*, *deca-*, *hecto-* and *kilo-* must be carefully noticed, as they are used also in other metric tables. The LATIN prefixes **milli-**, **centi-** and **deci-** belong to measures which are **less than** the unit (a gram); and the GREEK prefixes

kilo-, **hecto-**, **deca-** belong to **multiples** of the unit. As the prefixes *milli-* and *kilo-* refer to the number 1000, *centi-* and *hecto-* to the number 100, *deci-* and *deca-* to the number 10, the relation between a given measure and the unit should never be in doubt. For the larger measures,

deca- denotes 10 times the unit,
hecto- ..... 100 .....,
and *kilo-* ..... 1000 ..... ;

and for the smaller measures,

10 times a *deci-* is equal to the unit,
100 ..... *centi-* .....,
and 1000 ..... *milli-* .....

Thus 1000 mg. = 1 g., 100 cg. = 1 g.,
1000 g. = 1 Kg., 100 g. = 1 Hg.

The weights which are most commonly used are the **milligram**, the **centigram**, the **gram**, and the **kilogram**.

For heavy weights a **quintal** and a **tonne** are also used:

1 **quintal** (q.) = 100 **kilograms** ($= 10^5$ grams),
1 **tonne** (t.) = 1000 **kilograms**.

116. In the metric system of weights and measures, the underlying idea is to make full use of the relation which exists between the place-values of the digits in the common system of notation. The measures of money, in which there are only two denominations (francs and centimes, for example), do not shew this so fully as the measures of weight, length, etc.

One of the consequent advantages of the system is that reduction from one denomination to the next lower, or to the next higher, denomination is effected at once by multiplying or by dividing by 10, an operation which makes no change in the relative order of the digits.

For example,

7 g. 3 dg. = 70 dg. + 3 dg. = 73 dg. ;
7 g. 3 dg. 4 cg. = 700 cg. + 30 cg. + 4 cg. = 734 cg. ;
7 g. 3 dg. 4 mg. = 7000 mg. + 300 mg. + 4 mg. = 7304 mg.

In the last case it must be noticed how the absence of the denomination cg. is represented by the digit 0 in 7304 mg. Again,

57 g. 27 mg. = 57000 mg. + 27 mg. = 57027 mg.,

and 5 Kg. 3 Dg. 8 g. = 5 Kg. 0 Hg. 3 Dg. 8 g. = 5038 g.

Hence we see that to reduce to a lower denomination, we write the figures in the same order, inserting a 0 where any denomination is absent.

Conversely, to reduce to the next *higher* denomination we *divide* by 10. If, for example, we divide 57845 by 10, the quotient is 5784, and the remainder is 5; thus:

$$\begin{aligned} 57845 \text{ mg.} &= 5784 \text{ cg. } 5 \text{ mg.} \\ &= 578 \text{ dg. } 4 \text{ cg. } 5 \text{ mg.} \\ &= 57 \text{ g. } 8 \text{ dg. } 4 \text{ cg. } 5 \text{ mg.} \end{aligned}$$

The connection between the metric system of weights and measures and the common system of representing numbers may be seen also from the following comparison.

The number $5025 = 5 \times 10^3 + 0 \times 10^2 + 2 \times 10 + 5,$

the weight 5025 g. = 5 Kg. + 0 Hg. + 2 Dg. + 5 g.,

and the weight 5025 mg. = 5 g. + 0 dg. + 2 cg. + 5 mg.

NOTE.—The decigram, Decagram and Hectogram are seldom used.

For	5 dg. 3 cg.	it is customary to write	53 cg.;
for	3 Dg. 7 g.	,,	,,	,,	37 g.;
and for	2 Hg. 7 Dg. 5 g.	,,	,,	,,	275 g.

NOTE.—For comparison with the British system, it should be remembered that

1 Kilogram = 2⅕ lb. nearly.

EXAMPLES 37. *Oral.*

1. Express in centigrams:

   5 g. 25 cg.; 27 g. 3 dg. 7 cg.; 8 g. 5 cg.; 2 Dg. 3 g.; 9 Dg. 27 cg.; 5 Hg. 3 Dg. 5 g. 8 cg.; 35 g. 15 cg.; 10 g. 5 dg.

2. Express in milligrams:

   3 dg. 5 cg. 7 mg.; 15 dg. 25 mg.; 8 g. 2 cg. 9 mg.; 10 g. 5 mg.; 13 g. 7 dg. 5 cg.; 25 g. 5 dg.; 35 g. 15 cg. 5 mg.

3. Express in grams:

   5 Kg. 7 Hg. 5 Dg. 1 g.; 15 Kg. 7 Dg. 3 g.; 7 Kg. 25 g.; 12 Kg. 8 g.; 9 Hg. 8 g.; 4 Kg. 2 Hg.; 9 Kg. 3 Dg.; 14 Kg. 75 g.

4. Express in grams, decigrams, centigrams and milligrams:

   256 cg., 105 cg., 2056 cg., 1875 mg., 715 cg., 756 dg., 2157 mg., 7500 dg., 500 mg., 93471 mg.

5. Express in Kg., Hg., Dg. and g.:

   451 g., 5005 g., 2001 Dg., 575 Hg., 7564 g., 1500 g., 7568 Dg., 57645 g., 7684 Hg., 92180 g.

6. Express in t., q. and Kg.:

7564 Kg., 5764 Kg., 8000 Kg., 5300 Kg., 8467 Kg., 50105 Kg., 1057 Kg., 6740105 Kg.

XV. MEASURES OF LENGTH.—BRITISH AND METRIC.

117. In England **lengths** are commonly measured with reference to a unit which is called a **yard**.

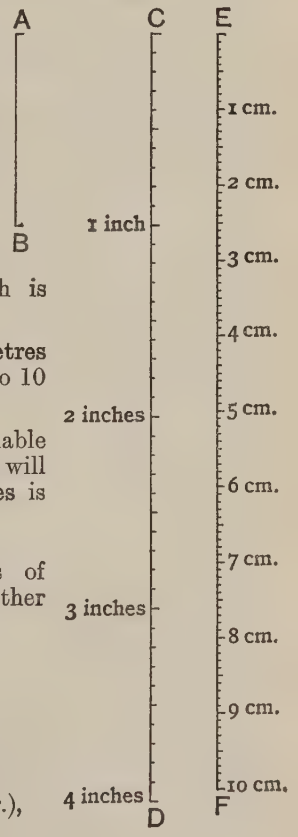
The *yard* is divided into 36 equal parts, each of which is called an **inch**. The line AB in the margin is exactly an inch in length.

In the Metric System lengths are measured with reference to a unit called a **metre**. This unit is divided into 100 equal parts, each of which is called a **centimetre**.

The straight line CD in the margin is **4 inches** in length, and each inch is divided into 10 equal parts.

The straight line EF is **10 centimetres** long, and each centimetre is divided into 10 equal parts called **millimetres**.

These two scales placed side by side enable one to compare the units employed. It will be seen that a length of **5 centimetres** is nearly equal to **2 inches**.



118. The other English measures of length, and their relations to one another are given in the following table:

LINEAR MEASURE.

12 **inches** (in.)	make	1 **foot** (ft.),
3 **feet**	.....	1 **yard** (yd.),
22 **yards**	.....	1 **chain** (ch.),
10 **chains**, or 220 **yards**	.....	1 **furlong** (fur.),
8 **furlongs**, or 1760 **yards**	.....	1 **mile** (mi.).

A measure called a **rod** or **pole** is also used, and is equivalent to $5\frac{1}{2}$ yards, or 5 yds. 1 ft. 6 in. Thus

4 **poles** make 1 **chain**,
and 40 **poles** ..... 1 **furlong**.

A chain is also divided into 100 equal parts, each of which is called a **link**; thus

100 **links** make 1 **chain**.

NOTE.—A halfpenny measures exactly one inch across.

119. The following measures of length are used but seldom, and only for special purposes:

3 **barleycorns** = 1 **inch**; 72 **points** = 1 inch; 12 **lines** = 1 inch; 1 **nail** = $2\frac{1}{4}$ inches; 1 **palm** = 3 inches; 1 **hand** = 4 inches; 1 **span** = 9 inches; 1 **cubit** = 18 inches: 1 **pace** (or 2 steps) = 5 feet; 1 **English ell** = $1\frac{1}{4}$ yards; 1 **fathom** = 6 feet; 1 **cable's length** = 120 fathoms; 1 **league** = 3 miles; 1 **knot** or **geographical mile** = 6080 feet; 1 **degree of latitude** = 60 knots.

120. The reduction of lengths is performed similarly to the reduction of money and weights (Arts. 83 and 114).

Example i. *Reduce* 3 *m.* 1 *fur.* 4 *ch.* 9 *yds.* 0 *ft.* 2 *in. to inches.*

m. fur. ch. yds. ft. in.
3 . 1 . 4 . 9 . 0 . 2
8
25 furlongs
10
254 chains
22
508
517
5597 yards
3
16791 feet
12
201494 inches.

Example ii. *Reduce* 201494 *inches to miles.*

12 | 201494 inches
3 | 16791 ft. 2 in.
22 { 11 | 5597 yds. 0 ft. 2 in.
2 | 508 . 9
10 | 254 ch. . 0 } 9 yds. 0 ft. 2 in.
8 | 25 fur. 4 ch. 9 yds. 0 ft. 2 in.
3 miles 1 fur. 4 ch. 9 yds. 2 in.

NOTE.—Since 1 *chain* = 4 *poles*, this can be expressed otherwise as

3 mi. 1 fur. 16 poles 9 yds. 2 in.,
or 3 mi. 1 fur. 17 poles 3 yds. 1 ft. 8 in.

121. The relations between the measures of length in the **Metric System** are given in the following table. It will be noticed that the same prefixes occur as in the measures of weight, and that they have the same meanings: the only difference between the table of lengths and the table of weights is that the word **metre** is substituted for the word *gram*.

Metric Measures of Length.

10 **millimetres** (mm.) make 1 **centimetre** (cm.).
10 **centimetres** ..... 1 *decimetre* (dm.).
10 *decimetres* ..... 1 **metre** (m.).
10 **metres** ..... 1 *decametre* (Dm.).
10 *decametres* ..... 1 *hectometre* (Hm.).
10 *hectometres* ..... 1 **kilometre** (Km.).

Hence 1000 mm. = 1 metre, 1 Dm. = 10 metres,
100 cm. = 1 metre, 1 Hm. = 100 metres,
10 dm. = 1 metre, 1 Km. = 1000 metres.

122. The process of reduction of lengths is the same as that of weights. The following examples will illustrate it.

Example i. *Reduce* 3 *m.* 2 *dm.* 3 *cm. to centimetres.*

Since 1 m. = 100 cm., and 1 dm. = 10 m., it follows that
3 m. 2 dm. 3 cm. = 300 cm. + 20 cm. + 3 cm. = 323 cm.

Example ii. *Reduce* 4 *Dm.* 5 *dm.* 3 *mm. to millimetres.*

Since 1 Dm. = 10 m. = 10000 mm., and 1 dm. = 100 mm., it follows that
4 Dm. 5 dm. 3 mm. = 40000 mm. + 500 mm. + 3 mm. = 40503 mm.

From these cases it will be seen that the rule for reduction is the same as before: *Write down the figures in the order in which they occur, putting* 0 *in the place of any denomination which is absent.*

Example iii. *Express* 56845 *cm. in metres etc.*

$$56845 \text{ cm.} = (5 \times 10^4 + 6 \times 10^3 + 8 \times 10^2 + 4 \times 10 + 5) \text{ cm.}$$
$$= 5 \text{ Hm. } 6 \text{ Dm. } 8 \text{ m. } 4 \text{ dm. } 5 \text{ cm.};$$

or thus, 56845 cm. = 568 m. 45 cm.
= 5 Hm. 6 Dm. 8 m. 4 dm. 5 cm.

123. Of the metric measures of length, the *decimetre*, *decametre* and *hectometre* are little used. The *decimetre*, however, is used indirectly in the construction of the unit of capacity (**litre**), and the *decametre* in that of the unit of area (**are**).

EXAMPLES. 38. *Oral.*

1. How many inches in 2 ft., 1 yd., 3 yds., 4 yds.?
2. How many feet in 48, 60, 72, 90, 100 inches?
3. How many yards in 72, 144, 108 inches?
4. How many yards in 1 furlong? How many feet?

5. How many chains in 757, 1015, 8465 links?
6. How many chains in 44, 110, 88, 66, 132 yards?
7. How many miles in 88, 40, 32, 440, 96 furlongs?
8. How many centimetres in
   3 m. 5 dm. 7 cm.; 3 m. 15 cm.; 10 m. 6 dm.; 20 m. 5 cm.; 7 m. 8 dm. 5 cm.; 10 m.; 8 m. 5 dm.; 30 m. 8 cm.?
9. How many metres in
   7 Km. 5 Hm.; 6 Hm. 3 Dm. 5 m.; 7 Km. 5 Dm.; 6 Dm. 5 m.; 7 Hm. 8 m.; 5 Km. 3 Hm. 5 Dm.; 70 Km. 13 m.?
10. Express in Km., Hm., etc.:
    57 m., 846 m., 8001 m., 5060 m., 8105 m. 375015 cm
11. Express in m., dm., cm., mm.:
    84 cm., 96 cm., 115 cm., 75 cm., 1184 cm., 1005 cm., 64 mm., 105 cm., 1256 mm., 3050 mm., 5000 mm.

EXAMPLES. 39.

Reduce

1. 3 yds. 1 ft. to inches.
2. 3 fur. 3 ch. 19 yds. to inches.
3. 4 fur. 4 yds. 2 ft. 5 in. to inches.
4. 1 mi. 3 fur. 2 ch. 11 yds. 2 ft. to feet.
5. 1 mi. 7 fur. 1 ft. to feet.
6. 2 mi. 3 fur. 18 yds. 1 ft. 6 in. to inches.
7. 2 mi. 5 fur. 3 yds. 1 ft. 7 in. to inches.
8. 5 mi. 2 fur. 2 yds. 10 in. to inches.
9. 13 mi. 6 fur. 1 yd. 2 ft. 9 in. to inches.
10. 45 mi. 7 fur. 3 yds. 11 in. to inches.
11. 51 mi. 2 yds. to inches.
12. 173 mi. 7 in. to inches.
13. 87354 in. to feet, etc.
14. 74310 in. to yards, etc.
15. 67501 in. to chains, etc.
16. 58793 in. to furlongs, etc.
17. 97412 in. to miles, etc.
18. 297610 in. to miles, etc.

Addition and Subtraction of Weights and Measures.

124. In the addition of weights and measures the principle is the same as in the addition of money. We write them one under another, arranging them so that all terms of the same denomination are in a column, and so that each column is of a lower denomination than the column to the left of it.

Example i. *Add together* 4 *cwt.* 2 *qrs.* 12 *lb.*, 17 *cwt.* 3 *qrs.* 10 *lb.*, *and* 12 *cwt.* 1 *qr.* 8 *lb.*

The sum of the pounds-column is 30 lb., or 1 qr. 2 lb. We write down **2** lb. at the foot, and transfer the 1 qr. to increase the next column.

The sum of the quarters-column is now 7 qrs., or 1 cwt. 3 qrs. We therefore write down **3** qrs. at the foot, and transfer the 1 cwt. as before.

ton	cwt.	qrs.	lb.
	4	2	12
	17	3	10
	12	1	8
1	14	3	2

Example ii. *Add together* 5 *ch.* 20 *yds.* 1 *ft.* 2 *in.*, 15 *yds.* 9 *in.*, 4 *ch.* 8 *yds.* 2 *ft.* 3 *in.*, 7 *ch.* 15 *yds.* 10 *in.*

The sum of the inches-column is 24 *inches*, or 2 *ft.* We write down **0** at the foot of the inches-column, and we transfer 2 *feet* to increase the feet-column.

The sum of the feet-column is now 5 *ft.*, or 1 *yard* 2 *ft.* We write down **2** at the foot of the feet-column, and we transfer 1 *yard* to the yards-column.

The sum of the yards-column is now 59 *yards*, or 2 *chains* 15 *yards*. We write down **15** at the foot of the yards-column, and we transfer 2 *chains* to increase the chains column.

fur.	ch.	yds.	ft.	in.
5	20	1	2	
	15	0	9	
4	8	2	3	
7	15	0	10	
1	8	15	2	0

The sum of the chains-column is 18 *chains*, or 1 *fur.* **8** *chains*. The result is therefore 1 *fur.* 8 *ch.* 15 *yds.* 2 *ft.*

Subtraction is performed in a similar way.

125. To add quantities expressed in the Metric System we may proceed as in the following example. In (*a*) the measures are placed one under another in columns; in (*b*) each is reduced to the lowest denomination.

In arranging the quantities in (*a*) the denominations are placed in the order in which they occur in the metric tables, the lowest denomination being placed at the right hand. If any denomination is absent, a figure **0** must be inserted in its place; and if any quantities occur such as 65 cm. or 5 Km. 86 m., they must be replaced by 6 dm. 5 cm. and 5 Km. 8 Dm. 6 m. respectively. In (*b*) the quantities are at once reduced to a common denomination. This is the safer and the better plan to adopt, especially for a beginner. In multiplication and in division it should always be done.

Example i. *Add together* 5 *m.* 3 *dm.* 5 *cm.*, 37 *dm.* 25 *mm.*, 3 *m.* 8 *mm.*

(*a*)

Dm.	m.	dm.	cm.	mm.
	5	3	5	0
	3	7	2	5
	3	0	0	8
Sum = 1	2	0	8	3

(*b*)

mm.
5350
3725
3008

Sum = 12083 mm.
= 1 Dm. 2 m. 8 cm. 3 mm.

Example ii. *Subtract* 5 *Kg.* 36 *g.* 4 *mg. from* 13 *Kg.* 23 *dg.* 2 *mg.*

	Kg.	Hg.	Dg.	g.	dg.	cg.	mg.
(*a*)	13	0	0	2	3	0	2
	5	0	3	6	0	0	4
Difference =	7	9	6	6	2	9	8

(*b*) 13002302 − 5036004 = Diff. = 7966298

The difference may be expressed as 7 Kg. 966 g. 298 mg.

EXAMPLES. 40.

Add together

1.

dwt.	grs.
1	10
13	5

2.

oz.	dwt.	grs.
3	6	12
5	9	10
1	2	15

3.

oz.	dwt.	grs.
5	11	21
10	5	18
8	19	23

4.

oz.	dwt.	grs.
9	16	3
10	9	17
9	15	23
11	13	22
7	4	19

5.

oz.	dwt.	grs.
1	15	10
5	10	0
11	17	22
3	15	21
10	4	15

6.

oz.	drs.
3	10
5	6
2	12
7	4
5	8

7.

oz.	drs.
5	1
6	3
1	14
2	15
8	5

8.

lb.	oz.	drs.
3	5	12
12	10	14
10	12	15
2	3	10
	2	8

9.

lb.	oz.	drs.
5	3	10
3	8	15
10	12	8
1	12	9
12	4	8

10.

cwt.	qrs.	lb.
5	1	17
13	1	5
		21
	2	19
15	3	8
12	2	16

11.

cwt.	qrs.	lb.
8	1	15
4	2	14
5	3	21
10	0	5
12	2	27
7	3	8

12.

tons.	cwt.	qrs.	lb.	oz.
3	15	0	12	5
5	6	1	5	11
1	19	3	24	13
8	8	0	26	9
21	17	2	2	15
35	14	1	0	4

13.

tons.	cwt.	qrs.	lb.	oz.
7	13	0	14	8
2	10	3	21	12
13	2	1	26	4
32	15	0	7	15
	3	1	10	0
			25	8

14.

g.	cg.
78	5
14	37
5	78
6	89
24	43
17	6

15.

cg.	mg.
54	5
32	8
4	6
12	4
23	3
10	2

16.

g.	dg.	cg.	mg.
2	0	5	8
9	4	6	3
7	8	4	8
5	2	3	7
4	1	0	3

17.

Hg.	Dg.	g.
5	8	7
6	3	5
	2	4
		8
4	0	6

18.

Kg.	g.
7	32
8	112
	85
3	520
4	675

19.

Kg.	Hg.	Dg.	g.
	3	5	4
7	8	6	0
4	0	1	5
	7	0	3
1	2	1	2

20.

t.	q.	Kg.
	6	15
3	4	9
	8	75
7	0	37
5	3	6

21.

t.	q.	Kg.	Hg.
	5	5	8
4	0	37	1
3	6	84	3
1	3	9	9
	8	92	4

EXAMPLES. 41.

Add together

1.

yds.	ft.	in.
5	2	10
	1	9
3	0	6
2	1	7

2.

yds.	ft.	in.
15	1	8
7	2	5
	2	11
8	1	4

3.

ch.	yds.	ft.	in.
5	9	1	3
	15	2	9
7	20	0	7
3	7	2	11

4.

mi.	fur.	ch.	yds.
1	3	9	19
	6	5	17
4	0	2	6
5	7	7	21

5.

mi.	fur.	yds.
3	4	125
	7	95
8	0	211
7	5	64
2	3	115

6.

fur.	ch.	yds.	ft.
3	5	15	2
7	4	9	1
5	8	13	2
3	7	20	0
	9	17	2

7.

m.	cm.	mm.
3	54	7
4	7	5
7	75	6
6	44	3

8.

m.	dm.	cm.	mm.
	3	7	5
3	7	5	6
6	5	7	0
4	0	3	1

9.

Km.	m.	cm.
3	115	25
7	69	74
5	521	0
9	672	84

10.

Km.	Hm.	m.
7	5	24
8	2	87
	7	95
9	2	15
5	3	76

11.

Km.	Hm.	m.	cm.
7	2	5	75
	3	7	15
8	7	6	56
	9	7	33
6	6	6	66

12.

Dm.	m.	cm.	mm.
5	4	27	8
	9	92	4
7	7	8	0
9	8	76	9
	2	54	7

EXAMPLES. 42.

Subtract

1. 5 dwt. 18 grs. from 12 dwt. 9 grs.
2. 13 dwt. 8 grs. from 5 oz. 11 dwt. 3 grs.
3. 8 oz. 7 dwt. 23 grs. from 11 oz. 2 dwt. 10 grs.
4. 6 oz. 15 dwt. 17 grs. from 12 oz.
5. 189 oz. 20 grs. from 391 oz. 12 dwt.
6. 14 oz. 10 drs. from 15 oz. 8 drs.
7. 4 lb. 3 oz. 4 drs. from 6 lb. 1 oz. 2 drs.
8. 22 lb. 10 oz. 14 drs. from 27 lb. 9 oz. 8 drs.
9. 8 cwt. 2 qrs. 15 lb. from 10 cwt. 2 qrs. 14 lb.
10. 2 cwt. 3 qrs. 21 lb. from 16 cwt. 5 lb.
11. 17 cwt. 2 qrs. 14 lb. from 1 ton.
12. 3 tons 15 cwt. 3 qrs. 21 lb. from 8 tons 1 qr.
13. 3 cg. 5 mg. from 5 cg. 2 mg.
14. 25 cg. 7 mg. from 50 cg.
15. 5 g. 15 cg. 3 mg. from 7 g. 25 cg. 1 mg.
16. 5 Kg. 7 Dg. 5 g. from 9 Kg. 2 Dg. 7 g.
17. 12 Kg. 35 g. from 15 Kg. 15 g.
18. 3 Kg. 2 dg. 5 cg. from 6 Kg. 2 g. 15 mg.
19. 2 t. 8 q. 54 Kg. from 5 t.

EXAMPLES. 43.

Subtract

1. 3 yds. 1 ft. 9 in. from 5 yds. 0 ft. 6 in.
2. 6 yds. 2 ft. 8 in. from 7 yds.
3. 8 ch. 17 yds. 1 ft. from 9 ch. 9 yds. 2 ft.
4. 4 mi. 3 fur. 7 ch. from 10 mi. 7 fur. 3 ch.
5. 7 fur. 5 ch. 19 yds. 2 ft. from 1 mi. 1 fur. 7 ch. 5 yds.
6. 2 mi. 5 fur. 175 yds. from 3 mi. 2 fur. 26 yds.
7. 9 ch. 20 yds. 2 ft. 10 in. from 1 fur.
8. 7 fur. 8 ch. 19 yds. 1 ft. 8 in. from 1 mi.
9. 10 cm. 5 mm. from 31 cm. 7 mm.
10. 3 m. 7 dm. 8 cm. 2 mm. from 4 m. 35 cm.
11. 2 Km. 8 Hm. 75 m. from 5 Km. 2 Hm. 7 m.

12. 19 Km. 7 Hm. 8 Dm. 7 m. from 20 Km. 32 m.
13. 36 m. 25 cm. from 5 Dm.
14. 14 Hm. 75 m. 12 cm. from 1 Km. 9 Hm. 2 Dm. 3 m. 5 cm.
15. 26 Km. 4 m. 1 cm. from 92 Km. 3 Hm. 5 mm.

Multiplication and Division of Weights and Measures.

126. The multiplication of weights and measures is effected in the same way as in the case of money (Art. 97).

Example i. *Multiply* 5 *cwt.* 3 *qrs.* 14 *lb.* *by* 8.

We see that 8 times 14 lb. = 112 lb., or exactly 4 qrs. Hence we write down 0 under the lb., and carry 4 qrs. forward.

Again, 8 times 3 qrs. = 24 qrs., which become 28 qrs. when we add the 4 qrs.: and 28 qrs. are exactly 7 cwt. Hence we write down 0 under the qrs., and carry forward 7 cwt.

Lastly, 8 times 5 cwt. = 40 cwt., which become 47 cwt. when we add the 7 cwt. brought forward.

cwt.	qrs.	lb.	
5	3	14	
		8	
47	0	0	= 2 tons 7 cwt.

NOTE.—A rough approximation gives an answer between 8 times 5 cwt. and 8 times 6 cwt., *i.e.* between 40 cwt. and 48 cwt.

Example ii. *Multiply*

15 *cwt.* 2 *qrs.* 20 *lb.* *by* 72.

The multiplier is the same as 8×9; therefore we multiply the given quantity by 8, and the product by 9.

tons	cwt.	qrs.	lb.
	15	2	20
			8
6	5	1	20
			9
56	8	3	12

Example iii. *Multiply*

4 *oz.* 12 *dwt.* 3 *grs.* *by* 1242.

oz.	dwt.	grs.	
4	12	3 × 2	
		10	
46	1	6 × 4	
		10	
460	12	12 × 2	
		10	
4606	5	0	= 1000 times
921	5	0	= 200 times
184	5	0	= 40 times
9	4	6	= 2 times
5720	19	6	= 1242 times

NOTE.—In (ii) a rough approximation gives an answer between 15 cwt. × 72 and 16 cwt. × 72, *i.e.* between 1080 cwt. and 1152 cwt., *i.e.* between 54 tons and 57 tons 12 cwt.

In (iii) a rough approximation gives an answer between 1242 times 4 *oz.* and 1242 times 5 *oz.*, *i.e.* between 4968 *oz.* and 6210 *oz.*

127. When the multiplier is a large number we may apply the method given in Art. 98.

Example. *Multiply* 3 *fur.* 7 *ch.* 18 *yds.* 1 *ft.* 6 *in. by* 451.

3 fur. 7 ch. 18 yds. 1 ft. 6 in.

451

12 | 2706 in.

225 ft. 6 in.

451 ft. = 1 ft. × 451

3 | 676 ft.

225 yds. 1 ft.

451 yds. = 10 yds. × 451

3608 yds. = 8 yds. × 451

Rough Approximation.— The answer must lie between 3 × 451 fur. and 4 × 451 fur., *i.e.* between 169 mi. 1 fur. and 225 mi. 4 fur.

22 { 2 | 8343 yds.
11 | 4171 . 1 } 5 yds.
379 ch. . 2

3157 ch. = 7 ch. × 451

10 | 3536 ch.

353 fur. 6 ch.

1353 fur. = 3 fur. × 451

8 | 1706 fur.

213 mi. 2 fur. 6 ch. 5 yds. 1 ft. 6 in.

128. To multiply weights and measures expressed in the metric system we reduce them to the lowest denomination mentioned, and we then proceed as with ordinary numbers.

Ex. i. *Multiply* 5 *g.* 5 *cg.* 2 *mg. by* 21.

5052 mg.
21
106092 mg. = 1 Hg. 6 g. 9 cg. 2 mg.

Ex. ii. *Multiply* 13 *Km.* 25 *m. by* 200.

13025 m.
200
2605000 m. = 2605 Km

EXAMPLES. 44.

Multiply

1. 164 oz. 7 dwt. by 8, 9, 12.
2. 41 oz. 16 dwt. 12 grs. by 3, 7, 10.
3. 5 oz. 12 dwt. 8 grs. by 28, 35, 55.
4. 190 oz. 12 grs. by 42, 96, 128.
5. 165 oz. 10 dwt. by 101, 103, 107.
6. 167 oz. 12 dwt. 17 grs. by 2134, 3268, 5001.
7. 21 lb. 10 oz. 5 drs. by 7, 9, 10.
8. 18 cwt. 3 qrs. 14 lb. by 6, 9, 11.

9. 10 cwt. 1 qr. 14 lb. by 72, 63, 220.
10. 14 tons 12 cwt. 14 lb. by 81, 540, 560.
11. 15 cwt. 2 qrs 14 oz. by 59, 204, 760.
12. 10 tons 10 cwt. 2 qrs. 8 lb. by 976, 1083, 2311.
13. 5 g. 7 dg. 5 cg. by 8, 11, 15.
14. 7 Dg. 8 dg. 3 cg. 4 mg. by 7, 18, 21.
15. 57 Kg. 45 g. by 230, 500, 630.
16. 8 t. 7 q. 75 Kg. 80 g. by 800, 944, 984.
17. 8 Kg. 7 Hg. 4 g. 5 dg. by 900, 1125, 7324.
18. 3 yds. 1 ft. 6 in. by 7, 8, 9.
19. 10 yds. 2 ft. 8 in. by 11, 13, 14, 19.
20. 5 ch. 15 yds. 1 ft. by 24, 30, 39.
21. 3 fur. 9 ch. 21 yds. by 54, 217, 315.
22. 4 mi 7 fur. 8 ch. 5 yds. by 7, 49, 343.
23. 24 mi 6 fur. 154 yds. by 341, 501, 722.
24. 5 fur 7 ch. 10 yds. 1 ft. by 25, 921, 1234.
25. 10 cm. 5 mm. by 6, 15, 24, 35.
26. 8 m. 54 cm. by 21, 54, 127.
27. 4 Hm. 27 m. 5 cm. by 92, 341, 5672.
28. 3 Km. 8 Hm. 7 Dm. 6 m. by 84, 481, 1842.

129. The following examples illustrate the division of weights and measures.

Example i. *Divide* 7709 *oz.* 6 *dwt.* *by* 48.

We know that $48 = 6 \times 8$.

	oz.	dwt.	grs.
6 |	7709 .	6 .	0
8 |	1284 .	17 .	16
	160 .	12 .	5

Example ii. *Divide* 1228 *cwt.* 2 *qrs.* 13 *lb.* *by* 365.

	cwt.	qr.	lb.
Quotient =	3 .	1 .	13
365 |	1228 .	2 .	13
	133		
	4		
	534 quarters		
	169		
	28		
	338		
	1365		
	4745 lb.		
	1095		

The quotient = 3 cwt. 1 qr. 13 lb.

Example iii. *Divide* 10 *Km.* 3 *Hm.* 74 *m.* *by* 91.

Quot. = 114 m. = 1 Hm. 1 Dm. 4m.

91 |	10374 m.
	127
	364

EXAMPLES. 45.

Divide

1. 1 yd. 2 ft. 3 in. by 7, 9.
2. 19 yds. 9 in. by 9, 11.
3. 1 fur. 3 ch. 2 yds. 2 ft. 3 in. by 7, 105, 385.
4. 2 mi. 5 fur. 2 ch. 7 yds. 1 ft. by 77, 154, 1274.
5. 8275 mi. 5 fur. 5 ch. by 1919, 6555.
6. 1 m. 82 cm. by 13, 14.
7. 200 cm. 2 mm. by 11, 77.
8. 65 m. 5 dm. 5 cm. by 19, 23, 95.
9. 14 Km. 14 m. by 91, 98, 182.
10. 66 Km. 2 Hm. 5 m. 5 dm. by 285, 1515, 5757.
11. 91 Km. 80 m. by 264, 345, 11385.
12. 41 Km. 6 Hm. 27 m. 30 cm. by 533, 2911, 10153.
13. 14 oz. 19 grs. by 23.
14. 47 oz. 16 dwt. 16 grs. by 41.
15. 196 oz. 4 dwt. 8 grs. by 61.
16. 1390 oz. 5 dwt. 7 grs. by 137.
17. 199879 oz. 17 dwt. 8 grs. by 647.
18. 1 qr. 9 lb. 7 oz. 1 dr. by 71.
19. 2 tons 1 cwt. 2 qrs. 8 lb. 8 oz. by 139.
20. 24 tons 5 cwt. 3 qrs. 21 lb. by 311.
21. 53 tons 9 cwt. 3 qrs. 14 lb. by 317.
22. 142 tons 2 cwt. 1 qr. 7 lb. 8 oz. by 1009.
23. 1757 tons 5 cwt. 3 qrs. 26 lb. by 4631.
24. 238 g. 9 cg. 5 mg. by 13, 15, 111.
25. 11 Kg. 9 Hg. 4 g. 75 cg. by 65, 555, 1443.
26. 8 t. 3 q. 85 Kg. 5 Hg. 2 Dg. 5 g. by 101, 369, 4141.
27. 69 t. 1 q. 71 Kg. 3 Hg. by 51, 411, 6987.

130. To find how often one weight is contained in another, we may reduce both to a common denomination (Art. 106), or we may proceed as follows (Art. 107):

Example. *How often is* 6 *tons* 1 *qr.* 27 *lb.* 10 *oz. contained in* 457 *tons* 17 *cwt.* 2 *qrs.* 27 *lb.* 8 *oz.?*

By the method of Art. 106 we first reduce the quantities to 215930 oz. and 16410680 oz., and we then divide the latter by the former. The quotient is 76. By the process of Art. 107 the work would appear as below:

tons	cwt.	qrs.	lb.	oz.	
6	0	1	27	10	= D.
				10	
60	4	3	24	4	= 10 D.

tons	cwt.	qrs.	lb.	oz.	
457	17	2	27	8	⌊76 = Quot.
421	14	3	1	12	= 70 D.
36	2	3	25	12	
36	2	3	25	12	= 6 D.

EXAMPLES. 46.

Divide

1. 308 oz. by 5 oz. 10 dwt.
2. 155 oz. 3 dwt. 12 grs. by 8 oz. 12 dwt. 10 grs.
3. 1787 oz. 3 dwt 18 grs. by 17 oz. 10 grs.
4. 10 lb. 4 oz. 3 drs. by 4 oz. 7 drs.
5. 3 qrs. 10 lb. 15 oz. by 1 lb. 8 oz. 8 drs.
6. 2 tons 5 cwt. 3 qrs. 24 lb. by 1 qr. 8 lb.
7. 5 tons 18 cwt. 1 qr. 24 lb. 8 oz. by 2 qrs. 3 lb. 8 oz.
8. 59 tons 9 cwt. 24 lb. by 2 qrs. 7 lb. 8 oz.
9. 3 Kg. 95 g. 57 cg. 5 mg. by 9 dg. 3 cg. 1 mg.
10. 11 Kg. 8 Hg. 7 Dg. 2 dg. 5 cg. by 12 g. 7 dg. 5 cg.
11. 30 t. 6 q. 92 Kg. 9 Hg. 7 Dg. by 10 Kg. 1 Hg. 9 Dg. 7 g.
12. 153 Kg. 4 Hg. 64 g. 85 cg. by 3 Dg. 3 g. 1 dg. 1 cg.
13. 18 m. 1 dm. 3 cm. 9 mm. by 1 m. 6 cm. 7 mm.
14. 19 Km. 9 Hm. 5 Dm. 2 m. 9 dm. by 18 m. 70 cm.

XVI. (a). FRACTIONS. INTRODUCTORY.

131. The abstract numbers which we have considered hitherto have all been *multiples* of the whole abstract unit; and all the concrete quantities have been numbers of *whole* tons, or pounds, or shillings, or other concrete units; but since we can divide a quantity into *parts*, we must have some mode of representing the relation which any part bears to the whole.

132. If, for example, we have the amount of a shilling in penny pieces, we have 12 pennies altogether; and if we divide it into 2 equal parts, we shall have 6 pennies in each part.

Each of these equal parts is called **one-half** of the whole shilling, and its relation to the shilling is expressed by the symbol $\frac{1}{2}$. Thus 6 pence $= \frac{1}{2}$ of 1 shilling.

Consequently $\frac{1}{2}$ of a quantity is that which is obtained when the quantity is divided by 2, or into 2 equal parts.

We may also define $\frac{1}{2}$ as that which, when taken twice over, or multiplied by 2, will reproduce the quantity.

Thus $\frac{1}{2}$ of 24 $= 12$, $\frac{1}{2}$ of £1 $= 10$ shillings,

$\frac{1}{2}$ of 1 shilling $= 6$ pence, $\frac{1}{2}$ of £2. 16*s*. 9*d*. $=$ £1. 8*s*. $4\frac{1}{2}$*d*.,

and $\frac{1}{2}$ of 3 cwt. 2 qrs. 16 lb. $=$ 1 cwt. 3 qrs. 8 lb.

EXAMPLES. 47. *Oral.*

What is $\frac{1}{2}$ of each of the following quantities?

1.	4.	2.	14.	3.	12036.
4.	20 shillings.	5.	£10.	6.	£246.
7.	6*s.* 8*d.*	8.	£16. 12*s.* 4*d.*	9.	£17. 11*s.* 8*d.*
10.	£287. 5*s.* 3*d.*	11.	£1391. 16*s.* $2\frac{1}{2}$*d.*	12.	£1305. 15*s.* $6\frac{1}{2}$*d.*
13.	£344621. 19*s.* $11\frac{1}{2}$*d.*			14.	36 cwt. 2 qrs. 10 lb.
15.	23 cwt. 2 qrs. 8 lb.			16.	3 tons 15 cwt. 24 lb.
17.	3 tons 1 cwt. 1 qr. 12 lb.			18.	14 tons 19 cwt. 6 lb. 8 oz.
19.	15 oz. 5 dwt. 6 grs.			20.	7 oz. 13 dwt. 16 grs.
21.	27 oz. 4 dwt.			22.	576 guineas.

133. If we divide a shilling, represented by 12 penny pieces, into three equal parts, there will be 4 pennies in each part.

Each of these parts is called *one third-part*, or briefly **one-third** of the whole shilling, and its relation to the shilling is represented by the symbol $\frac{1}{3}$.

If we take 2 parts together, there will be 8 pennies, and their relation to the whole shilling is described as 2 *third-parts*, or briefly as **two-thirds**, and is expressed by the symbol $\frac{2}{3}$.

Thus $\frac{1}{3}$ **of a shilling** = 4 pence,

$\frac{2}{3}$ ..... = 8 pence,

and $\frac{3}{3}$ ..... = 12 pence.

Consequently $\frac{1}{3}$ of a quantity is that which we obtain when we divide the quantity by 3, or into 3 equal parts; and we obtain $\frac{2}{3}$ of it by multiplying $\frac{1}{3}$ of the quantity by 2.

The following examples illustrate the meaning of $\frac{1}{3}$ and $\frac{2}{3}$:

(i) $\frac{1}{3}$ of 1*s.* = $\frac{1}{3}$ of 12 pence = 4 pence,

$\therefore$ $\frac{2}{3}$ of 1*s.* = $\frac{2}{3}$ ..... = 8 pence.

(ii) $\frac{1}{3}$ of £1 = 6*s.* 8*d.*,

$\therefore$ $\frac{2}{3}$ ..... = 6*s.* 8*d.* × 2 = 13*s.* 4*d.*

(iii) $\frac{1}{3}$ of £2. 16*s.* 9*d.* = 18*s.* 11*d.*,

$\therefore$ $\frac{2}{3}$ ..... = 18*s.* 11*d.* × 2 = £1. 17*s.* 10*d.*

134. Again, since $\frac{2}{3}$ of 1*s.* = 8 pence, and also $\frac{1}{3}$ of 2*s.* = 8 pence,

$\therefore$ $\frac{2}{3}$ of 1*s.* = $\frac{1}{3}$ of 2*s*;

and, generally, $\frac{2}{3}$ denotes

1 *third-part* of 2 *units*, as well as 2 *third-parts* of 1 *unit*.

EXAMPLES. 48.

Find $\frac{1}{3}$, and also $\frac{2}{3}$, of each of the following quantities:

1.	39.	2.	561.	3.	18327.
4.	17*s.* 6*d.*	5.	£12. 3*s.* 6*d.*	6.	£17. 5*s.* 6*d.*
7.	£1863. 14*s.* 7½*d.*	8.	£4000.	9.	£5845. 17*s.* 9*d.*
10.	£24299. 14*s.* 9*d.*	11.	6 cwt.	12.	975 tons.
13.	3 cwt. 2 qrs. 22 lb.	14.	1 ton 2 qrs. 20 lb.		
15.	2 tons 5 cwt. 2 qrs. 16 lb.	16.	19 tons 7 cwt. 1 qr. 8 lb.		
17.	34 cwt. 1 qr. 11 lb. 14 oz.	18.	4 oz. 7 dwt. 12 grs.		
19.	10 oz. 15 dwt. 15 grs.	20.	86 oz. 5 dwt. 18 grs.		

135. If we divide the sum of one shilling into four equal parts, each of the parts is called *one fourth-part* or, briefly, **one-fourth** of the whole shilling, and is represented by the symbol $\frac{1}{4}$.

If we take two parts together, we may represent their relation to the whole by the symbol $\frac{2}{4}$, and three parts by the symbol $\frac{3}{4}$.

Thus $\frac{1}{4}$ of 1 shilling = 3 pence,
$\frac{2}{4}$ ..... = 6 .....,
$\frac{3}{4}$ ..... = 9 .....,
and $\frac{4}{4}$ ..... = 12 .....

136. The division into equal parts, and the representation by symbols of the relation which a part bears to the whole, may be carried as far as we please.

If a quantity be divided into 5 equal parts, each is called **one-fifth** of the whole, and is represented by the symbol $\frac{1}{5}$; and what we get by taking 2, 3, 4 of these parts together is represented by the symbols $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$ respectively. Thus

$\frac{1}{5}$ of £1 = $\frac{1}{5}$ of 20*s.* = 4*s.*,
$\frac{2}{5}$ ..... = $\frac{2}{5}$ ..... = 8*s.*,
$\frac{3}{5}$ ..... = $\frac{3}{5}$ ..... = 12*s.*,
$\frac{4}{5}$ ..... = $\frac{4}{5}$ ..... = 16*s.*,
and similarly $\frac{5}{5}$ ..... = $\frac{5}{5}$ ..... = 20*s.* = £1.

137. Examining the symbols $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, we see that each consists of two parts, (i) the number above the line, and (ii) the number below it. It will be seen too that the number below the line denotes the number of equal parts into which the whole quantity has been divided, and that the number above the line denotes how many of these parts have been taken.

In this manner we may represent any portion of a quantity. If, for example, a quantity be divided into 11792 equal parts, the relation which each part bears to the whole will be represented by $\frac{1}{11792}$, and that of the sum of 6437 parts by $\frac{6437}{11792}$.

138. It will be seen also that the number *below* the line gives a *name* to the parts. If, for example, this number be 15, each of the equal parts is a *fifteenth* part of the quantity. For this reason the number *below* the line is called the **Denominator.**

Again, since the number *above* the line denotes the *number* of equal parts that are taken, it is called the **Numerator.**

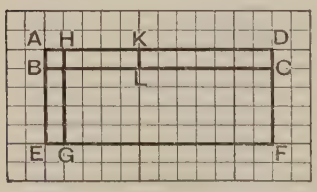
Thus in the symbol $\frac{6437}{11792}$, 11792 is the Denominator and 6437 the Numerator.

139. If a quantity be divided into parts, each part is called a **FRACTION** of the quantity; and when fractions are represented in the manner here described they are called **VULGAR**, or **COMMON, FRACTIONS.**

Also, just as 1, 2, 3, 4, etc., are called *abstract numbers* when they are taken by themselves without reference to any particular unit, so may $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, etc., be described as *abstract fractions*, or fractions of an abstract unit.

140. As we have seen (Art. 134) that $\frac{2}{3}$ may stand for $\frac{1}{3}$ of 2 units, so also may $\frac{3}{4}$ stand for $\frac{1}{4}$ of 3 units; and generally a fraction denotes a division, in which the numerator is the dividend and the denominator the divisor.

141. That the two ways of regarding a fraction are consistent, may be illustrated graphically as follows. We will show that the result is the same whether we regard the fraction $\frac{5}{12}$ as representing 5 twelfths of 1 unit, or 1 twelfth of 5 units.



In the accompanying diagram we will take the figure ABCD, made up of 12 squares, to represent the unit; the figure AEFD will consequently represent 5 units.

If 12 such strips as AEGH be placed in a row, they will make up the whole figure AEFD;

$\therefore$ AEGH $= \frac{1}{12}$ of AEFD $= \frac{1}{12}$ of 5 units.

And ABLK contains 5 squares, of which 12 make up ABCD;

$\therefore$ ABLK $= \frac{5}{12}$ of ABCD $= \frac{5}{12}$ of 1 unit.

But the figures AEGH and ABLK are equal, since each contains 5 squares; $\therefore \frac{1}{12}$ of 5 units $= \frac{5}{12}$ of 1 unit.

The pupil should verify other cases in this way for himself.

XVI. (b). TO FIND THE VALUE OF A FRACTION OF A GIVEN QUANTITY.

142. To do this we divide the quantity by the denominator, and multiply the result by the numerator; or we may first multiply the quantity by the numerator, and then divide the product by the denominator. Thus:

Example i. $\frac{5}{12}$ of £1 = 1*s.* 8*d.* × 5 = 8*s.* 4*d.*

Example ii. $\frac{3}{10}$ of 5*s.* 10*d.* = $\frac{1}{10}$ of 17*s.* 6*d.* = 1*s.* 9*d.*

NOTE.—When the numerator or the denominator is large, the work cannot be done mentally, but must be shown as a compound division followed by a compound multiplication.

EXAMPLES. 49. *Oral.*

What is

1. $\frac{1}{2}$ of £15; of £8. 19*s.*; of £57. 18*s.* 6*d.*; of £482. 19*s.* 6*d.*; of £1087. 12*s.* 9*d.*; of 756 f. 30 c.; of 481 m. 10 pf.; 862 dol. 80 c.?

2. $\frac{1}{3}$ of £16. 10*s.*; of £42. 18*s.* 9*d.*; of £57. 12*s.* 9*d.*; of £2168. 12*s.* 6*d.*; of 75 f. 15 c.; of 105 m. 75 pf.?

3. $\frac{1}{4}$ of £6. 17*s.* 4*d.*; of £28. 15*s.* 6*d.*; of £341. 14*s.* 9*d.*; of £3241. 18*s.* 9*d.*; of 841 f. 24 c.; of 571 m. 60 pf.?

4. $\frac{1}{5}$ of £10. 5*s.* 10*d.*; of £462. 4*s.* 7*d.*; of £361. 19*s.* 7*d.*; of £989. 12*s.* 6*d.*; of 831 dol. 75 c.; of 857 m. 50 pf.?

5. $\frac{1}{6}$ of £4. 8*s.* 6*d*; of £32. 7*s.* 6*d.*; of £58. 17*s.* 6*d.*; of £153. 12*s.* 9*d.*; of £627. 8*s.* 3*d.*; of 63 f. 42 c.; of 182 dol. 10 c.?

6. $\frac{1}{7}$ of 2 lb. 3 oz.; of 5 tons 12 cwt.; of 3 cwt. 1 qr. 14 lb.; of 3 Kg. 24 g.; of 14 Km. 791 m.; of 7 g. 56 mg.?

7. $\frac{1}{8}$ of 13 lb.; of 26 lb.; of 4 cwt.; of 5 cwt. 1 qr. 20 lb.?

8. $\frac{1}{9}$ of £3. 4*s.* 6*d.*; of £672. 6*s.*; of 3 cwt. 3 qrs. 12 lb.; of 15 tons 3 cwt. 3 qrs.; of 3 m. 6 mm.; of 5 Kg. 4 g.?

EXAMPLES. 50.

Find the value of each of the following quantities:

1. $\frac{2}{3}$ of £4. 16*s.* 2. $\frac{5}{6}$ of £70. 4*s.* 3. $\frac{3}{10}$ of £77.
4. $\frac{5}{8}$ of £63. 17*s.* 6*d.* 5. $\frac{7}{8}$ of a guinea. 6. $\frac{5}{8}$ of £27. 13*s.* 4*d.*

Find the value of each of the following quantities:

7. $\frac{4}{11}$ of £92. 19*s*. 11*d*.
8. $\frac{2}{15}$ of £24. 16*s*. 3*d*.
9. $\frac{4}{9}$ of £3. 17*s*. 3*d*.
10. $\frac{2}{5}$ of 7 cwt. 3 qrs. 7 lb.
11. $\frac{3}{7}$ of 1 qr. 21 lb.
12. $\frac{7}{9}$ of 3 tons 5 cwt. 10 lb
13. $\frac{4}{5}$ of 13 lb. 2 oz.
14. $\frac{5}{8}$ of 40 oz. 16 dwt. 8 grs.
15. $\frac{7}{12}$ of 188 oz. 12 dwt.
16. $\frac{11}{13}$ of £478. 18*s*. 10*d*.
17. $\frac{3}{14}$ of 56 oz. 14 dwt.
18. $\frac{5}{22}$ of 19 cwt. 2 qr. 16 lb.
19. $\frac{14}{27}$ of £5762. 16*s*. 3*d*.
20. $\frac{39}{140}$ of £4896. 10*s*.
21. $\frac{90}{187}$ of 9 Km. 537 m.
22. $\frac{16}{255}$ of 21 Kg. 675 g.
23. $\frac{5}{18}$ of 126 f. 10 c.
24. $\frac{80}{97}$ of 504 dol. 40 c.

XVI. (c). REDUCTION TO LOWEST TERMS.

143. We can pay a debt of 10*s*. by paying over either 1 half-sovereign, or 4 half-crowns, or 5 florins, or 10 shilling pieces, or 20 sixpenny pieces, or 40 threepenny pieces, and in many other ways too. But

1 half-crown $= \frac{1}{8}$ of £1, $\therefore$ 4 half-crowns $= \frac{4}{8}$ of £1.

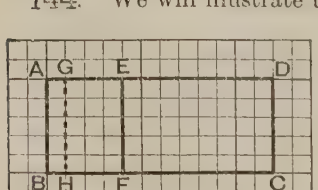
Similarly, 5 florins $= \frac{5}{10}$ of £1, 10 shillings $= \frac{10}{20}$ of £1,

and 20 sixpences $= \frac{20}{40}$ of £1.

Hence $\frac{4}{8}$ of £1, $\frac{5}{10}$ of £1, $\frac{10}{20}$ of £1, $\frac{20}{40}$ of £1 are all equal to $\frac{1}{2}$ of £1, and therefore the symbols $\frac{1}{2}$, $\frac{4}{8}$, $\frac{5}{10}$, $\frac{10}{20}$, $\frac{20}{40}$, are equivalent to one another. Thus the *same* fraction may be represented by many *different* symbols.

Again, if we divide a quantity into 12 equal parts, we may group the parts in 3 equal sets of 4, 4 and 4; and each set is $\frac{1}{3}$ of the quantity. But each set contains 4 *twelfth*-parts, and its relation to the whole may therefore be represented by $\frac{4}{12}$. Hence $\frac{4}{12}$, and similarly $\frac{5}{15}$, $\frac{6}{18}$, $\frac{70}{210}$, etc., are each equivalent to $\frac{1}{3}$.

144. We will illustrate this graphically with squared paper.



The figure ABFE is a part of ABCD. It is made up of 20 squares, of which 60 make up ABCD; we may therefore say that ABFE is $\frac{20}{60}$ of ABCD.

Or we may consider ABFE as made up of 4 strips such as AGHB, of which 12 make up the whole ABCD; we may therefore say that ABFE is $\frac{4}{12}$ of ABCD.

Or again we may suppose each of the small squares to be divided into 4 equal parts. The figure ABFE, which contains 20 of the squares, will then contain 80 of the parts; and the figure ABCD, which contains 60 of the squares, will contain 240 of the parts; we may therefore say that ABFE is $\frac{80}{240}$ of ABCD.

Hence $\frac{20}{60}$, $\frac{4}{12}$ and $\frac{80}{240}$ must have the same meaning.

145. From the preceding articles we see that *the Numerator and the Denominator of a fraction may be multiplied or divided by any, the same, number without altering the value of the fraction.*

146. The general theorem may be stated in the form

$$\frac{a}{b} = \frac{a \times m}{b \times m},$$

where $\frac{a}{b}$ represents any fraction, and m is any number whatever. It will be sufficient to consider the case in which a, b and m are all integers.

The fraction $\frac{a}{b}$ denotes that part of a unit which, when multiplied by b, produces a units. Consequently $\frac{a}{b} \times b = a$;

$\therefore$ by multiplication, $\frac{a}{b} \times b \times m = a \times m$; $\therefore$ by division, $\frac{a}{b} = \frac{a \times m}{b \times m}$;

from which it follows that, when both the numerator and the denominator are *multiplied* by m, the result is still equal to $\frac{a}{b}$.

Conversely it follows that, when we *divide* both numerator and denominator by the same number, the result is equal to the original fraction.

The simple rules for the treatment of fractions are based upon these two properties.

147. It must be noticed that we may not **add** equal numbers to the numerator and the denominator, and that we may not **subtract** equal numbers from the numerator and the denominator. Either of these operations would change the value of the fraction.

148. When there are no factors common to numerator and denominator, the fraction is said to be in its **lowest terms**, or in its **simplest form.** When there are factors common to numerator and denominator, a fraction may be **reduced to its lowest terms** by dividing both numerator and denominator by their G.C.M.

Example i. *Reduce* $\frac{63}{135}$ *to its lowest terms.*

$$\frac{63}{135}=\frac{7\times 9}{15\times 9}$$
$$=\frac{7}{15}.$$

Example ii. *Express* $\frac{10812}{22800}$ *in its simplest form.*

The G.C.M. of 10812 and 22800 is 12;

$$\therefore\ \frac{10812}{22800}=\frac{901\times 12}{1900\times 12}=\frac{901}{1900}.$$

EXAMPLES. 51. *Oral.*

Reduce each of the following fractions to its lowest terms:

1.	$\frac{2}{4}$.	2.	$\frac{3}{6}$.	3.	$\frac{3}{9}$.	4.	$\frac{4}{12}$.	5.	$\frac{5}{15}$.
6.	$\frac{6}{24}$.	7.	$\frac{7}{28}$.	8.	$\frac{8}{40}$.	9.	$\frac{9}{108}$.	10.	$\frac{10}{100}$.
11.	$\frac{12}{132}$.	12.	$\frac{13}{26}$.	13.	$\frac{13}{52}$.	14.	$\frac{15}{135}$.	15.	$\frac{4}{6}$.
16.	$\frac{6}{28}$.	17.	$\frac{14}{21}$.	18.	$\frac{10}{15}$.	19.	$\frac{10}{14}$.	20.	$\frac{9}{27}$.
21.	$\frac{12}{28}$.	22.	$\frac{46}{58}$.	23.	$\frac{24}{87}$.	24.	$\frac{56}{94}$.	25.	$\frac{18}{27}$.
26.	$\frac{21}{28}$.	27.	$\frac{18}{30}$.	28.	$\frac{24}{27}$.	29.	$\frac{18}{24}$.	30.	$\frac{24}{28}$.
31.	$\frac{9}{30}$.	32.	$\frac{24}{32}$.	33.	$\frac{18}{32}$.	34.	$\frac{24}{36}$.	35.	$\frac{9}{36}$.
36.	$\frac{32}{36}$.	37.	$\frac{18}{48}$.	38.	$\frac{30}{48}$.	39.	$\frac{36}{48}$.	40.	$\frac{45}{48}$.
41.	$\frac{42}{48}$.	42.	$\frac{26}{39}$.	43.	$\frac{39}{52}$.	44.	$\frac{24}{60}$.	45.	$\frac{36}{60}$.

EXAMPLES. 52.

Reduce each of the following fractions to its lowest terms:

1.	$\frac{187}{836}$.	2.	$\frac{188}{729}$.	3.	$\frac{315}{924}$.	4.	$\frac{280}{945}$.
5.	$\frac{256}{1024}$.	6.	$\frac{1260}{1647}$.	7.	$\frac{1232}{1925}$.	8.	$\frac{1125}{1350}$.
9.	$\frac{291}{3201}$.	10.	$\frac{768}{2592}$.	11.	$\frac{444}{703}$.	12.	$\frac{1476}{1764}$.
13.	$\frac{1236}{4764}$.	14.	$\frac{1995}{7035}$.	15.	$\frac{6435}{7293}$.	16.	$\frac{2340}{8748}$.
17.	$\frac{3366}{96880}$.	18.	$\frac{6816}{10656}$.	19.	$\frac{7497}{15729}$.	20.	$\frac{5400}{30105}$.
21.	$\frac{372725}{473325}$.	22.	$\frac{2640}{73458}$.	23.	$\frac{422237}{755882}$.	24.	$\frac{32436}{684400}$.
25.	$\frac{18176}{191808}$.	26.	$\frac{7200}{704075}$.	27.	$\frac{890274}{1213643}$.		

XVI. (d). COMPARISON OF FRACTIONS.

149. To compare one fraction with another we first substitute for them equivalent fractions in which the equal parts have the *same name*, that is, we determine equivalent fractions with equal *denominators*.

For example, in the case of $\frac{6}{7}$ the equal parts are *sevenths*, and in the case of $\frac{11}{13}$ they are *thirteenths*. But

$$\frac{6}{7}=\frac{6\times13}{7\times13}=\frac{78}{91}, \text{ and } \frac{11}{13}=\frac{11\times7}{13\times7}=\frac{77}{91}.$$

The parts are now 91*ths* in both cases, and we see at once that the first fraction is the greater.

The common denominator may be *any* common multiple of the original denominators, but we ought to take the **Least.**

Example i. *In $\frac{1}{3}$, how many 63rds?*

We have $\frac{1}{3}=\frac{1\times21}{3\times21}=\frac{21}{63}$; $\therefore$ no. of 63rds$=21$.

Example ii. *Which is the greater of the fractions $\frac{21}{40}$ and $\frac{34}{63}$?*

L.C.M. of denominators $=63\times40=2520$;

$$\frac{21}{40}=\frac{21\times63}{40\times63}=\frac{1323}{2520}, \text{ and } \frac{34}{63}=\frac{34\times40}{63\times40}=\frac{1360}{2520};$$

$\therefore$ $\frac{34}{63}$ is the greater.

150. To compare, and arrange in order of magnitude, three or more fractions, we find the L.C.M. of the denominators and replace each symbol by one which has for its denominator this L.C.M.

Before finding the L.C.M. of the denominators we should take care that all the fractions are in their lowest terms.

Example. *Reduce $\frac{31}{63}$, $\frac{11}{14}$, $\frac{5}{21}$, $\frac{1}{42}$ to equivalent fractions having a common denominator, and arrange them in order of magnitude.*

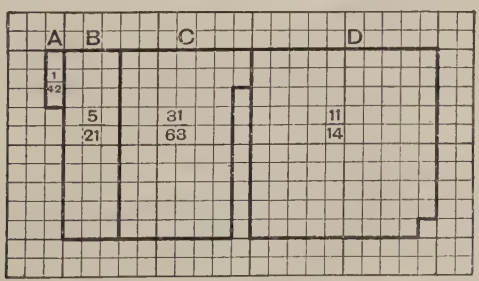
We find that the L.C.M. of the denominators $=126$; and that

$$\frac{31}{63}=\frac{62}{126}, \quad \frac{11}{14}=\frac{99}{126}, \quad \frac{5}{21}=\frac{30}{126}, \quad \frac{1}{42}=\frac{3}{126};$$

hence their ascending order of magnitude is

$$\frac{3}{126}, \quad \frac{30}{126}, \quad \frac{62}{126}, \quad \frac{99}{126}, \quad i.e. \quad \frac{1}{42}, \quad \frac{5}{21}, \quad \frac{31}{63}, \quad \frac{11}{14}.$$

151. The four fractions in the preceding example may for comparison be represented graphically by the figures A, B, C, D



in the diagram. These figures contain 3, 30, 62 and 99 squares respectively.

The unit in *each* case is made up of 126 small squares. It is only by using the same unit that comparison becomes possible. When we use an Atlas, for example, we cannot compare the size of a country taken from one map with the size of another country taken from another map, because the scales are usually different. The page of the map is not made to fit the country; but the country is drawn of such a size as will fit into the page. The page given to England is the same size as that given to Africa or Australia. Consider, for example, the sizes of England and Australia as given on separate maps, and again as given on a map of the World.

152. Fractions may also be compared by reducing them to others with equal *numerators*; the one with the *smaller* denominator then represents the *larger* fraction. This method should always be applied when the L.C.M. of the denominators is large, and that of the numerators is small.

Example. *Arrange* $\frac{8}{13}$, $\frac{32}{97}$, $\frac{16}{57}$ *and* $\frac{2}{7}$ *in order of magnitude.*

The L.C.M. of the numerators is 32, but that of the denominators is $13 \times 97 \times 57 \times 7$. It is best therefore to reduce the fractions to equivalent fractions *with equal numerators*. We see that the fractions are equivalent to $\frac{32}{52}$, $\frac{32}{97}$, $\frac{32}{114}$ and $\frac{32}{112}$; and their ascending order of magnitude is

$$\frac{32}{114}, \frac{32}{112}, \frac{32}{97}, \frac{32}{52}, \quad i.e. \quad \frac{16}{57}, \frac{2}{7}, \frac{32}{97}, \frac{8}{13}.$$

EXAMPLES. **53.** *Oral.*

1. In $\frac{1}{2}$, how many 6ths, 8ths, 12ths, 24ths, 28ths?
2. In $\frac{1}{3}$, how many 9ths, 12ths, 15ths, 18ths, 39ths?
3. In $\frac{1}{4}$, how many 8ths, 12ths, 16ths, 20ths, 24ths?
4. In $\frac{1}{5}$, how many 10ths, 20ths, 25ths, 35ths, 50ths?
5. Change $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ to 12ths.
6. Change $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{7}$, $\frac{3}{4}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{6}{7}$, $\frac{3}{14}$, $\frac{9}{14}$ to 28ths.
7. Change $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{9}$, $\frac{1}{18}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{12}$, $\frac{11}{18}$, $\frac{17}{18}$ to 36ths.
8. Change $\frac{1}{3}$, $\frac{1}{19}$, $\frac{2}{3}$, $\frac{5}{19}$, $\frac{7}{19}$, $\frac{18}{19}$ to 57ths.

EXAMPLES. **54.** *Oral.*

Which is the greater in each of the following pairs of fractions?

1. $\frac{1}{2}$ and $\frac{2}{3}$.
2. $\frac{3}{10}$ and $\frac{7}{20}$.
3. $\frac{3}{6}$ and $\frac{7}{12}$.
4. $\frac{2}{3}$ and $\frac{7}{12}$.
5. $\frac{7}{8}$ and $\frac{23}{24}$.
6. $\frac{31}{42}$ and $\frac{15}{21}$.
7. $\frac{5}{6}$ and $\frac{7}{9}$.
8. $\frac{5}{6}$ and $\frac{17}{24}$.
9. $\frac{4}{9}$ and $\frac{5}{12}$.
10. $\frac{7}{11}$ and $\frac{20}{33}$.
11. $\frac{5}{14}$ and $\frac{8}{21}$.
12. $\frac{7}{18}$ and $\frac{17}{45}$.

EXAMPLES. 55.

Arrange in ascending order of magnitude:

1. $\frac{4}{5}$, $\frac{3}{10}$ and $\frac{7}{20}$.
2. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{5}{6}$ and $\frac{7}{12}$.
3. $\frac{5}{6}$, $\frac{11}{12}$ and $\frac{1}{18}$.
4. $\frac{4}{9}$, $\frac{3}{5}$ and $\frac{8}{15}$.
5. $\frac{5}{7}$, $\frac{17}{42}$, $\frac{9}{14}$ and $\frac{16}{21}$.
6. $\frac{2}{7}$, $\frac{11}{14}$, $\frac{3}{8}$ and $\frac{11}{28}$.
7. $\frac{3}{5}$, $\frac{7}{10}$, $\frac{9}{16}$ and $\frac{5}{8}$.
8. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{17}{24}$ and $\frac{3}{4}$.
9. $\frac{1}{5}$, $\frac{3}{4}$, $\frac{7}{10}$ and $\frac{13}{25}$.
10. $\frac{2}{3}$, $\frac{7}{8}$, $\frac{11}{12}$ and $\frac{19}{20}$.
11. $\frac{5}{6}$, $\frac{5}{8}$, $\frac{2}{9}$ and $\frac{13}{24}$.
12. $\frac{5}{12}$, $\frac{2}{3}$, $\frac{19}{24}$ and $\frac{4}{5}$.
13. $\frac{5}{8}$, $\frac{17}{20}$, $\frac{19}{30}$ and $\frac{29}{32}$.
14. $\frac{5}{7}$, $\frac{11}{14}$, $\frac{19}{35}$, $\frac{15}{28}$ and $\frac{13}{21}$.
15. $\frac{3}{25}$, $\frac{7}{10}$, $\frac{3}{50}$ and $\frac{7}{65}$.
16. $\frac{9}{10}$, $\frac{11}{20}$, $\frac{13}{15}$, $\frac{7}{30}$ and $\frac{5}{36}$.
17. $\frac{3}{4}$, $\frac{11}{12}$, $\frac{15}{36}$ and $\frac{37}{48}$.
18. $\frac{1}{2}$, $\frac{2}{9}$, $\frac{3}{11}$, $\frac{7}{18}$ and $\frac{17}{33}$.
19. $\frac{7}{16}$, $\frac{19}{24}$, $\frac{4}{5}$ and $\frac{15}{32}$.
20. $\frac{3}{8}$, $\frac{9}{16}$, $\frac{13}{27}$ and $\frac{19}{36}$.

XVI. (e). IMPROPER FRACTIONS AND MIXED NUMBERS.

153. Hitherto the numerators have all been *less than* the denominators, which indeed seems proper and essential to the nature of a fraction. But it is convenient to represent quantities *greater than* the unit in the same fractional form. For example,

1 sixpence = 1 *fortieth*-part of £1,

$\therefore$ 43 sixpences = 43 *fortieth*-parts of £1;

and we may conveniently represent the relation of 43 sixpences to £1 by the symbol $\frac{43}{40}$, which has its numerator *greater* than its denominator. To distinguish between the two cases, we give the name **proper** fractions to those, such as $\frac{4}{13}$, in which the numerator is *less than* the denominator, and the name **improper** fractions to those, such as $\frac{13}{4}$, in which the numerator is *greater than* the denominator.

154. Again, since $\frac{13}{4}$ represents 13 *fourth*-parts of a unit, and 12 *fourth*-parts make up the unit 3 times, it follows that $\frac{13}{4}$ is equivalent to 3 units and 1 *fourth*-part of a unit. We may therefore represent it by $3\frac{1}{4}$, which is read "*three and a quarter*," and means $3+\frac{1}{4}$. In the form $3\frac{1}{4}$ it is called a **mixed number.**

155. The student will now understand why we used the symbols $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ to denote 1 *farthing*, 2 *farthings* and 3 *farthings* respectively, and why the symbol for the farthings was placed closely adjacent to the figure denoting pence, as in £3. 12*s.* $7\frac{3}{4}d.$ The $7\frac{3}{4}d.$ is really a *mixed* number denoting 7 whole pence, and 3 *fourth-parts* of a penny.

The word *farthing* means the same as *fourth*-part, and is used specially to denote a *fourth*-part of a *penny.* Before the time of

Edward I. the silver penny was indented with a cross, so that it could easily be broken into 2 parts, 3 parts, or 4 parts.

156. In the following examples we will show how to express an *improper fraction* in the form of a *mixed number*.

Example i. *Express* $\frac{25}{3}$ *as a mixed number.*

$\frac{25}{3}$ = 25 *third-parts*
= 24 *third-parts* + 1 *third-part*
= 8 *units* + 1 *third-part*
= $8\frac{1}{3}$.

Hence, *if we divide the numerator by the denominator, the quotient will be the integral part of the mixed number; the fractional part will have for its numerator the remainder after the division, and for its denominator the original denominator.*

NOTE.—If there should be *no remainder* after the division, the quantity, although expressed in fractional form, represents a whole number.

Example ii. *Express* $\frac{60}{12}$ *as a whole number.*

The numerator can be divided by the denominator without remainder, and the quotient is 5. Hence $\frac{60}{12} = 5$.

Example iii. *Express* $\frac{185}{7}$ *as a mixed number.*

If we divide 185 by 7, the quotient is 26, and the remainder is 3. Hence

$$\frac{185}{7} = 26\frac{3}{7}.$$

EXAMPLES. 56. *Oral.*

Express each of the following improper fractions as a mixed number, or as a whole number.

1. $\frac{4}{3}$.	2. $\frac{6}{5}$.	3. $\frac{9}{2}$.	4. $\frac{10}{3}$.	5. $\frac{15}{8}$.
6. $\frac{21}{5}$.	7. $\frac{17}{5}$.	8. $\frac{29}{7}$.	9. $\frac{31}{4}$.	10. $\frac{39}{10}$.
11. $\frac{11}{5}$.	12. $\frac{15}{2}$.	13. $\frac{16}{3}$.	14. $\frac{11}{6}$.	15. $\frac{12}{7}$.
16. $\frac{13}{3}$.	17. $\frac{14}{5}$.	18. $\frac{14}{9}$.	19. $\frac{9}{4}$.	20. $\frac{26}{7}$.
21. $\frac{25}{2}$.	22. $\frac{41}{3}$.	23. $\frac{57}{2}$.	24. $\frac{34}{3}$.	25. $\frac{37}{12}$.
26. $\frac{30}{7}$.	27. $\frac{42}{5}$.	28. $\frac{39}{8}$.	29. $\frac{33}{10}$.	30. $\frac{47}{11}$.

EXAMPLES. 57.

Express each of the following improper fractions as a mixed number in its lowest terms, or as a whole number.

1. $\frac{549}{29}$.	2. $\frac{561}{34}$.	3. $\frac{637}{42}$.	4. $\frac{789}{63}$.	5. $\frac{457}{91}$.
6. $\frac{706}{103}$.	7. $\frac{1942}{105}$.	8. $\frac{45223}{241}$.	9. $\frac{69841}{481}$.	10. $\frac{56273}{373}$.

11.	$\frac{19684}{566}$.	12.	$\frac{18522}{567}$.	13.	$\frac{23082}{720}$.	14.	$\frac{52504}{572}$.
15.	$\frac{51637}{152}$.	16.	$\frac{39812}{545}$.	17.	$\frac{59956}{1251}$.	18.	$\frac{77913}{3522}$.
19.	$\frac{10027}{50}$.	20.	$\frac{9999}{347}$.	21.	$\frac{43053}{709}$.	22.	$\frac{2901109}{2909}$.

157. To express a mixed number in the form of an improper fraction is the converse of the preceding.

Example i. *Express* $3\frac{4}{5}$ *as an improper fraction.*

Each *unit* = 5 *fifth-parts*,

$\therefore$ 3 *units* = 15 ..... ;

$\therefore$ 3 *units* and 4 *fifth parts* = 19 *fifth-parts*,

$\therefore 3\frac{4}{5} = \frac{19}{5}$.

Hence, *if we multiply the integral part of a mixed number by the denominator of the fractional part, and add to the product the numerator of the fractional part, the result will be the numerator of the improper fraction; and the original denominator will be its denominator.*

Example ii. *Express* $5\frac{17}{45}$ *as an improper fraction.*

$$5\tfrac{17}{45} = \frac{5\times45+17}{45} = \frac{225+17}{45} = \frac{242}{45}.$$

EXAMPLES. 58. *Oral.*

Express each of the mixed numbers in questions 1–6 as an improper fraction:

1.	$1\frac{1}{2}$,	$1\frac{1}{3}$,	$1\frac{1}{4}$,	$1\frac{1}{5}$,	$1\frac{1}{6}$,	$2\frac{1}{2}$,	$2\frac{1}{3}$,	$3\frac{1}{4}$,	$4\frac{1}{5}$,	$5\frac{1}{6}$.
2.	$1\frac{2}{3}$,	$2\frac{2}{3}$,	$2\frac{3}{4}$,	$3\frac{3}{5}$,	$4\frac{3}{5}$,	$5\frac{5}{6}$,	$6\frac{7}{8}$,	$5\frac{8}{9}$,	$4\frac{5}{9}$,	$2\frac{5}{8}$.
3.	$4\frac{7}{8}$,	$3\frac{2}{9}$,	$4\frac{3}{4}$,	$5\frac{7}{9}$,	$3\frac{8}{9}$,	$4\frac{6}{7}$,	$5\frac{11}{12}$,	$4\frac{8}{9}$.		
4.	$8\frac{1}{2}$,	$7\frac{1}{3}$,	$4\frac{3}{11}$,	$5\frac{4}{7}$,	$8\frac{9}{10}$,	$10\frac{4}{7}$,	$12\frac{7}{10}$,	$9\frac{4}{9}$.		
5.	$5\frac{2}{7}$,	$8\frac{4}{5}$,	$4\frac{1}{12}$,	$6\frac{5}{12}$,	$8\frac{2}{11}$,	$9\frac{6}{7}$,	$4\frac{3}{7}$,	$10\frac{11}{12}$.		
6.	$7\frac{12}{17}$,	$12\frac{3}{10}$,	$17\frac{2}{3}$,	$20\frac{4}{5}$,	$10\frac{8}{9}$,	$14\frac{3}{5}$,	$15\frac{5}{6}$.			

7. In 5 how many 9ths? 8. In 11 how many 5ths?

9. In 14 ..... 3rds? 10. In 15 ..... 4ths?

11. Among how many persons must 10 apples be divided in order that each may receive $\frac{1}{2}$ of an apple? $\frac{1}{3}$? $\frac{1}{10}$? $\frac{1}{4}$?

EXAMPLES. 59.

Express each of the following 16 mixed numbers as an improper fraction:

1.	$12\frac{14}{25}$.	2.	$23\frac{17}{41}$.	3.	$13\frac{28}{51}$.	4.	$45\frac{2}{33}$.
5.	$14\frac{31}{43}$.	6.	$11\frac{11}{112}$.	7.	$14\frac{8}{95}$.	8.	$13\frac{16}{17}$.
9.	$36\frac{15}{38}$.	10.	$71\frac{31}{43}$.	11.	$127\frac{13}{18}$.	12.	$183\frac{8}{21}$.
13.	$206\frac{29}{50}$.	14.	$317\frac{11}{15}$.	15.	$620\frac{13}{14}$.	16.	$2607\frac{6}{29}$.

17. Express 17 as a fraction having 7 for its denominator.
18. ..... 16 ..... 16 .....
19. ..... 31 ..... 19 .....
20. ..... 501 ..... 21 .....

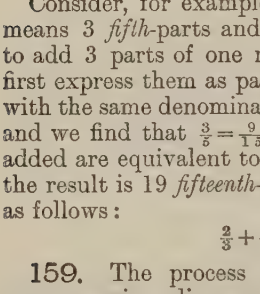
XVI. (f). ADDITION.

158. We will now consider how two or more fractions may be added together, and the result expressed by a *single* fraction.

Consider, for example, the addition of $\frac{3}{5}$ and $\frac{2}{3}$. The former means 3 *fifth*-parts and the latter 2 *third*-parts, so that we have to add 3 parts of one name, and 2 parts of another. We must first express them as parts having the *same* name, *i.e.* as fractions with the same denominator. The L.C.M. of the denominators is 15, and we find that $\frac{3}{5}=\frac{9}{15}$, and $\frac{2}{3}=\frac{10}{15}$. Hence the fractions to be added are equivalent to 9 *fifteenth*-parts and 10 *fifteenth*-parts, and the result is 19 *fifteenth*-parts. The process may be stated briefly as follows:

$$\frac{2}{3}+\frac{3}{5}=\frac{10}{15}+\frac{9}{15}=\frac{19}{15}=1\frac{4}{15}.$$

159. The process may be illustrated graphically by the accompanying diagram. We will add together the fractions $\frac{4}{9}$ and $\frac{2}{15}$.



The figure ABCD represents the whole unit, and contains 45 squares.

The figure ABFE contains 20 squares, and therefore represents the fraction $\frac{20}{45}$, or $\frac{4}{9}$.

The figure EFLKHG contains 6 squares, and therefore represents the fraction $\frac{6}{45}$, or $\frac{2}{15}$.

Hence ABLKHG represents the sum of the two fractions. It contains, however, 26 squares and it therefore represents also the fraction $\frac{26}{45}$. Hence the sum of $\frac{4}{9}$ and $\frac{2}{15}$ is $\frac{26}{45}$.

The pupil may draw figures in this way to illustrate Nos. 1 to 20 in the following set of examples.

NOTE.—The number of squares in the unit should be one which each of the denominators will divide (Art. 151). Generally the L.C.M. of the denominators will be the best number to take.

160. In the same manner we add together 3 or more fractions, first substituting for them equivalent fractions with equal denominators, each of which is the L.C.M. of the original denominators.

Example. *Add together* $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{7}$, $\frac{9}{14}$ and $\frac{5}{21}$.

The L.C.M. of the denominators is 84. Hence

$$\frac{1}{2}+\frac{3}{4}+\frac{4}{7}+\frac{9}{14}+\frac{5}{21}=\frac{42}{84}+\frac{63}{84}+\frac{48}{84}+\frac{54}{84}+\frac{20}{84}$$
$$=\frac{42+63+48+54+20}{84}=\frac{227}{84}=2\frac{59}{84}.$$

161. If some of the fractions are *improper* fractions, we express them as *mixed numbers* and proceed as in Art. 162.

162. If some of the fractions are *mixed numbers*, we add together the integral parts at once, and treat the fractional parts as in Art. 160. Thus

$$\frac{3}{5}+2\frac{1}{6}+3\frac{3}{4}+1\frac{7}{8}=2+3+1+\frac{3}{5}+\frac{1}{6}+\frac{3}{4}+\frac{7}{8}$$
$$=6+\frac{72}{120}+\frac{20}{120}+\frac{90}{120}+\frac{105}{120}\quad\ldots\ldots\ldots\ldots(\alpha)$$
$$=6+\frac{72+20+90+105}{120}$$
$$=6+\frac{287}{120}$$
$$=6+2\frac{47}{120}$$
$$=8\frac{47}{120}.$$

The process may be shortened by omitting the line marked (α) and the one above it. This may easily be done after a little practice, and without lessening the clearness of the process.

EXAMPLES. 60. *Oral.*

Add together the following fractions:

1.	$\frac{2}{7}+\frac{3}{7}$.	2.	$\frac{1}{6}+\frac{5}{6}$.	3.	$\frac{2}{9}+\frac{4}{9}+\frac{1}{9}$.	4.	$\frac{1}{2}+\frac{1}{8}$.
5.	$\frac{1}{2}+\frac{1}{5}$.	6.	$\frac{1}{3}+\frac{1}{4}$.	7.	$\frac{1}{3}+\frac{1}{5}$.	8.	$\frac{1}{4}+\frac{1}{6}$.
9.	$\frac{1}{7}+\frac{1}{14}$.	10.	$\frac{1}{2}+\frac{1}{6}$.	11.	$\frac{1}{2}+\frac{1}{3}+\frac{1}{6}$.	12.	$\frac{1}{4}+\frac{1}{6}$.
13.	$\frac{1}{2}+\frac{1}{4}+\frac{1}{6}$.	14.	$\frac{1}{2}+\frac{1}{6}+\frac{1}{12}$.	15.	$\frac{1}{3}+\frac{1}{4}+\frac{1}{12}$.		
16.	$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{6}$.	17.	$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{12}$.				
18.	$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{6}+\frac{1}{12}$.	19.	$\frac{2}{3}+\frac{1}{4}+\frac{5}{6}$.	20.	$\frac{2}{3}+\frac{7}{9}+\frac{1}{18}$.		
21.	$\frac{1}{2}+\frac{3}{4}+\frac{5}{6}$.	22.	$\frac{3}{4}+\frac{5}{6}+\frac{7}{12}$.	23.	$\frac{2}{3}+\frac{3}{4}+\frac{11}{12}$.		
24.	$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{5}{6}$.	25.	$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{5}{6}+\frac{7}{12}$.				
26.	$\frac{4}{5}+\frac{3}{10}+\frac{7}{20}$.	27.	$\frac{3}{5}+\frac{7}{10}+\frac{11}{20}$.	28.	$\frac{2}{5}+\frac{9}{10}+\frac{13}{20}$.		
29.	$\frac{5}{6}+\frac{5}{8}+\frac{2}{3}$.	30.	$\frac{1}{6}+\frac{7}{8}+\frac{1}{3}$.	31.	$2\frac{1}{2}+1\frac{1}{3}$.		
32.	$3\frac{1}{4}+4\frac{1}{3}$.	33.	$5\frac{1}{3}+3\frac{1}{5}$.	34.	$6\frac{2}{3}+4\frac{1}{2}$.		
35.	$2\frac{1}{4}+4\frac{3}{4}+2\frac{1}{2}$.	36.	$3\frac{5}{6}+2\frac{1}{3}+1\frac{1}{2}$.	37.	$4\frac{1}{8}+5\frac{1}{2}+6\frac{7}{12}$.		

EXAMPLES. 61.

Find the value of the following expressions:

1. $\frac{2}{7}+\frac{3}{21}+\frac{31}{42}+\frac{1}{2}$.
2. $\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{7}{12}$.
3. $\frac{8}{11}+\frac{5}{6}+\frac{1}{2}+\frac{9}{22}$.
4. $\frac{3}{5}+\frac{9}{4}+\frac{7}{10}+\frac{13}{26}$.
5. $\frac{3}{8}+\frac{5}{32}+\frac{1}{3}+\frac{5}{12}$.
6. $\frac{3}{16}+\frac{2}{9}+\frac{5}{12}+\frac{1}{2}$.
7. $\frac{2}{3}+\frac{7}{8}+\frac{11}{12}+\frac{19}{20}$.
8. $\frac{1}{2}+\frac{3}{5}+\frac{5}{6}+\frac{7}{9}+\frac{1}{3}$.
9. $32\frac{2}{5}+\frac{1}{9}+\frac{22}{3}+\frac{1}{2}+\frac{2}{15}$.
10. $\frac{7}{18}+\frac{7}{30}+\frac{11}{15}+\frac{3}{2}+\frac{2}{3}$.
11. $\frac{7}{10}+\frac{11}{12}+\frac{4}{9}+\frac{2}{3}$.
12. $\frac{5}{12}+\frac{7}{9}+\frac{3}{5}+\frac{3}{10}$.
13. $\frac{5}{12}+\frac{3}{25}+\frac{1}{6}+\frac{7}{10}$.
14. $\frac{7}{9}+\frac{4}{11}+\frac{5}{6}+\frac{10}{33}$.
15. $\frac{2}{9}+\frac{8}{11}+\frac{1}{3}+\frac{7}{22}$.
16. $\frac{23}{27}+\frac{3}{5}+\frac{1}{3}+\frac{4}{15}+\frac{1}{9}$.
17. $\frac{10}{11}+\frac{16}{7}+\frac{5}{14}+\frac{3}{2}+\frac{1}{22}$.
18. $\frac{4}{7}+\frac{1}{10}+\frac{9}{28}+\frac{5}{16}$.
19. $\frac{4}{7}+\frac{5}{18}+\frac{9}{14}+\frac{23}{84}$.
20. $\frac{5}{11}+\frac{2}{15}+\frac{16}{33}+\frac{17}{60}$.
21. $\frac{5}{18}+\frac{6}{7}+\frac{5}{36}+\frac{2}{21}$.
22. $\frac{1}{27}+\frac{5}{18}+\frac{4}{81}+\frac{11}{36}$.
23. $\frac{5}{17}+\frac{1}{4}+\frac{7}{8}+\frac{18}{51}$.
24. $\frac{6}{7}+\frac{7}{8}+\frac{48}{49}+\frac{279}{392}$.
25. $\frac{9}{16}+\frac{5}{11}+\frac{8}{33}+\frac{1}{4}$.
26. $\frac{11}{17}+\frac{10}{221}+\frac{7}{13}+\frac{14}{39}$.
27. $\frac{50}{39}+\frac{1}{2}+\frac{1}{3}+\frac{11}{15}+\frac{4}{5}$.
28. $\frac{1}{22}+\frac{9}{4}+\frac{8}{11}+\frac{2}{5}+\frac{3}{9}$.
29. $\frac{3}{20}+\frac{6}{17}+\frac{13}{50}+\frac{15}{34}$.
30. $\frac{14}{33}+\frac{3}{40}+\frac{25}{11}+\frac{1}{8}+\frac{49}{22}$.
31. $\frac{5}{13}+\frac{1}{7}+\frac{6}{11}+\frac{17}{21}$.
32. $2\frac{1}{2}+1+5\frac{1}{4}$.
33. $1\frac{1}{3}+5+3\frac{2}{9}$.
34. $5\frac{1}{6}+3\frac{1}{2}+4\frac{1}{3}+\frac{5}{4}$.
35. $15\frac{2}{3}+8+12\frac{7}{8}+11\frac{2}{9}$.
36. $6\frac{1}{5}+\frac{7}{9}+2\frac{4}{6}+1\frac{1}{3}$.
37. $\frac{3}{4}+2\frac{14}{15}+\frac{3}{8}+8\frac{1}{5}$.
38. $3\frac{7}{8}+4+7\frac{1}{2}+6\frac{1}{3}$.
39. $20\frac{4}{9}+17\frac{2}{3}+6+24\frac{5}{6}$.
40. $5\frac{2}{35}+\frac{6}{5}+\frac{6}{7}+\frac{39}{70}$.

XVI. (g). SUBTRACTION.

163. We will next consider how one fraction may be subtracted from another, and their *difference* expressed by a *single* fraction. As in addition, we replace the given fractions by equivalent fractions having equal denominators, each of which must as before be the L.C.M. of the original denominators.

Example i. *Find the difference between* $\frac{2}{3}$ *and* $\frac{3}{5}$.

$$\frac{2}{3}-\frac{3}{5}=\frac{10}{15}-\frac{9}{15}=\frac{10-9}{15}=\frac{1}{15}.$$

Example ii. *Find the difference between* $\frac{11}{14}$ *and* $\frac{13}{21}$.

The L.C.M. of the denominators $=2\times7\times3=42$;

$$\therefore\ \frac{11}{14}-\frac{13}{21}=\frac{33-26}{42}=\frac{7}{42}=\frac{1}{6}.$$

Example iii. *Find the difference between* $2\frac{1}{3}$ *and* $1\frac{3}{4}$.

$$2\tfrac{1}{3} - 1\tfrac{3}{4} = 1\tfrac{4-9}{12} = 1 - \tfrac{5}{12} = \tfrac{7}{12}.$$

Example iv. *Find the difference between* $16\frac{4}{9}$ *and* $12\frac{1}{6}$.

$$16\tfrac{4}{9} - 12\tfrac{1}{6} = 4\tfrac{8-3}{18} = 4\tfrac{5}{18}.$$

EXAMPLES. 62. *Oral.*

State the results of the following subtractions:

1.	$1 - \frac{1}{4}$.	2.	$1 - \frac{1}{2}$.	3.	$1 - \frac{3}{4}$.	4.	$1 - \frac{1}{3}$.
5.	$1 - \frac{2}{3}$.	6.	$1 - \frac{7}{8}$.	7.	$2 - \frac{4}{5}$.	8.	$3 - \frac{5}{16}$.
9.	$8 - \frac{43}{48}$.	10.	$\frac{3}{4} - \frac{1}{4}$.	11.	$\frac{1}{2} - \frac{1}{4}$.	12.	$\frac{5}{6} - \frac{1}{6}$.
13.	$\frac{7}{9} - \frac{1}{9}$.	14.	$\frac{7}{9} - \frac{4}{9}$.	15.	$\frac{1}{2} - \frac{1}{3}$.	16.	$\frac{1}{2} - \frac{1}{5}$.
17.	$\frac{1}{3} - \frac{1}{5}$.	18.	$\frac{1}{2} - \frac{1}{8}$.	19.	$\frac{1}{4} - \frac{1}{8}$.	20.	$\frac{1}{2} - \frac{1}{12}$.
21.	$\frac{1}{2} - \frac{1}{6}$.	22.	$\frac{2}{3} - \frac{1}{2}$.	23.	$\frac{1}{2} - \frac{2}{5}$.	24.	$\frac{3}{5} - \frac{1}{2}$.
25.	$\frac{4}{5} - \frac{1}{2}$.	26.	$\frac{2}{3} - \frac{3}{5}$.	27.	$\frac{3}{4} - \frac{2}{3}$.	28.	$\frac{3}{4} - \frac{2}{5}$.
29.	$\frac{1}{2} - \frac{3}{8}$.	30.	$\frac{3}{8} - \frac{1}{4}$.	31.	$\frac{5}{8} - \frac{1}{2}$.	32.	$\frac{3}{4} - \frac{5}{8}$.
33.	$\frac{7}{8} - \frac{3}{4}$.	34.	$\frac{7}{9} - \frac{2}{3}$.	35.	$\frac{5}{6} - \frac{1}{2}$.	36.	$\frac{5}{6} - \frac{1}{3}$.
37.	$\frac{2}{3} - \frac{1}{6}$.	38.	$\frac{11}{12} - \frac{1}{2}$.	39.	$\frac{8}{15} - \frac{2}{5}$.	40.	$\frac{13}{54} - \frac{1}{6}$.
41.	$3\frac{2}{3} - 2\frac{1}{2}$.	42.	$2\frac{1}{3} - 1\frac{1}{6}$.	43.	$5\frac{1}{2} - 2\frac{1}{3}$.	44.	$3\frac{4}{5} - 2\frac{1}{10}$.
45.	$5\frac{7}{8} - 3\frac{1}{4}$.	46.	$4\frac{9}{10} - 3\frac{9}{20}$.	47.	$13\frac{7}{12} - 3\frac{1}{2}$.	48.	$5\frac{7}{9} - \frac{5}{18}$.

EXAMPLES. 63.

Find the value of each of the following expressions:

1.	$\frac{14}{25} - \frac{3}{20}$.	2.	$\frac{11}{27} - \frac{5}{18}$.	3.	$\frac{12}{35} - \frac{5}{21}$.	4.	$\frac{12}{14} - \frac{1}{21}$.
5.	$\frac{35}{36} - \frac{5}{27}$.	6.	$\frac{9}{10} - \frac{7}{15}$.	7.	$\frac{8}{9} - \frac{8}{13}$.	8.	$\frac{15}{16} - \frac{2}{7}$.
9.	$\frac{49}{50} - \frac{4}{9}$.	10.	$\frac{11}{20} - \frac{8}{15}$.	11.	$\frac{16}{21} - \frac{8}{15}$.	12.	$16\frac{3}{32} - 4\frac{1}{40}$.
13.	$161\frac{7}{8} - 128\frac{11}{36}$.	14.	$6\frac{4}{7} - \frac{6}{7}$.	15.	$20\frac{1}{8} - 2\frac{1}{4}$.		
16.	$12\frac{3}{4} - 11\frac{7}{8}$.	17.	$10\frac{1}{8} - 5\frac{3}{16}$.	18.	$4\frac{1}{6} - \frac{13}{24}$.		
19.	$10\frac{1}{9} - 3\frac{1}{6}$.	20.	$6\frac{5}{21} - \frac{4}{7}$.	21.	$11\frac{1}{40} - 6\frac{5}{8}$.		
22.	$14\frac{2}{9} - 9\frac{5}{8}$.	23.	$456\frac{1}{2} - 125\frac{19}{20}$.	24.	$153\frac{1}{6} - 109\frac{5}{24}$.		
25.	$8\frac{3}{7} - 6\frac{4}{5}$.	26.	$12\frac{3}{20} - 9\frac{7}{15}$.	27.	$20\frac{11}{30} - 13\frac{7}{12}$.		
28.	$4\frac{1}{21} - 3\frac{9}{14}$.	29.	$5\frac{3}{16} - 2\frac{7}{12}$.	30.	$16\frac{5}{8} - 13\frac{9}{14}$.		

Find the value of each of the following expressions:

31. $10\frac{12}{143} - 7\frac{8}{11}$. 32. $19\frac{5}{432} - 12\frac{11}{54}$. 33. $15\frac{1}{10} - 11\frac{8}{11}$.

34. $11\frac{34}{35} - 9\frac{273}{280}$. 35. $10\frac{1}{19} - 5\frac{3}{20}$. 36. $15\frac{9}{17} - 14\frac{14}{15}$.

37. $30\frac{7}{30} - 22\frac{5}{13}$. 38. $65\frac{1}{60} - 19\frac{2}{39}$. 39. $30\frac{1}{70} - 24\frac{6}{55}$.

164. The preceding examples have involved either addition alone, or subtraction alone. We will now consider some which involve both.

Example i. *Express* $\frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \frac{1}{6} - \frac{3}{8}$ *by a single fraction.*

We see, by inspection, that the L.C.M. of the denominators is 24. Hence

$$\frac{1}{2} + \frac{2}{3} - \frac{3}{4} + \frac{1}{6} - \frac{3}{8} = \frac{12+16-18+4-9}{24}.$$

The sum of the terms in the numerator which are preceded by + is 32, and the sum of those preceded by − is 27;

$$\therefore \text{ the given expression} = \frac{32-27}{24} = \frac{5}{24}.$$

Example ii. *Simplify*

$$2\frac{1}{2} + 3\frac{1}{3} - 4\frac{1}{5} - 1\frac{2}{3} + 6\frac{5}{6}.$$

$$\text{Exp.} = 6 + \frac{15+10-6-20+25}{30}$$
$$= 6 + \frac{50-26}{30}$$
$$= 6\frac{24}{30}$$
$$= 6\frac{4}{5}.$$

Example iii. *Simplify*

$$5\frac{1}{4} - 1\frac{1}{12} - 1\frac{7}{8} - \frac{5}{6}.$$

$$\text{Exp.} = 4 + \frac{6-22-21-20}{24}$$
$$= 4 + \frac{6-63}{24}$$
$$= 4 - \frac{57}{24}$$
$$= 4 - \frac{19}{8}$$
$$= 4 - 2\frac{3}{8}$$
$$= 1\frac{5}{8}.$$

EXAMPLES. 64.

Simplify:

1. $1\frac{1}{2} + 3\frac{5}{6} - 2\frac{5}{12}$. 2. $3\frac{1}{3} + 4\frac{1}{5} - 3\frac{2}{15}$. 3. $13\frac{4}{5} - 9\frac{3}{9} - 1\frac{8}{45}$.

4. $5\frac{1}{2} + 3\frac{1}{24} - 1\frac{1}{8}$. 5. $12\frac{1}{5} - 10\frac{3}{8} + \frac{8}{15}$. 6. $15\frac{3}{4} + 1\frac{1}{3} - 3\frac{1}{9}$.

7. $15\frac{2}{3} + 1\frac{9}{10} - \frac{7}{15} - 10\frac{17}{80}$. 8. $3\frac{5}{9} - 1\frac{3}{27} + 3\frac{1}{3} + 5\frac{7}{8}$.

9. $9\frac{7}{16} + 16\frac{11}{80} - 10\frac{1}{2} + 26 - 1\frac{11}{20}$. 10. $3\frac{8}{13} - \frac{6}{23} - 1\frac{3}{26} + \frac{5}{46}$.

11. $7\frac{1}{3} + 5\frac{1}{13} - 2\frac{9}{16}$. 12. $6\frac{1}{4} - 2\frac{3}{8} + \frac{5}{9} - 1\frac{3}{7}$.

13. $6\frac{5}{8} - 2\frac{8}{25} + 1\frac{9}{104}$. 14. $7\frac{4}{7} - 2\frac{3}{5} + 10 - 1\frac{5}{32}$.

15. $47\frac{5}{27} - 3\frac{2}{5} - 31\frac{5}{54} - 2\frac{5}{8}$. 16. $7\frac{2}{3} + 6\frac{3}{4} - 3\frac{4}{15} - 2\frac{17}{25} + \frac{11}{20}$.

17. $10\frac{6}{29} - 3\frac{95}{174} - 2\frac{1}{30}$. 18. $12\frac{1}{2} - 22 + 1\frac{9}{20} - \frac{13}{65} + 17\frac{57}{64}$.

19. $4\frac{7}{12} - 2\frac{4}{9} - \frac{25}{144} + 14\frac{1}{7}$. 20. $4\frac{1}{9} - 1\frac{5}{18} + 3\frac{5}{8} + 12\frac{9}{16} - \frac{11}{27} - \frac{1}{4}$.

XVI. (h). TO MULTIPLY BY A WHOLE NUMBER.

165. Multiplication by an integer is a short way of obtaining the sum of a number of *equal* quantities. For example:

$$4 \text{ times } \tfrac{3}{17} = \tfrac{3}{17} + \tfrac{3}{17} + \tfrac{3}{17} + \tfrac{3}{17}$$
$$= \tfrac{3+3+3+3}{17} = \tfrac{4\times 3}{17} = \tfrac{12}{17}.$$

Otherwise: $\tfrac{3}{17} =$ 3 *seventeenth*-parts;

$\therefore$ 4 times $\tfrac{3}{17} = 12$ ..... $= \tfrac{12}{17}$.

Hence, *to multiply a fraction by a whole number we multiply the numerator by the number, and leave the denominator unaltered.*

166. The resulting fraction must always be expressed in its lowest terms. If, therefore, the multiplier should introduce into the numerator a factor which is already in the denominator, this factor must be removed from both. For example,

$$3 \text{ times } \tfrac{4}{15} = \tfrac{3\times 4}{15} = \tfrac{3\times 4}{3\times 5} = \tfrac{4}{5}.$$

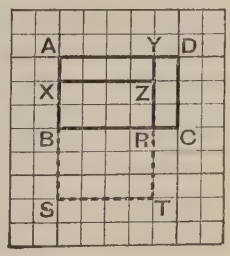
Hence, *if the multiplier be a factor of the denominator, the result may be obtained more easily by dividing the denominator by the multiplier, and leaving the numerator unaltered.*

Again, 6 times $\tfrac{4}{15} = 2 \times 3 \times \tfrac{4}{15} = 2 \times \tfrac{4}{5} = \tfrac{8}{5} = 1\tfrac{3}{5}$.

Hence, *if the multiplier consist of factors, some of which are also factors of the denominator and others not, the result may be obtained at once, in its lowest terms, by removing common factors from the multiplier and the denominator, and multiplying the numerator by the factors of the multiplier that remain.*

This operation of removing common factors from numerator and denominator is called **cancelling.**

167. The diagram in the margin illustrates the process graphically. The figure ABCD contains 15 squares, and the figure AXZY contains 4 squares. The figure AXZY represents therefore $\tfrac{4}{15}$ of the figure ABCD. Three such strips as AXZY make up the figure ABRY, which therefore represents the result of multiplying $\tfrac{4}{15}$ by 3. It contains 12 squares, and is therefore $\tfrac{12}{15}$ of the figure ABCD.

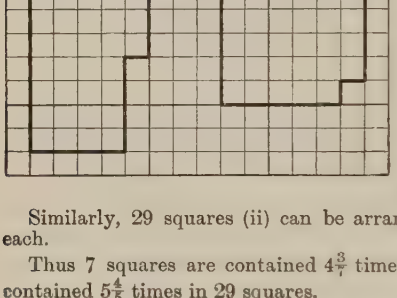


If we take 6 strips such as AXZY we make up the figure ASTY, which contains 24 squares and is

therefore $\frac{24}{15}$ of the figure ABCD. Thus the result of multiplying $\frac{4}{15}$ by 6 is $\frac{24}{15}$, or $\frac{8}{5}$, or $1\frac{3}{5}$.

168. To multiply a mixed number, we multiply the integral part and the fractional part separately, and we add the products. For example, $3 \times 2\frac{1}{2} = 6 + \frac{3}{2} = 6 + 1\frac{1}{2} = 7\frac{1}{2}$.

NOTE.—We may notice here that since, for example, $7 \times 4\frac{3}{7} = 31$, it follows that when we divide 31 by 7, the *complete* quotient is $4\frac{3}{7}$. That is, if a division does not terminate without remainder, we may *complete* the quotient by annexing to its integral part a fraction which has the remainder for numerator and the divisor for denominator.



Similarly, if we divide 29 by 5, the quotient is 5 and the remainder 4; therefore the *complete quotient* is $5\frac{4}{5}$.

The figure (i) in the margin shows how 31 squares can be arranged in 4 columns, each containing 7 squares, with 3 squares remaining over. These 3 squares form $\frac{3}{7}$ of a column, so that we may say that altogether there are $4\frac{3}{7}$ columns of 7 squares each.

Similarly, 29 squares (ii) can be arranged in $5\frac{4}{5}$ columns of 5 squares each.

Thus 7 squares are contained $4\frac{3}{7}$ times in 31 squares and 5 squares are contained $5\frac{4}{5}$ times in 29 squares.

EXAMPLES. 65. *Oral.*

Multiply

1.	$\frac{1}{4}$ by 3.	2.	$\frac{2}{11}$ by 2.	3.	$\frac{4}{13}$ by 3.	4.	$\frac{2}{9}$ by 4.
5.	$\frac{7}{38}$ by 3.	6.	$\frac{3}{19}$ by 4.	7.	$\frac{5}{28}$ by 5.	8.	$\frac{5}{36}$ by 5.
9.	$\frac{5}{61}$ by 11.	10.	$\frac{3}{4}$ by 3.	11.	$\frac{4}{5}$ by 4.	12.	$\frac{8}{9}$ by 2.
13.	$\frac{3}{4}$ by 5.	14.	$\frac{7}{10}$ by 7.	15.	$\frac{5}{12}$ by 7.	16.	$\frac{24}{29}$ by 2.
17.	$\frac{55}{91}$ by 2.	18.	$\frac{13}{23}$ by 3.	19.	$\frac{13}{63}$ by 8.	20.	$\frac{6}{11}$ by 9.
21.	$\frac{1}{4}$ by 2.	22.	$\frac{3}{4}$ by 4.	23.	$\frac{2}{3}$ by 3.	24.	$\frac{5}{24}$ by 2, 48

EXAMPLES. 66.

Multiply

1.	$\frac{2}{15}$ by 21 and 25.	2.	$\frac{5}{21}$ by 48 and 49.
3.	$\frac{14}{33}$ by 88 and 90.	4.	$\frac{13}{34}$ by 51 and 40.

5. $\frac{98}{100}$ by 15 and 50.
6. $\frac{39}{147}$ by 539 and 56.
7. $\frac{60}{148}$ by 110 and 121.
8. $\frac{41}{112}$ by 128 and 168.
9. $\frac{43}{45}$ by 108 and 153.
10. $\frac{19}{56}$ by 3514 and 4016.
11. $2\frac{1}{2}$ by 4 and 5.
12. $4\frac{2}{3}$ by 7 and 11.
13. $6\frac{4}{5}$ by 3 and 12.
14. $7\frac{2}{7}$ by 4 and 15.
15. $9\frac{1}{7}$ by 8 and 16.
16. $8\frac{5}{14}$ by 13 and 19.
17. $3\frac{17}{22}$ by 15 and 21.
18. $2\frac{27}{35}$ by 12 and 22.
19. $8\frac{5}{14}$ by 2 and 7.
20. $2\frac{27}{35}$ by 5 and 7.
21. $3\frac{17}{22}$ by 2 and 11.
22. $10\frac{13}{34}$ by 2 and 17.

XVI. (i). TO DIVIDE BY A WHOLE NUMBER.

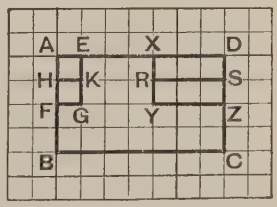
169. If we divide a quantity into 14 equal parts, each part is $\frac{1}{14}$ of the whole quantity. And if we divide each of these *fourteenth*-parts again into 2 equal parts, we get 28 equal parts altogether, each of which is $\frac{1}{28}$ of the whole quantity. Hence

$$\frac{1}{14} \div 2 = \frac{1}{28}.$$

Similarly, if we have $\frac{3}{14}$ of the whole quantity, and divide each fourteenth-part into 2 equal parts, we get 6 *twenty-eighth* parts; and if we divide this by 2, we get 3 *twenty-eighth* parts. Thus

$$\frac{3}{14} \div 2 = \frac{6}{28} \div 2 = \frac{3}{28} = \frac{3}{2 \times 14}.$$

Hence, *to divide a fraction by a whole number we multiply the denominator by the number, and leave the numerator unaltered.*



The figure ABCD in the margin contains 28 squares, or 14 double squares. The figure AFGE is therefore $\frac{1}{14}$ of the whole figure ABCD. If we divide the figure AFGE by 2, we get the single square AHKE, which is $\frac{1}{28}$ of the whole figure ABCD.

Similarly, the figure XYZD contains 3 double-squares, and is therefore $\frac{3}{14}$ of the figure ABCD. If we divide it by 2, we get the figure XRSD, which contains 3 squares and is $\frac{3}{28}$ of the figure ABCD. Thus

$$\frac{1}{14} \div 2 = \frac{1}{28}, \text{ and } \frac{3}{14} \div 2 = \frac{3}{28}.$$

170. The resulting fraction must always be reduced to its lowest terms; that is, all factors common to numerator and denominator must be removed. Thus

$\frac{6}{7} \div 2 = \frac{6}{14} = \frac{3}{7}$, and $\frac{210}{1573} \div 378 = \frac{3 \times 7 \times 2 \times 5}{1573 \times 2 \times 3 \times 7 \times 9} = \frac{5}{14157}$.

Hence, in practice, we simplify the application of the rule by noticing that *if the divisor be a factor of the numerator we divide the numerator by it, and leave the denominator unchanged;* and *if the divisor can be separated into factors, some of which are factors of the numerator and others not, we cancel all those common factors and multiply the denominator only by those that remain.*

EXAMPLES. 67. *Oral.*

Divide

1. $\frac{1}{2}$ by 2, 3, 4, 5.
2. $\frac{1}{3}$ by 2, 3, 4, 5.
3. $\frac{3}{5}$ by 4, 5, 7, 10.
4. $\frac{2}{9}$ by 3, 5, 7, 9.
5. $\frac{3}{7}$ by 2, 4, 5, 8.
6. $\frac{12}{13}$ by 2, 3, 4, 6.
7. $\frac{36}{43}$ by 3, 4, 6.
8. $\frac{63}{94}$ by 7, 9, 21.
9. $\frac{120}{359}$ by 6, 20, 40.
10. $\frac{42}{151}$ by 3, 6, 7.
11. $\frac{112}{121}$ by 4, 7, 28.
12. $\frac{243}{560}$ by 9, 27, 81.
13. $\frac{42}{45}$ by 6, 7, 21.
14. $\frac{171}{520}$ by 9, 19, 57.
15. $\frac{255}{641}$ by 5, 15, 17, 51.
16. $\frac{4}{19}$ by 2, 4, 8, 16.
17. $\frac{5}{18}$ by 5, 10, 15, 30.
18. $\frac{3}{20}$ by 6, 12, 24, 36.
19. $\frac{6}{7}$ by 6, 18, 36, 60.
20. $\frac{15}{28}$ by 3, 5, 30, 45.

EXAMPLES. 68.

Divide

1. $\frac{14}{15}$ by 6, 8, and 10.
2. $\frac{40}{67}$ by 16, 24, and 36.
3. $\frac{35}{36}$ by 14, 21, and 28.
4. $\frac{33}{100}$ by 11, 22, and 24.
5. $\frac{683}{1211}$ by 28, 35, and 54.
6. $\frac{1485}{3422}$ by 25, 75, and 87.
7. $\frac{57}{140}$ by 18, 24, and 38.
8. $\frac{54}{65}$ by 16, 18, and 135.
9. $\frac{1255}{1256}$ by 55, 75, and 95.
10. $\frac{64}{71}$ by 48, 96, and 160.
11. $2\frac{1}{2}$ by 2, 6, and 14.
12. $3\frac{3}{4}$ by 4, 9, and 11.
13. $7\frac{1}{3}$ by 5, 7, and 9.
14. $16\frac{2}{3}$ by 5, 7, and 15
15. $1\frac{5}{9}$ by 7, 21, and 28.
16. $2\frac{1}{6}$ by 13, 65, and 117
17. $3\frac{2}{5}$ by 17, 34, and 85.
18. $4\frac{2}{9}$ by 19, 38, and 95.
19. $3\frac{5}{6}$ by 5, 11, and 46
20. $16\frac{2}{7}$ by 19, 25, and 32.

XVI. (j). TO MULTIPLY BY A FRACTION.

171. Hitherto we have dealt only with fractions of the whole unit. We will now consider how to determine the relation which a *part of a part* bears to the whole unit, and how we may represent this relation by a single fraction. We will find, for example, single fractions equivalent to such expressions as $\frac{1}{2}$ *of* $\frac{3}{4}$ and $\frac{2}{3}$ *of* $\frac{5}{7}$.

172. We know that $\frac{1}{2}$ of a quantity means the result of dividing the quantity by 2. But if we divide $\frac{3}{4}$ by 2, we get $\frac{3}{2\times 4}$;

hence $\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{2\times 4} = \frac{3}{8}$ .....(1);

and, similarly, $\frac{1}{3}$ of $\frac{5}{7} = \frac{5}{3\times 7} = \frac{5}{21}$ .....(2).

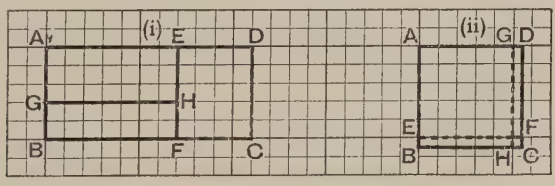
Again, since $\frac{2}{3} = 2 \times \frac{1}{3}$,

$\therefore$ $\frac{2}{3}$ of $\frac{5}{7} = 2 \times \frac{1}{3}$ of $\frac{5}{7} = 2 \times \frac{5}{3\times 7} = \frac{2\times 5}{3\times 7}$ .....(3);

and, similarly, $\frac{3}{5}$ of $\frac{2}{41} = \frac{3\times 2}{5\times 41} = \frac{6}{205}$ .....(4).

Hence *the symbol that represents the relation which a fraction of a fraction bears to the whole quantity has for its numerator the product of the numerators of the two fractions, and for its denominator the product of the two denominators.* For this reason the operation is described as the *multiplication* of one fraction by another.

173. The diagram (i) illustrates the multiplication of $\frac{3}{5}$ by $\frac{7}{11}$.



The figure ABCD contains 55 squares in 11 columns of 5 squares each, and the figure ABFE is made up of 7 of these columns. The figure ABFE is therefore $\frac{7}{11}$ of the figure ABCD.

Again the figure ABFE is made up of 5 rows of 7 squares each, and the figure AGHE is made up of 3 of these rows. The figure AGHE is therefore $\frac{3}{5}$ of the figure ABFE; but ABFE is $\frac{7}{11}$ of ABCD;

$\therefore$ AGHE $= \frac{3}{5}$ of $\frac{7}{11}$ of ABCD.

But AGHE $= \frac{21}{55}$ of ABCD, since AGHE contains 21 squares and ABCD contains 55. Hence we see that $\frac{3}{5}$ of $\frac{7}{11}$ of a thing is the same as $\frac{21}{55}$ of it.

The second diagram (ii) illustrates the multiplication of $5\frac{1}{2}$ by $5\frac{1}{2}$, the result being $30\frac{1}{4}$.

174. If there are any factors common to either numerator and either denominator, they should be removed before multiplication.

Thus

$$\frac{105}{286} \times \frac{198}{350} = \frac{3 \times \cancel{5} \times \cancel{7}}{2 \times \cancel{7} \times \cancel{11} \times 13} \times \frac{\cancel{9} \times \cancel{2} \times \cancel{11}}{\cancel{5} \times \cancel{7} \times \cancel{2} \times \cancel{5}} = \frac{27}{130}.$$

Or thus

$$\text{the expression} = \frac{\overset{3}{\cancel{21}}}{\underset{26}{\cancel{286}}\ 13} \times \frac{\overset{9}{\cancel{18}}}{\underset{7}{\cancel{35}}\ 10} = \frac{27}{130}.$$

175. It follows from Art. 172 that such an expression as

$$\tfrac{1}{2} \text{ of } \tfrac{3}{5} \text{ of } \tfrac{7}{11} = \tfrac{1}{2} \text{ of } \tfrac{3 \times 7}{5 \times 11}, \text{ and that this} = \frac{1 \times 3 \times 7}{2 \times 5 \times 11};$$

We proceed in the same way whatever be the number of fractions involved. Thus

$$\frac{3}{5} \times \frac{2}{7} \times \frac{1}{13} \times \frac{11}{10} = \frac{3 \times 2 \times 1 \times 11}{5 \times 7 \times 13 \times \underset{5}{10}} = \frac{33}{2275};$$

and $\frac{2}{3}$ of $\frac{5}{7}$ of $\frac{6}{15}$ of $\frac{21}{56}$ of $\frac{4}{9} = \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{5}}{7} \times \frac{\cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{5}} \times \frac{\cancel{3} \times \cancel{7}}{\cancel{7} \times \cancel{4} \times \cancel{2}} \times \frac{\cancel{4}}{\cancel{9}} = \frac{2}{63}$.

Hence *the product of any number of fractions is a fraction which has for its numerator the product of the original numerators, and for its denominator the product of the original denominators, the resulting fraction being reduced to its lowest terms by the removal of common factors.*

176. The fractions to be multiplied may be either proper or improper, or any of them may be whole or mixed numbers, for which improper fractions can be substituted.

Example i. *Simplify* $\frac{2}{3} \times 5\frac{1}{2} \times 2 \times 7\frac{5}{6} \times \frac{9}{11}$.

$$\text{The product} = \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{11}}{\cancel{2}} \times \frac{\cancel{2}}{1} \times \frac{47}{\cancel{6}} \times \frac{\cancel{9}}{\cancel{11}} = 47.$$

Example ii. *Simplify* $3\frac{1}{5}$ of $5\frac{5}{8}$ of $3\frac{5}{9}$ of $\frac{3}{22}$ of $\frac{33}{8}$ of $\frac{4}{39}$.

$$\text{The expression} = \frac{\cancel{16}}{\cancel{5}} \times \frac{\cancel{9}\ \cancel{8}}{\cancel{8}} \times \frac{\overset{16}{\cancel{32}}}{\cancel{9}} \times \frac{\cancel{8}}{\cancel{22}} \times \frac{\overset{3}{\cancel{33}}}{\cancel{8}} \times \frac{\cancel{4}}{\underset{13}{\cancel{39}}} = \frac{48}{13} = 3\frac{9}{13}.$$

Example iii. *Find the value of* $\frac{1}{5}$ of $\frac{4}{7}$ of $\frac{8}{1\,9}$ of $4\frac{3}{4}$ of 247 *guineas.*

$$\text{The value} = \tfrac{1}{5} \times \tfrac{\cancel{4}}{7} \times \tfrac{8}{\cancel{1\,9}} \times \tfrac{\cancel{1\,9}}{\cancel{4}} \times 247 \text{ guineas}$$

$$= \tfrac{8}{35} \times 247 \times 21 \text{ shillings}$$

$$= \tfrac{8}{5} \times 247 \times 3 \text{ shillings}$$

$$= \tfrac{5928}{5} \text{ shillings}$$

$$= 1185\tfrac{3}{5} \text{ shillings}$$

$$= £59.\ 5s.\ 7\tfrac{1}{5}d.$$

EXAMPLES. 69. *Oral.*

Multiply

1. $\frac{1}{2}$ by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$.
2. $\frac{1}{3}$ by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$.
3. $\frac{1}{4}$ by $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{12}$.
4. $\frac{1}{6}$ by $\frac{1}{7}$, $\frac{1}{6}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{10}$, $\frac{1}{15}$.
5. $\frac{1}{8}$ by $\frac{1}{10}$, $\frac{1}{6}$, $\frac{1}{2}$, $\frac{1}{12}$, $\frac{1}{5}$, $\frac{1}{9}$.
6. $\frac{1}{8}$ by $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{13}$, $\frac{1}{11}$, $\frac{1}{3}$, $\frac{1}{7}$.
7. $\frac{1}{3}$ by $\frac{2}{5}$, $\frac{1}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{4}{5}$, $\frac{4}{7}$.
8. $\frac{1}{2}$ by $\frac{3}{7}$, $\frac{1}{9}$, $\frac{3}{5}$, $\frac{1}{5}$, $\frac{5}{7}$, $\frac{7}{8}$.
9. $\frac{1}{4}$ by $\frac{5}{8}$, $\frac{3}{5}$, $\frac{7}{9}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{8}$.
10. $\frac{1}{2}$ by $\frac{7}{9}$, $\frac{5}{8}$, $\frac{3}{5}$, $\frac{3}{7}$, $\frac{7}{10}$, $\frac{1}{12}$.
11. $\frac{2}{3}$ by $\frac{4}{5}$, $\frac{5}{9}$, $\frac{2}{3}$, $\frac{1}{3}$, $\frac{4}{7}$, $\frac{2}{5}$.
12. $\frac{3}{7}$ by $\frac{2}{5}$, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{5}$, $\frac{4}{7}$, $\frac{5}{8}$.
13. $\frac{4}{11}$ by $\frac{7}{13}$, $\frac{4}{7}$, $\frac{8}{9}$, $\frac{6}{11}$, $\frac{10}{13}$, $\frac{5}{7}$.
14. $\frac{6}{11}$ by $\frac{7}{11}$, $\frac{3}{5}$, $\frac{1}{13}$, $\frac{6}{7}$, $\frac{5}{11}$, $\frac{3}{7}$.
15. $\frac{5}{12}$ by $\frac{1}{2}$, $\frac{5}{4}$, $\frac{7}{8}$, $\frac{7}{9}$, $\frac{11}{12}$, $\frac{5}{7}$.
16. $\frac{13}{15}$ by $\frac{2}{5}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{9}$, $\frac{7}{8}$, $\frac{8}{11}$.
17. $\frac{8}{9}$ by $\frac{5}{13}$, $\frac{2}{5}$, $\frac{4}{7}$, $\frac{8}{9}$, $\frac{7}{9}$, $\frac{2}{3}$.
18. $\frac{2}{3}$ by $\frac{9}{7}$; $\frac{3}{4}$ by $\frac{8}{9}$; $\frac{7}{8}$ by $\frac{16}{21}$; $\frac{16}{49}$ by $\frac{7}{8}$; $\frac{22}{51}$ by $\frac{17}{2}$.

EXAMPLES. 70.

Multiply

1. $\frac{22}{35}$ by $\frac{16}{19}$, $\frac{18}{45}$, $\frac{13}{25}$.
2. $\frac{17}{19}$ by $\frac{18}{45}$, $\frac{27}{89}$, $\frac{31}{45}$.
3. $\frac{39}{44}$ by $\frac{11}{13}$, $\frac{22}{39}$, $\frac{66}{13}$.
4. $\frac{14}{27}$ by $\frac{12}{35}$, $\frac{5}{28}$, $\frac{36}{35}$.
5. $\frac{39}{44}$ by $\frac{3}{26}$, $\frac{55}{42}$, $\frac{80}{91}$.
6. $\frac{6}{25}$ by $\frac{35}{54}$, $\frac{65}{66}$, $\frac{75}{92}$.
7. $\frac{18}{65}$ by $\frac{25}{27}$, $\frac{80}{81}$, $\frac{39}{80}$.
8. $\frac{36}{65}$ by $\frac{35}{152}$, $\frac{39}{108}$, $\frac{75}{144}$.
9. $2\frac{1}{2}$ by $\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$.
10. $3\frac{2}{3}$ by $\frac{2}{5}$, $\frac{1}{7}$, $\frac{5}{6}$.
11. $5\frac{3}{4}$ by $\frac{7}{11}$, $\frac{4}{16}$, $\frac{32}{46}$.
12. $9\frac{7}{9}$ by $\frac{12}{11}$, $\frac{15}{42}$, $\frac{36}{88}$.
13. $6\frac{7}{8}$ by $\frac{3}{7}$, $\frac{5}{8}$, $\frac{1}{6}$.
14. $3\frac{3}{4}$ by $2\frac{1}{2}$, $5\frac{1}{4}$, $7\frac{2}{3}$.
15. $7\frac{1}{2}$ by $1\frac{5}{6}$, $4\frac{1}{2}$, $3\frac{4}{5}$.
16. $4\frac{7}{8}$ by $\frac{4}{13}$, $\frac{10}{91}$, $\frac{16}{39}$.
17. $5\frac{5}{12}$ by $3\frac{3}{5}$, $2\frac{7}{10}$, $5\frac{1}{4}$.
18. $8\frac{15}{16}$ by $6\frac{2}{11}$, $3\frac{1}{9}$, $2\frac{2}{3}$.
19. $3\frac{3}{8}$ by $1\frac{81}{133}$, $4\frac{5}{6}$, $7\frac{2}{3}$.
20. $1\frac{8}{55}$ by $2\frac{1}{49}$, $17\frac{1}{3}$, $6\frac{2}{7}$.

Find the value of

21. $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{5}$.
22. $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{1}{7}$.
23. $\frac{2}{3}$ of $\frac{7}{9}$ of $\frac{5}{11}$.
24. $\frac{2}{3} \times \frac{11}{7} \times 2\frac{1}{4} \times \frac{4}{11}$.
25. $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7}$.
26. $4\frac{1}{2} \times \frac{1}{3} \times \frac{4}{8}$.
27. $\frac{2}{3}$ of $\frac{6}{13}$ of $\frac{39}{4}$.
28. $2\frac{1}{2} \times \frac{18}{35} \times \frac{10}{3}$.
29. $\frac{8}{11} \times 16\frac{1}{2} \times \frac{5}{61}$.
30. $\frac{3}{7} \times 14 \times \frac{3}{2} \times 3\frac{5}{8}$.
31. $1\frac{1}{2} \times 1\frac{1}{3} \times 1\frac{1}{5} \times 1\frac{1}{8} \times 1\frac{1}{7}$.
32. $\frac{11}{5} \times 3\frac{1}{2} \times \frac{8}{5}$ of $\frac{2}{3}$ of 10.
33. $\frac{3}{9}$ of $\frac{4}{7} \times \frac{9}{11} \times \frac{2}{3}$ of 18.
34. $\frac{4}{7}$ of $8\frac{3}{10} \times \frac{5}{7}$ of $9\frac{4}{5}$.
35. $3\frac{2}{3}$ of $\frac{9}{52} \times \frac{3}{8} \times \frac{3}{5} \times 4\frac{7}{8} \times \frac{5}{13} \times 11\frac{1}{4} \times \frac{2}{15} \times 9\frac{5}{11}$ of $\frac{64}{243}$.
36. $\frac{2}{3}$ of $\frac{9}{16}$ of 6*s*.
37. $\frac{3}{2}$ of $\frac{8}{9}$ of 4*s*. 6*d*.
38. $\frac{1}{12}$ of $\frac{3}{4}$ of 5*s*.
39. $15\frac{3}{5}$ of $\frac{5}{39}$ of £1.
40. $\frac{3}{8}$ of $\frac{1}{2}$ of £1.
41. $2\frac{4}{9}$ of $5\frac{1}{8}$ of $\frac{2}{55}$ of £1. 2*s*. 6*d*.
42. $\frac{1}{5}$ of $\frac{13}{18}$ of 1*s*.
43. $2\frac{1}{10}$ of $\frac{3}{2}$ of a guinea.
44. $2\frac{5}{8}$ of 1*s*.
45. $3\frac{4}{7}$ of 10 guineas.
46. $3\frac{9}{16}$ of 1 lb.
47. $3\frac{1}{27}$ of 2 qrs. 25 lb.
48. $1\frac{2}{9}$ of £1. 6*s*. 3*d*.
49. $2\frac{4}{9}$ of £16. 16*s*. $4\frac{1}{2}$*d*.
50. $5\frac{1}{4}$ of £3. 7*s*. 8*d*.

XVI. (k). TO DIVIDE BY A FRACTION.

177. It is essential to the definition of division that the divisor should be a whole number, but if we extend the definition and consider *dividend*, *divisor* and *quotient* merely as quantities, such that *Dividend* = *Divisor* × *Quotient*, we may allow the divisor to be a fraction or a mixed number, and we may consider the quotient as merely answering the question—"*By what must the divisor be multiplied in order that the result may be the dividend?*"

Thus the quotient of 7 divided by 8 is $\frac{7}{8}$; that of $3\frac{1}{2}$ divided by $\frac{1}{2}$ is 7; that of 7 by $3\frac{1}{2}$ is 2; and that of 28 by 3 is $9\frac{1}{3}$.

We may therefore consider a fraction to represent such a quantity that *when it is multiplied by the denominator the result is the numerator.*

178. A fraction which has a fractional numerator or a fractional denominator is called a **complex** fraction. When both are integers it is called a **simple** fraction.

179. The practical method of simplifying a complex fraction is as follows.

We have seen (Art. 145) that the *value* of a fraction is not altered if we multiply both the numerator and the denominator by the same number. For this purpose we choose such a multiplier as will make both the numerator and the denominator whole numbers. If we consider the fraction $3\frac{1}{3} \div 7\frac{3}{4}$, we see that both $3\frac{1}{3}$ and $7\frac{3}{4}$ become whole numbers when multiplied by **12**. Hence 12 is the multiplier to use in this case, and we find that

$$\frac{3\frac{1}{3}}{7\frac{3}{4}} = \frac{3\frac{1}{3} \times 12}{7\frac{3}{4} \times 12} = \frac{40}{93}.$$

Similarly,
$$\frac{\frac{1}{2}}{6\frac{5}{6}} = \frac{\frac{1}{2} \times 6}{6\frac{5}{6} \times 6} = \frac{3}{41}.$$

In the first case the multiplier must be a multiple of 3 and of 4, and in the second case it must be a multiple of 2 and of 6. As a rule it should be the L.C.M. of the two denominators.

When the two denominators are large, it is better to arrange the work as follows, but the principle is the same:

$$\frac{4\frac{2}{5}\frac{5}{7}}{7\frac{4}{2}\frac{}{3}} = \frac{4\frac{2}{5}\frac{5}{7}}{1} \times \frac{1}{7\frac{4}{2}\frac{}{3}} = \frac{253}{57} \times \frac{23}{165} = \frac{23 \times 23}{57 \times 15} = \frac{529}{855}.$$

Hence, *to obtain a simple fraction equivalent to a complex fraction we may express the numerator and the denominator in the form of proper or improper fractions, and multiply the fractional numerator by the fractional denominator* **inverted.**

Example i. $5 \div \frac{7}{10} = 5 \times \frac{10}{7} = \frac{50}{7} = 7\frac{1}{7}$.

Example ii. $\frac{9}{16} \div \frac{5}{7} = \frac{9}{16} \times \frac{7}{5} = \frac{63}{80}$.

Example iii. $\frac{3}{8} \div 5\frac{1}{2} = \frac{3}{8} \div \frac{11}{2} = \frac{3}{8} \times \frac{2}{11} = \frac{3}{44}$.

Example iv. $\frac{3\frac{1}{2}}{2\frac{1}{4}} \div \frac{6}{4\frac{1}{2}} = \frac{14}{9} \div \frac{12}{9} = \frac{14}{12} = \frac{7}{6} = 1\frac{1}{6}$.

180. The difference in meaning between

$$3\tfrac{1}{3} \div 2\tfrac{1}{2} \times 7\tfrac{1}{4}$$

and
$$3\tfrac{1}{3} \div 2\tfrac{1}{2} \text{ of } 7\tfrac{1}{4}$$

should be noticed. **In the former the sign ÷ extends only to the next quantity $2\frac{1}{2}$, and no farther. But, in the latter, $2\frac{1}{2}$ of $7\frac{1}{4}$ is a single quantity.**

Thus the former $= \frac{10}{3} \times \frac{2}{5} \times \frac{29}{4} = 9\frac{2}{3}$,

but the latter $= \frac{10}{3} \times \frac{2}{5} \times \frac{4}{29} = \frac{16}{87}$.

EXAMPLES. 71. *Oral.*

Divide

1. $\frac{1}{7}$ by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$.
2. $\frac{1}{9}$ by $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{7}$.
3. $\frac{1}{5}$ by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$.
4. $\frac{1}{2}$ by $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{9}$.
5. $\frac{1}{3}$ by $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{7}$.
6. $\frac{1}{4}$ by $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{9}$.
7. $\frac{1}{2}$ by $\frac{2}{3}$, $\frac{3}{5}$, $\frac{4}{7}$, $\frac{5}{9}$.
8. $\frac{2}{3}$ by $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{5}$, $\frac{5}{7}$.
9. $\frac{3}{5}$ by $\frac{2}{7}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{2}{3}$.
10. $\frac{2}{7}$ by $\frac{3}{5}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{3}{4}$.
11. $\frac{7}{9}$ by $\frac{5}{4}$, $\frac{2}{5}$, $\frac{4}{5}$, $\frac{1}{8}$.
12. $\frac{5}{3}$ by $\frac{7}{5}$, $\frac{1}{4}$, $\frac{2}{5}$, $\frac{4}{7}$.
13. $\frac{2}{3}$ by $\frac{4}{5}$, $\frac{5}{7}$, $\frac{1}{4}$, $\frac{5}{6}$.
14. $\frac{2}{5}$ by $\frac{6}{7}$, $\frac{2}{3}$, $\frac{4}{7}$, $\frac{1}{5}$.
15. 5 by $\frac{7}{10}$, $\frac{1}{5}$, $\frac{2}{3}$, $\frac{4}{7}$.

EXAMPLES. 72.

Divide

1. $\frac{9}{16}$ by $\frac{3}{4}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{7}{9}$.
2. 29 by $\frac{3}{4}$, $\frac{2}{3}$, $\frac{4}{7}$, $\frac{12}{13}$.
3. 50 by $\frac{4}{13}$, $\frac{5}{12}$, $\frac{4}{17}$, $\frac{11}{21}$.
4. $\frac{13}{14}$ by $\frac{13}{15}$, $\frac{3}{14}$, $\frac{5}{7}$, $\frac{5}{8}$.
5. $\frac{5}{10}$ by $\frac{5}{100}$, $\frac{8}{7}$, $\frac{3}{10}$, $\frac{4}{5}$.
6. $\frac{8}{15}$ by $\frac{9}{20}$, $\frac{8}{9}$, $\frac{4}{15}$, $\frac{6}{13}$.
7. 55 by $\frac{11}{24}$, $\frac{15}{82}$, $\frac{25}{53}$, $\frac{2}{9}$.
8. $\frac{7}{8}$ by $\frac{14}{19}$, $\frac{3}{8}$, $\frac{4}{7}$, $\frac{5}{16}$.
9. $\frac{21}{25}$ by $\frac{14}{15}$, $\frac{17}{25}$, $\frac{14}{25}$, $\frac{8}{19}$.
10. $\frac{7}{22}$ by $\frac{112}{375}$, $\frac{6}{19}$, $\frac{14}{9}$, $\frac{3}{11}$.
11. $\frac{31}{60}$ by $\frac{23}{60}$, $\frac{7}{4}$, $\frac{7}{18}$, $\frac{3}{20}$.
12. $\frac{15}{10000}$ by $\frac{3}{10}$, $\frac{3}{200}$, $\frac{13}{1000}$, $\frac{4}{5}$.
13. $\frac{9}{19}$ by $1\frac{7}{38}$, $5\frac{2}{5}$, $4\frac{1}{2}$, $1\frac{8}{19}$.
14. $4\frac{3}{4}$ by $5\frac{1}{2}$, $3\frac{4}{5}$, $9\frac{1}{4}$, $4\frac{1}{4}$.

Simplify the expressions:

15. $4\frac{1}{5} \div \frac{7}{15}$.
16. $1\frac{5}{12} \div 1\frac{8}{9}$.
17. $3\frac{1}{4} \div 4\frac{1}{3}$.
18. $1\frac{43}{65} \div \frac{144}{85}$.
19. $\frac{\frac{7}{8}}{\frac{3}{11}}$.
20. $\frac{7\frac{1}{2}}{\frac{5}{17}}$.
21. $\frac{28\frac{1}{2}}{7\frac{1}{8}}$.
22. $\frac{19\frac{1}{17}}{32\frac{4}{7}}$.
23. $\frac{1078}{47\frac{3}{28}}$.
24. $\frac{\frac{5}{7} \times 1\frac{6}{13}}{2\frac{5}{7} \times \frac{4}{13}}$.
25. $\frac{2\frac{1}{2} \div 5\frac{1}{8}}{5\frac{4}{7} \div 1\frac{1}{4}}$.
26. $\frac{3\frac{2}{3} \div 1\frac{3}{8}}{6\frac{2}{5} \div 5\frac{1}{3}}$.
27. $\frac{4\frac{2}{3} \text{ of } 5\frac{1}{7} \times 8}{28\frac{1}{2} \times \frac{8}{57}}$.
28. $\frac{\frac{1}{9} \text{ of } 1\frac{1}{4} \times 4\frac{1}{2}}{\frac{5}{56} \times 1\frac{1}{3} \times 3\frac{1}{2}}$.
29. $\frac{\frac{7}{8} \times \frac{11}{3} \times 12\frac{1}{2}}{\frac{1}{3} \times \frac{2}{15} \times 8\frac{3}{4}}$.
30. $\frac{3}{14}$ of $\frac{4\frac{5}{9}}{6\frac{1}{8}}$ of $\frac{6\frac{8}{11}}{11\frac{5}{7}}$.
31. $\frac{\frac{2}{3} \times \frac{5}{7} \div \frac{1}{18}}{\frac{4}{5} \times 6 \div 1\frac{7}{12}}$.
32. $\frac{\frac{7}{8} \times 3\frac{2}{3} \div \frac{2}{25}}{\frac{1}{3} \times 8\frac{3}{4} \div 7\frac{1}{2}}$.
33. $2\frac{1}{4} \times 1\frac{1}{2} \div 1\frac{1}{8}$ of $2\frac{2}{5}$.
34. $1\frac{1}{2} \times \frac{7}{8} \div 5\frac{1}{3}$ of $2\frac{2}{5}$.
35. $2\frac{1}{4}$ of $1\frac{1}{2} \div 1\frac{1}{8} \times 2\frac{2}{5}$.
36. $1\frac{1}{2}$ of $\frac{7}{8} \div 5\frac{1}{3} \times 2\frac{2}{5}$.
37. $2\frac{1}{4}$ of $1\frac{1}{2} \div 1\frac{1}{8}$ of $2\frac{2}{5}$.
38. $1\frac{1}{2}$ of $\frac{7}{8} \div 5\frac{1}{3}$ of $2\frac{2}{5}$.
39. $3\frac{2}{5} \times \frac{10}{11} \div 3\frac{1}{7}$ of $3\frac{3}{7}$ of $\frac{5}{44}$ of $1\frac{16}{33}$.

XVI. (1). G.C.M. AND L.C.M. OF TWO OR MORE FRACTIONS.

181. In Arts, 56-71 we considered the processes of determining the G.C.M. and the L.C.M. of whole numbers. We will now extend these to *fractions.*

182. The G.C.M. of two or more fractions is the greatest number (integral or fractional) which is contained in each of the fractions an *integral* number of times. It follows that, if we divide each fraction by the G.C.M., all the quotients will be integers; and if we *invert* the required G.C.M. and multiply each fraction by it, all the products will be integers.

Hence the denominator of the required G.C.M. must be a common multiple of the given denominators, and the numerator a common measure of the given numerators, and moreover the former must be as small, and the latter as great as possible.

Consequently *the* G.C.M. *of two or more fractions has for its numerator the* G.C.M. *of the given numerators, and for its denominator the* L.C.M. *of the given denominators.*

Example. i. *Find the* G.C.M. *of* $\frac{1}{2}$, $\frac{3}{5}$, $\frac{4}{7}$ and $\frac{5}{21}$.

The G.C.M. of the given numerators $=1$, and the L.C.M. of the given denominators $=210$;

$\therefore$ the G.C.M. required $=\frac{1}{210}$.

Example ii. *Find the* G.C.M. *of* $1\frac{1}{24}$, $1\frac{2}{3}$, $\frac{5}{6}$, and $2\frac{1}{7}$.

The fractions are equivalent to $\frac{35}{24}$, $\frac{5}{3}$, $\frac{5}{6}$, $\frac{15}{7}$;

$\therefore$ the G.C.M. required $=\frac{5}{168}$.

183. The L.C.M. of two or more fractions is the *least* number (integral or fractional) which contains each of the fractions an *integral* number of times.

It follows that, if we divide the L.C.M. by each fraction in turn, all the quotients will be integers; and if we multiply the L.C.M. successively by each fraction *inverted*, all the products will be integers. Consequently the required numerator is a common multiple of the given numerators, and the required denominator a common measure of the given denominators; and as the former must be as small, and the latter as great as possible, it follows that the L.C.M. *of two or more fractions, proper or improper, is a fraction whose numerator is the* L.C.M. *of the given numerators, and denominator the* G.C.M. *of the given denominators.*

Example i. *Find the* L.C.M. *of* $\frac{1}{2}$, $\frac{3}{5}$, $\frac{4}{7}$, $\frac{5}{21}$.

The L.C.M. of the numerators = 60, and the G.C.M. of the denominators = 1;

$\therefore$ the L.C.M. required = 60.

Example ii. *Find the* L.C.M. *of* $1\frac{11}{24}$, $1\frac{2}{3}$, $1\frac{1}{6}$, and $2\frac{7}{9}$.

The fractions are equivalent to $\frac{35}{24}$, $\frac{5}{3}$, $\frac{7}{6}$, and $\frac{25}{9}$.

The L.C.M. of the numerators = 175, and the G.C.M. of the denominators = 3;

$\therefore$ the L.C.M. required $= \frac{175}{3} = 58\frac{1}{3}$.

EXAMPLES. 73.

Find the G.C.M. and the L.C.M. of the following fractions:

1. $2\frac{1}{4}$, $\frac{1}{3}$, $7\frac{1}{5}$, $\frac{1}{5}$, $\frac{3}{4}$.
2. $\frac{1}{3}$, $\frac{1}{6}$, $\frac{4}{3}$, $\frac{8}{21}$.
3. $\frac{1}{4}$, $\frac{2}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{3}$.
4. $\frac{7}{9}$, $6\frac{5}{12}$, $\frac{1}{3}$.
5. $\frac{1}{9}$, $\frac{1}{12}$, $\frac{1}{16}$.
6. $\frac{2}{35}$, $\frac{1}{63}$, $\frac{3}{14}$, $\frac{5}{21}$.
7. $\frac{4}{15}$, $4\frac{4}{9}$, $3\frac{1}{5}$, $13\frac{1}{3}$.
8. $7\frac{1}{2}$, $1\frac{13}{14}$, $1\frac{1}{5}$, $4\frac{1}{5}$, $\frac{3}{4}$.
9. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{6}{7}$, $\frac{8}{9}$.
10. $6\frac{4}{5}$, $2\frac{11}{20}$, $1\frac{2}{15}$, $13\frac{3}{5}$.
11. $3\frac{2}{7}$, $1\frac{2}{3}$, $1\frac{3}{8}$, $2\frac{2}{9}$, $3\frac{10}{12}$.
12. $1\frac{1}{6}$, $1\frac{1}{12}$, $3\frac{1}{3}$, $1\frac{7}{9}$.
13. $1\frac{5}{28}$, $1\frac{1}{21}$, $3\frac{1}{7}$.
14. $19\frac{1}{3}$, $19\frac{5}{6}$, $21\frac{1}{4}$.
15. $7\frac{1}{3}$, $19\frac{9}{25}$, $20\frac{1}{7}$.
16. $1\frac{23}{55}$, $2\frac{2}{77}$, $3\frac{31}{33}$, $8\frac{19}{22}$.

EXAMPLES. 74.

Determine the missing figures in the following 9 examples:

1. $\frac{4}{27} + \frac{1}{*} + \frac{5}{63} = \frac{10}{27}$.
2. $1\frac{2}{3} + 2\frac{3}{4} - \frac{*}{*} - \frac{1}{12} = 3\frac{1}{2}$.
3. $7\frac{1}{5*} - *\frac{1}{19} = 2\frac{**}{5*}$.
4. $4\frac{3}{17} + *\frac{*}{**} - 7\frac{9}{187} - 2\frac{5}{33} = 3\frac{3}{187}$.
5. $*\frac{1}{5} \times 2\frac{1}{*} = 9$.
6. $5\frac{8}{27} \times *\frac{6}{13} = 18\frac{1}{*}$.
7. $4\frac{*}{27} \times *\frac{*}{16} = 35$.
8. $30\frac{*}{17} \div *\frac{20}{51} = 6\frac{*}{7}$.
9. $10\frac{4}{**} \div *\frac{2}{27} = 9\frac{9}{17}$.
10. Reduce $\frac{231567}{374141}$ to its lowest terms.
11. Find the sum of $\frac{38}{60}$, $\frac{3}{4}$ and $\frac{17}{100}$.
12. Find the price of 268 quarts of milk at $4\frac{1}{2}d.$ per quart.
13. Find the total weight of 165 boxes, each $16\frac{4}{5}$ lb.
14. Find the price of $14\frac{3}{5}$ lb. of meat at 1*s.* $2\frac{1}{2}d.$ per lb.
15. £140 is $\frac{7}{15}$ of what amount?
16. What must be added to $\frac{1}{2}\frac{1}{3}\frac{5}{4}$ that the sum may be 2?
17. What must we take from $\frac{5}{8}$, to leave $\frac{2}{5}$?
18. What number multiplied by $6\frac{2}{3}$ gives a product 24?

19. By what do we divide $\frac{3}{9}\frac{1}{7}$, if the quotient is $3\frac{10}{57}$?

20. How many times does $\frac{1}{12}+\frac{1}{4}+\frac{1}{8}$ contain $\frac{1}{3}+\frac{1}{6}+\frac{1}{9}$?

21. What number do we divide by $7\frac{5}{8}$, if the quotient is $\frac{2}{5}$?

22. If the divisor be $\frac{4}{7}$ and the quotient $\frac{3}{8}$ of the divisor, what must the dividend be?

23. If $\frac{3}{4}$ of a lb. of butter cost 1*s.* 3*d.*, what would $\frac{1}{4}$ of a lb. cost? and what would 1 lb. cost?

24. A sum of money is divided among A, B and C. A receives $\frac{5}{16}$ of it, and B receives $\frac{7}{12}$. How much does C get?

25. A gives away $\frac{2}{9}$ of a guinea. He gives $\frac{1}{2}$ of this to B, $\frac{3}{7}$ of it to C, and the remainder to D. How much does each get?

26. A copy-book contains 24 pages, and a boy has written 16 pages. What fraction of the whole has he yet to write?

27. A sum of money is divided among 3 men. If the first has $\frac{1}{7}$ of it, and the second $\frac{5}{9}$, how much of it has the third? If the third receives £1. 18*s.*, what did the first receive?

28. A boy after giving away $\frac{1}{4}$ of his pocket money to one companion, and $\frac{7}{8}$ of the remainder to another, has 9 pence left. How much had he at first?

29. A field is owned by 3 people; A has $\frac{3}{7}$ of it, and B has twice as much as C. What fractions of the field have B and C?

30. From a pole 18 ft. long, as many pieces as possible are cut off, each $3\frac{1}{3}$ ft. in length: how much of the pole will be left?

31. A cask contains $46\frac{2}{3}$ gallons of beer; if $\frac{3}{4}$ of a gallon leak out each week, in how many weeks will the barrel be empty?

32. A, having £34. 17*s.* 6*d.*, has $4\frac{1}{2}$ times as much as B has; what has B?

33. A brig started for Valparaiso with 608 tons of coal. During a storm $\frac{5}{19}$ of the cargo was thrown overboard. How many tons of coal did the brig deliver?

34. A man travelled $\frac{5}{16}$ of his journey by coach, and $\frac{7}{20}$ by rail; he walked the remaining 10 miles: how far did he go altogether?

35. A man owns $\frac{7}{25}$ of a ship, and sells $\frac{5}{14}$ of his share; what fraction of the ship does he still own?

36. A merchant owned $\frac{13}{16}$ of a ship's cargo, and sold $\frac{4}{5}$ of his share: what fraction of the whole cargo had he left?

37. If I pay away $\frac{2}{3}$ of my money, and then $\frac{3}{7}$ of what remains, how much of the whole is left?

38. One quarter of an estate is left to the eldest daughter, $\frac{1}{3}$ to the second daughter, and $\frac{2}{3}$ of the remainder to a son. How much more was there of the estate?

39. A boy spent $\frac{3}{5}$ of his money, and then found that $\frac{3}{5}$ of the remainder was half-a-crown: what had he at first?

40. A person at his first game loses $\frac{1}{3}$ of his money, at the second $\frac{3}{5}$ of the remainder, at the third $\frac{4}{7}$ of the rest; what fraction of his original money has he left?

XVII. THE USE OF BRACKETS.

184. We may represent the division of $2\frac{1}{2}$ *by* $1\frac{2}{3}$ by $2\frac{1}{2} \div 1\frac{2}{3}$; and again we may represent the sum of $3\frac{4}{5}$ and $2\frac{1}{2} \div 1\frac{2}{3}$ by

$$3\tfrac{4}{5} + 2\tfrac{1}{2} \div 1\tfrac{2}{3}.$$

If now we have to divide $3\frac{4}{5} + 2\frac{1}{2}$ by $1\frac{2}{3}$, how can we represent the operation? Not by $3\frac{4}{5} + 2\frac{1}{2} \div 1\frac{2}{3}$, for this means that $2\frac{1}{2}$ *only* is divided by $1\frac{2}{3}$, the result being added to $3\frac{4}{5}$. We have indeed no mode yet of representing it except

$$\frac{3\frac{4}{5} + 2\frac{1}{2}}{1\frac{2}{3}}.$$

It will therefore be convenient to introduce some new forms ( ), { }, [ ], called **brackets**, which are used to *bind together* two or more quantities into one. With such brackets we represent the division of $3\frac{4}{5} + 2\frac{1}{2}$ by $1\frac{2}{3}$ concisely in the form $(3\frac{4}{5} + 2\frac{1}{2}) \div 1\frac{2}{3}$. But we must not write the expression in this form *without* brackets, for $3\frac{4}{5} + 2\frac{1}{2} \div 1\frac{2}{3}$ means $3\frac{4}{5}$ added to $2\frac{1}{2} \div 1\frac{2}{3}$.

185. In the same manner, the multiplication of $3\frac{4}{5} + 2\frac{1}{2}$ by $1\frac{2}{3}$ is represented by $(3\frac{4}{5} + 2\frac{1}{2}) \times 1\frac{2}{3}$, and we must again be careful not to omit the brackets, for $3\frac{4}{5} + 2\frac{1}{2} \times 1\frac{2}{3}$ means $2\frac{1}{2} \times 1\frac{2}{3}$ added to $3\frac{4}{5}$.

186. Again, the multiplication of $3\frac{4}{5} + 2\frac{1}{2}$ by $1\frac{2}{3} + 5\frac{6}{7}$ is represented by

$$(3\tfrac{4}{5} + 2\tfrac{1}{2}) \times (1\tfrac{2}{3} + 5\tfrac{6}{7});$$

and here as before the brackets must not be omitted, for without them we should have

$$3\tfrac{4}{5} + 2\tfrac{1}{2} \times 1\tfrac{2}{3} + 5\tfrac{6}{7},$$

which means the sum of the three quantities $3\frac{4}{5}$, $2\frac{1}{2} \times 1\frac{2}{3}$ and $5\frac{6}{7}$. We may, however, omit the sign $\times$ between the brackets.

187. The need of brackets, or some sign of notation equivalent to them, and their use in marking out the distinction between

$$2+3\times5 \text{ and } (2+3)\times5,$$

and between

$$2+3\times5+6,\ (2+3)\times(5+6),\ (2+3)\times5+6 \text{ and } 2+3\times(5+6),$$

may be clearer if we compare the more general expressions

$$a+b\times c \text{ and } (a+b)\times c,$$

and also

$$a+b\times c+d,\ (a+b)\times(c+d),\ (a+b)\times c+d, \text{ and } a+b\times(c+d).$$

The signs $\times$ and $\div$ are sometimes called **strong** signs, and the signs $+$ and $-$ **weak** signs. The meanings of these words are shown in the above illustrations. In the expression $2+3\times5$, for example, the sign $\times$ links the numbers 3 and 5 into one term, and draws the 3 away from the 2. The sign $+$ by itself is too *weak* to link the 2 to the 3 when opposed to the *stronger* sign $\times$. If such linking is desired, it can only be effected by bringing brackets to the assistance of the sign.

188. The division of $3\frac{1}{4}+2\frac{1}{2}$ by $1\frac{2}{3}+5\frac{6}{7}$ is represented by $(3\frac{1}{4}+2\frac{1}{2})\div(1\frac{2}{3}+5\frac{6}{7})$, in which neither the brackets nor the sign $\div$ may be omitted.

189. Again, we represent the sum of $3\frac{1}{4}$ and $1\frac{2}{3}-\frac{5}{6}$ by $3\frac{1}{4}+(1\frac{2}{3}-\frac{5}{6})$, but we may leave out the brackets and write it thus, $3\frac{1}{4}+1\frac{2}{3}-\frac{5}{6}$, without affecting the value of the expression.

190. Lastly, we represent the difference between $1\frac{2}{3}+\frac{5}{6}$ and $3\frac{1}{4}$ by $3\frac{1}{4}-(1\frac{2}{3}+\frac{5}{6})$; but if we omit the brackets the expression must be written $3\frac{1}{4}-1\frac{2}{3}-\frac{5}{6}$.

Again, the difference between $1\frac{2}{3}-\frac{5}{6}$ and $3\frac{1}{4}$ is represented by $3\frac{1}{4}-(1\frac{2}{3}-\frac{5}{6})$. If we omit the brackets we must write the expression thus, $3\frac{1}{4}-1\frac{2}{3}+\frac{5}{6}$; for if we subtract $1\frac{2}{3}$ from $3\frac{1}{4}$ we get $3\frac{1}{4}-1\frac{2}{3}$, and, as we have subtracted $\frac{5}{6}$ too much, we must now *add* $\frac{5}{6}$. Hence

$$3\tfrac{1}{4}-(1\tfrac{2}{3}-\tfrac{5}{6})=3\tfrac{1}{4}-1\tfrac{2}{3}+\tfrac{5}{6}.$$

191. From a consideration of these cases we see that, in addition and subtraction, the brackets may be removed subject to certain changes which may be summed up as follows. *If the sign + precedes an expression in brackets, the brackets may be omitted without affecting the value of the expression; and if the sign − precedes,*

the brackets may be omitted, provided we change the sign of every term within them.

For example, $6\frac{2}{3} - 1\frac{1}{2} + (4\frac{1}{4} - 2\frac{3}{5} - 1\frac{7}{8} + \frac{3}{4})$
$= 6\frac{2}{3} - 1\frac{1}{2} + 4\frac{1}{4} - 2\frac{3}{5} - 1\frac{7}{8} + \frac{3}{4} = \text{etc.};$

and $6\frac{2}{3} - 1\frac{1}{2} - (4\frac{1}{4} - 2\frac{3}{5} - 1\frac{7}{8} + \frac{3}{4})$
$= 6\frac{2}{3} - 1\frac{1}{2} - 4\frac{1}{4} + 2\frac{3}{5} + 1\frac{7}{8} - \frac{3}{4} = \text{etc.}$

192. We need brackets of different forms, for we may have one pair enclosed within another, as in the expression,

$$6 - [\tfrac{5}{6} + \{3\tfrac{7}{8} - (2\tfrac{1}{3} - 1\tfrac{7}{9})\}].$$

To simplify such an expression as this, we may reduce the terms within the *innermost* pair of brackets to a single term, and then remove the brackets, carefully making the necessary changes when the sign – occurs in front of the pair. We proceed in this way (**a**), removing the brackets in turn, until all have disappeared.

When only additions and subtractions are involved, we may remove the brackets in turn, beginning with the *outermost*. By this method all the brackets can be removed in one step as in (**b**); but great care must be taken with the signs.

Thus, the above expression

(**a**)
$= 6 - [\tfrac{5}{6} + \{3\tfrac{7}{8} - 2\tfrac{1}{3} + 1\tfrac{7}{9}\}]$
$= 6 - [\tfrac{5}{6} + 3\tfrac{7}{8} - 2\tfrac{1}{3} + 1\tfrac{7}{9}]$
$= 6 - \tfrac{5}{6} - 3\tfrac{7}{8} + 2\tfrac{1}{3} - 1\tfrac{7}{9}$
$= 4\frac{-60-63+24-56}{72}$
$= 4 - \frac{155}{72}$
$= 4 - 2\frac{11}{72}$
$= 1\frac{61}{72}.$

(**b**) or
$= 6 - \tfrac{5}{6} - 3\tfrac{7}{8} + 2\tfrac{1}{3} - 1\tfrac{7}{9}$
$= 4\frac{-60-63+24-56}{72}$
$= 4 - \frac{155}{72}$
$= 4 - 2\frac{11}{72}$
$= 1\frac{61}{72}.$

NOTE.—In all cases such as Examples 8 and 14 in the next set, the multiplications and divisions must be effected before the additions and subtractions.

EXAMPLES. 75.

Simplify the following expressions:

1. $15\frac{1}{4} + 7\frac{3}{4} + 6\frac{2}{3}$.
2. $15\frac{1}{4} - 7\frac{3}{4} + 6\frac{2}{3}$.
3. $15\frac{1}{4} + 7\frac{3}{4} - 6\frac{2}{3}$.
4. $15\frac{1}{4} - (7\frac{3}{4} + 6\frac{2}{3})$.
5. $15\frac{1}{4} - (7\frac{3}{4} - 6\frac{2}{3})$.
6. $15\frac{1}{4} + \frac{2}{3} \times 7\frac{3}{4}$.
7. $(15\frac{1}{4} + \frac{2}{3}) \times 7\frac{3}{4}$.
8. $15\frac{1}{4} + \frac{2}{3} \times 7\frac{3}{4} - 6\frac{2}{3}$.
9. $(15\frac{1}{4} + \frac{2}{3}) \times 7\frac{3}{4} - 6\frac{2}{3}$.
10. $15\frac{1}{4} + \frac{2}{3} \times (7\frac{3}{4} - 6\frac{2}{3})$.
11. $(15\frac{1}{4} + \frac{2}{3})(7\frac{3}{4} - 6\frac{2}{3})$.
12. $15\frac{1}{4} - \frac{2}{3} \times (7\frac{3}{4} - 6\frac{2}{3})$.

13. $(15\frac{1}{4}-\frac{2}{3})(7\frac{3}{4}-6\frac{2}{3})$.
14. $15\frac{1}{4}+\frac{2}{3}\div 7\frac{3}{4}$.
15. $(15\frac{1}{4}-\frac{2}{3})\div 7\frac{3}{4}$.
16. $15\frac{1}{4}-\frac{2}{3}\div 7\frac{3}{4}$.
17. $(15\frac{1}{4}+\frac{2}{3})\div 7\frac{3}{4}$.
18. $15\frac{1}{4}-\frac{2}{3}\div 7\frac{3}{4}-6\frac{2}{3}$.
19. $15\frac{1}{4}+\frac{2}{3}\div(7\frac{3}{4}-6\frac{2}{3})$.
20. $(15\frac{1}{4}+\frac{2}{3})\div(7\frac{3}{4}-6\frac{2}{3})$.
21. $15\frac{1}{4}-\frac{2}{3}\div(7\frac{3}{4}-6\frac{2}{3})$.
22. $(15\frac{1}{4}-\frac{2}{3})\div(7\frac{3}{4}-6\frac{2}{3})$.
23. $10+\{2\frac{1}{2}+(4\frac{1}{2}+1\frac{1}{4})\}$.
24. $10+\{2\frac{1}{2}+(4\frac{1}{2}-1\frac{1}{4})\}$.
25. $10+\{2\frac{1}{2}-(4\frac{1}{2}+1\frac{1}{4})\}$.
26. $10+\{2\frac{1}{2}-(4\frac{1}{2}-1\frac{1}{4})\}$.
27. $10-\{2\frac{1}{2}+(4\frac{1}{2}+1\frac{1}{4})\}$.
28. $10-\{2\frac{1}{2}+(4\frac{1}{2}-1\frac{1}{4})\}$.
29. $10-\{2\frac{1}{2}-(4\frac{1}{2}+1\frac{1}{4})\}$.
30. $10-\{2\frac{1}{2}-(4\frac{1}{2}-1\frac{1}{4})\}$.
31. $23\frac{1}{2}-\{15\frac{1}{4}+\frac{2}{3}(7\frac{3}{4}-6\frac{2}{3})\}$.
32. $3\frac{1}{2}-\{5\frac{3}{4}-\frac{2}{3}(3\frac{2}{3}+2\frac{1}{5})\}$.
33. $6-\{1\frac{1}{2}-\frac{3}{5}(1\frac{7}{9}-\frac{1}{2})\}$.
34. $7\frac{3}{4}-\{3\frac{4}{7}-\frac{3}{8}(1\frac{7}{9}-\frac{1}{2})\}$.
35. $(2\frac{1}{4}-\frac{3}{4})(13\frac{5}{6}-\frac{2}{3})(1\frac{7}{9}-\frac{1}{2})$.
36. $1\div[4-1\div\{2-1\div(1-\frac{5}{13})\}]$
37. $(\frac{5}{7}$ of $1\frac{6}{13})\div(2\frac{5}{7}\div 3\frac{1}{4})$.
38. $(3\frac{1}{5}+4\frac{1}{3}-5\frac{1}{4}$ of $\frac{6}{7})\div(3\frac{1}{2}-4\frac{1}{3}$ of $\frac{7}{26}+4\frac{1}{5}$ of $\frac{3}{7})$.

XVIII. COMPLEX FRACTIONS.

193. We will now work out a few examples of a more complex kind.

Example i. *Simplify* $\dfrac{3\frac{1}{4}}{4\frac{1}{2}}$.

We multiply numerator and denominator by 4, which is the L.C.M. of the denominators 4 and 2.

Thus the fraction $=\frac{13}{18}$.

Example ii. *Simplify* $\dfrac{3\frac{1}{2}-2\frac{1}{3}}{3\frac{1}{4}+\frac{5}{6}}$.

We multiply numerator and denominator by 12, the L.C.M. of denominators 2, 3, 4 and 6.

Thus the fraction

$$=\frac{42-28}{39+10}=\frac{14}{49}=\frac{2}{7}.$$

Example iii. *Simplify*

$$1\div\{\tfrac{1}{2}+\tfrac{1}{3}+\tfrac{1}{6}\div(\tfrac{3}{4}-\tfrac{1}{3})\}.$$

The expression $=1\div\{\frac{5}{6}+\frac{1}{6}\div\frac{5}{12}\}$

$=1\div\{\frac{5}{6}+\frac{1}{6}\times\frac{12}{5}\}$

$=1\div\{\frac{5}{6}+\frac{2}{5}\}$

$=1\div\frac{37}{30}$

$=1\times\frac{30}{37}$

$=\frac{30}{37}$.

Example iv. *Simplify* $\dfrac{1\frac{1}{2}}{1+\dfrac{1}{1+\frac{1}{4}}}$.

We begin by multiplying numerator and denominator of $\dfrac{1}{1+\frac{1}{4}}$ by 4. Thus the expression

$$=\frac{1\frac{1}{2}}{1+\frac{4}{5}}=\frac{15}{10+8}=\frac{5}{6}.$$

Example v. *Simplify* $\dfrac{2\frac{1}{5}-1\frac{1}{6}}{3\frac{5}{6}-2\frac{4}{5}}$ of 5*s*.

Multiplying numerator and denominator by 30, we find that

the expression $=\dfrac{66-35}{115-84}$ of 5*s*. $=\frac{31}{31}$ of 5*s*. $=5s$.

Example vi. *Simplify* $\dfrac{1\frac{1}{8}\div 1\frac{1}{5}}{\frac{2}{3}\text{ of }\frac{5}{9}\div 10\frac{1}{3}}\times\dfrac{1\frac{1}{2}\text{ of }4\frac{1}{9}}{6\frac{15}{16}\text{ of }5\frac{1}{3}}$ of £1.

The expression $=\dfrac{\frac{9}{8}\times\frac{5}{6}}{\frac{2}{3}\times\frac{5}{9}\times\frac{3}{31}}\times\dfrac{\frac{3}{2}\times\frac{37}{9}}{\frac{111}{16}\times\frac{16}{3}}$ of £1

$=\frac{9}{8}\times\frac{5}{2\times 5}\times\frac{3}{2}\times\frac{9}{5}\times\frac{31}{3}\times\frac{3}{2}\times\frac{37}{9}\times\frac{3}{3\times 37}$ of £1

$=\frac{2709}{64}$ of £1

$=42\frac{23}{64}$ of £1

$=$£4. 7*s*. $2\frac{1}{4}d$.

To find the value of $\frac{23}{64}$ of £1, we divide £23 by 64 in the usual way.

EXAMPLES. 76.

Simplify the following expressions:

1. $\dfrac{1}{1+\frac{1}{2}}$.
2. $\dfrac{\frac{1}{2}}{1-\frac{1}{2}}$.
3. $6+\dfrac{1}{6+\frac{1}{6}}$.
4. $\dfrac{\frac{3}{4}}{\frac{6}{5}\times\frac{5}{7}}$.
5. $3\frac{7}{8}\div\frac{9}{10}-1\frac{3}{16}$.
6. $1\frac{5}{11}+2\frac{1}{8}-(2\frac{9}{16}-\frac{1}{8}-\frac{1}{22})$.
7. $\dfrac{6\frac{1}{3}}{12\frac{4}{9}}+\dfrac{6\frac{1}{2}}{9\frac{1}{3}}-\dfrac{4\frac{1}{2}}{7}$.
8. $\dfrac{2\frac{1}{2}+3\frac{1}{4}}{1\frac{1}{3}-\frac{3}{8}}$.
9. $\dfrac{6\frac{3}{4}-1\frac{5}{14}}{2\frac{1}{6}+1\frac{3}{7}}$.
10. $\dfrac{\frac{1}{2}\text{ of }1\frac{2}{5}}{3\frac{3}{8}+1\frac{14}{16}}$.
11. $\dfrac{2\frac{2}{7}\text{ of }2\frac{5}{8}}{2\frac{5}{8}-2\frac{2}{7}}$.
12. $\dfrac{4\frac{1}{5}-\frac{2}{3}\text{ of }\frac{5}{8}}{1\frac{2}{3}+2\frac{3}{7}-\frac{2}{5}}$.
13. $\dfrac{5\frac{2}{7}-3\frac{3}{4}}{11\frac{13}{15}+24\frac{6}{17}}$.
14. $\dfrac{4\frac{4}{15}-2\frac{1}{3}}{6\frac{1}{2}\times 2\frac{1}{5}}$.
15. $\dfrac{\frac{2}{9}\text{ of }1\frac{2}{3}\text{ of }2\frac{2}{5}}{\frac{7}{9}+\frac{5}{12}-\frac{3}{4}}$.
16. $\dfrac{1}{10+\dfrac{1}{2+\frac{1}{30}}}$.
17. $\dfrac{1}{3+\dfrac{1}{7+\frac{1}{16}}}$.
18. $(3\frac{1}{3}+7\frac{1}{4})\text{ of }\frac{2}{5}+9\frac{3}{4}$.
19. $\dfrac{\frac{9}{7}-\frac{2}{3}+\frac{1}{5}}{\frac{6}{7}+\frac{3}{5}-\frac{1}{6}}$.
20. $\dfrac{2\frac{1}{3}-\frac{7}{9}+\frac{2}{7}}{1+\frac{6}{7}-\frac{5}{6}}$.
21. $\dfrac{15\frac{3}{4}-\frac{14}{3}\times 1\frac{5}{6}}{\frac{1}{6}\times 23\frac{1}{3}+\frac{91}{36}}$.
22. $\dfrac{\frac{13}{42}\times\frac{1}{8}-\frac{11}{28}\times\frac{1}{9}}{\frac{16}{21}\times\frac{1}{12}-1\frac{6}{7}\times\frac{1}{36}}$.

23. $\dfrac{2\frac{1}{4}-\frac{2}{3}\text{ of }1\frac{5}{6}}{\frac{1}{5}\text{ of }3\frac{1}{3}+\frac{13}{36}}$.

24. $\dfrac{3\frac{1}{12}-8\frac{2}{9}+5\frac{1}{6}}{4\frac{1}{16}\text{ of }2\frac{6}{13}}$.

25. $2\frac{37}{53}\times\frac{4308\,9}{7750\,6}$.

26. $\dfrac{12\frac{15}{22}-8\frac{3}{4}}{3\frac{1}{2}+5\frac{5}{22}}$.

27. $\dfrac{\frac{1}{40}+\frac{8}{27}+\frac{64}{125}}{\frac{1}{4}+\frac{4}{9}+\frac{16}{25}-\frac{19}{125}}$.

28. $\dfrac{7\frac{2}{5}\times7\frac{2}{5}\times7\frac{2}{5}-27}{7\frac{2}{5}\times7\frac{2}{5}-9}$.

29. $(\frac{5}{7}\text{ of }1\frac{6}{13})\div\dfrac{2\frac{5}{7}}{3\frac{1}{4}}$.

30. $\dfrac{(\frac{3}{4}+\frac{4}{5})\text{ of }\frac{5}{6}+\frac{7}{8}}{\frac{3}{4}+\frac{4}{5}\text{ of }(\frac{5}{6}+\frac{7}{8})}$.

31. $(\frac{1}{2}+\frac{1}{3})\text{ of }(1\frac{1}{3}+2\frac{3}{4})\text{ of }(2\frac{1}{14}-1\frac{1}{2})\text{ of }(3\frac{1}{10}-\frac{3}{7})$.

32. $\dfrac{1}{\frac{1}{2}+\frac{1}{3}+\frac{\frac{1}{6}}{\frac{3}{4}-\frac{1}{3}}}$.

33. $\dfrac{5\frac{4}{5}-2\frac{1}{8}}{3\frac{3}{4}+\frac{9}{10}}\text{ of }\dfrac{4\frac{1}{2}+5\frac{19}{25}}{4\frac{1}{20}}$.

34. $\dfrac{1\frac{7}{9}\text{ of }\frac{27}{64}}{\frac{11}{12}\text{ of }9\frac{9}{11}}\div\dfrac{4\frac{4}{7}\text{ of }\frac{21}{160}}{2\frac{5}{8}\text{ of }\frac{15}{34}}$.

35. $\dfrac{6\frac{1}{2}-3\frac{1}{4}}{7\frac{1}{3}+1\frac{4}{7}}\div\dfrac{4\frac{1}{3}\text{ of }1\frac{1}{4}}{2\frac{3}{7}\text{ of }3\frac{2}{3}}$ of 1 acre.

36. $\dfrac{3\frac{7}{11}-2\frac{5}{8}}{3\frac{7}{11}\times2\frac{5}{8}}\times\dfrac{3\frac{7}{11}\div2\frac{5}{8}}{3\frac{7}{11}\text{ of }2\frac{5}{8}}$.

37. $\dfrac{6+\dfrac{1}{6-\frac{1}{6}}}{4-\dfrac{1}{4-\frac{1}{4}}}\times10\frac{8}{9}\text{ of £1. 10}s.$

38. $\dfrac{1}{4-\dfrac{1}{2-\dfrac{1}{1-\frac{5}{13}}}}$ of 3 cwt. 16 lb.

39. $\left\{\dfrac{1\frac{2}{3}\text{ of }1\frac{1}{4}}{3\frac{2}{5}+\dfrac{5}{1\frac{19}{41}}}+\dfrac{4}{5+\dfrac{6}{7+\frac{8}{9}}}\right\}$ of 15/6.

XIX. (a). DECIMALS.

194. The characteristic of the Arabic system of notation is that each digit has a **local** value depending upon its position relative to the others, as well as an **intrinsic** value depending only upon itself.

In the number 5555, for example, all the digits have the same *intrinsic* value 5, but the first of them represents 5 *thousands*, the second, 5 *hundreds*, the third, 5 *tens*, and the fourth, 5 *units*.

When, however, we consider *parts* of a unit we see that the system does not go far enough, but we shall find that we may represent any fraction whatever by placing digits to the *right* of the units' place, and extending to them the same Principle of Local Value. For example, a digit 5 placed to the right of the units' place may be considered to represent $\frac{5}{10}$ of the unit, and a

digit 5 placed to the right of this to represent $\frac{5}{100}$ of the unit, and so on.

The position of a digit is sometimes spoken of as its **order.** Thus in 62758, the digit 6 is of the order 4, 2 of the order 3, 7 of the order 2, 5 of the order 1, and 8 of the order 0, because they represent multiples of 10^4, 10^3, 10^2, 10^1, and 10^0 respectively

195. In the case of a whole number, since the last digit always denotes units, we can at once from its position tell what each of the other digits denotes; but, when the principle is carried on indefinitely beyond the units' place, it becomes necessary to distinguish the digits in some way. This is done by placing a point immediately after the figure in the units' place.

If, for example, in 62758 the units' digit is 2, we write the number thus, 62·758; and we see at once that it represents

$$60 + 2 + \tfrac{7}{10} + \tfrac{5}{100} + \tfrac{8}{10000}.$$

Unless however *one* digit is distinctly marked in this or some other way, we cannot tell the local value of any.

In 62·758, the digit 7 is said to be of the order -1, 5 of the order -2, 8 of the order -3, and so on, because they represent multiples of $\frac{1}{10}$, $\frac{1}{10^2}$, and $\frac{1}{10^3}$ respectively.

196. If there are no whole units in the number, we either place a zero before the point, or we begin with the point itself.

For example, we represent such a number in the form 0·768, or in the form ·768. For many purposes the former is the better.

197. An expression of the form ·768 is called a **Decimal.** The digits that successively follow the point represent the numerators of vulgar fractions whose denominators are 10, 100, 1000, etc., respectively; and the expression itself represents the sum of these fractions. The point is called a **Decimal Point.**

198. An expression such as 24·768, which consists of a whole number and a decimal, is called a *mixed number.*

199. We read a decimal by naming the digits in order. Thus, ·15 is "*decimal, one, five,*" not "*decimal, fifteen.*"

200. With this notation we can express a quantity measured by the metric system at once in terms of *any* denomination. Thus,

$$24 \text{ m. } 7 \text{ dm. } 6 \text{ cm. } 8 \text{ mm.} = (24 + \tfrac{7}{10} + \tfrac{6}{100} + \tfrac{8}{1000}) \text{ metres}$$
$$= 24{\cdot}768 \text{ m.};$$

and, conversely, it will be seen that 24·768 Km. may be written either as 24 Km. 7 Hm. 6 Dm. 8 m., or as 24 Km. 768 m.

Similarly, 24·15 f. = 24 f. 15 c., and 15 dol. 5 c. = 15·05 dol.

The process may be described in words thus: *We write down the figures in the order in which they occur, absent denominations being represented by 0's, and we put the decimal point immediately after the digit which is in the chosen units' place.*

The student should see clearly that the following measures denote the same quantity, but are expressed in terms of different units: 32 dm., 320 cm., 3200 mm., 3·2 m., ·32 Dm., ·032 Hm., and ·0032 Km.

201. The following peculiarity of decimals must be noticed:

$$\cdot760 = \tfrac{7}{10} + \tfrac{6}{100} + \tfrac{0}{1000} = \tfrac{7}{10} + \tfrac{6}{100} = \cdot76,$$

and $$\cdot7600 = \tfrac{7}{10} + \tfrac{6}{100} + \tfrac{0}{1000} + \tfrac{0}{10000} = \tfrac{7}{10} + \tfrac{6}{100} = \cdot76.$$

Hence *we may place any number of 0's at the end of a decimal, or we may remove them from the end, without altering the value of the decimal.*

The pupil will easily see the reason for this if we consider a concrete quantity, such as ·760 m. This denotes 7 dm. 6 cm. 0 mm., which is clearly the same as 7 dm. 6 cm., *i.e.* as ·76 m.

Similarly,

·7600 Km. = 7 Hm. 6 Dm. 0 m. 0 dm. = 7 Hm. 6 Dm. = ·76 Km.

It is quite otherwise however with 0's which occur at the beginning, immediately after the decimal point; for example, ·76 m. = 7 dm. 6 cm., but ·076 m. = 0 dm. 7 cm. 6 mm.

We may, in like manner, place a decimal point followed by any number of zeros at the end of a whole number, without affecting its value; thus, 3 = 3·0000..., and 126 = 126·00000....

EXAMPLES. **77.** *Oral.*

Express each of the following decimals as the sum of certain vulgar fractions, with denominators 10, 100, 1000, etc.:

1.	4·6.	**2.**	7·7.	**3.**	157·9.	**4.**	361·1.	**5.**	·5.
6.	·8.	**7.**	4·19.	**8.**	15·79.	**9.**	·87.	**10.**	·15.
11.	1·05.	**12.**	·04.	**13.**	·09.	**14.**	·05.	**15.**	5·345.
16.	1·579.	**17.**	·628.	**18.**	·015.	**19.**	·005.	**20.**	·5283.
21.	2·006.	**22.**	·1579.	**23.**	1·1003.	**24.**	6·0052		

Express in terms of separate denominations:

25. ·3245 m. 26. 35·674 Kg. 27. ·0012 Km.
28. ·5001 Hg. 29. 35·00125 Km. 30. 3·006 g.
31. ·003467 Kg. 32. 3·0075 m.

EXAMPLES. 78. *Oral.*

Express as decimals:

1. $\frac{8}{10}$. 2. $\frac{1}{10}+\frac{3}{100}$. 3. $\frac{3}{10}+\frac{7}{100}$. 4. $\frac{2}{10}+\frac{4}{100}$.
5. $\frac{5}{10}+\frac{8}{100}$. 6. $\frac{3}{100}$. 7. $\frac{1}{10}+\frac{2}{100}+\frac{3}{1000}$.
8. $\frac{2}{10}+\frac{6}{100}+\frac{4}{1000}$. 9. $\frac{3}{10}+\frac{5}{100}+\frac{7}{1000}$.
10. $\frac{3}{100}+\frac{5}{1000}$. 11. $\frac{4}{100}+\frac{8}{1000}$. 12. $\frac{3}{100}+\frac{1}{1000}$.
13. $\frac{1}{10}+\frac{6}{1000}$. 14. $\frac{2}{10}+\frac{5}{1000}$. 15. $\frac{9}{10}+\frac{2}{1000}$.
16. $\frac{3}{1000}$. 17. $\frac{5}{1000}$. 18. $\frac{1}{1000}$.
19. $\frac{6}{10}+\frac{5}{100}+\frac{4}{1000}+\frac{1}{10000}$. 20. $\frac{4}{10}+\frac{9}{100}+\frac{3}{1000}+\frac{5}{10000}$.
21. $\frac{4}{10}+\frac{1}{1000}+\frac{4}{10000}$. 22. $\frac{1}{10}+\frac{5}{100}+\frac{3}{10000}$.
23. $\frac{7}{10}+\frac{4}{1000}+\frac{6}{10000}$. 24. $\frac{3}{100}+\frac{7}{1000}+\frac{2}{10000}$.
25. $\frac{2}{10}+\frac{4}{1000}+\frac{8}{10000}$. 26. $\frac{7}{10}+\frac{5}{1000}+\frac{3}{10000}$.
27. $\frac{2}{100}+\frac{7}{10000}$. 28. $\frac{3}{10}+\frac{1}{10000}$.
29. $\frac{7}{10000}$. 30. $\frac{6}{1000000}$

Express the following as decimals:

31. 3 Dm. 2 m. 3 dm. 5 cm., of a metre.
32. 2 Dg. 2 dg. 5 mg., of a gram.
33. 8 Km. 5 Dm. 7 m. 2 cm., of a kilometre
34. 5 Kg. 3 Hg. 7 g. 2 mg., of a kilogram.
35. 7 f. 5 c., of a franc. 36. 8 dol. 15 c., of a dollar.
37. 7 g. 8 cg. 4 mg., of a centigram.
38. 8 Km. 5 Hm. 3 m. 5 dm., of a decametre.
39. 7 Kg. 8 Dg. 5 g. 5 cg., of a hectogram.
40. 10 Km. 256 m., of a hectometre.

XIX. (b). TO MULTIPLY A DECIMAL BY 10, 100, 1000, etc.

202. If ·325 m. be multiplied by 10, we have

$$\begin{aligned} \cdot 325 \text{ m.} \times 10 &= 3 \text{ dm. } 2 \text{ cm. } 5 \text{ mm.} \times 10 \\ &= 30 \text{ dm. } 20 \text{ cm. } 50 \text{ mm.} \\ &= 3 \text{ m. } 2 \text{ dm. } 5 \text{ cm.} \\ &= 3{\cdot}25 \text{ m.} \end{aligned}$$

Again,

$$\begin{aligned} \cdot 325 \text{ g.} \times 100 &= 3 \text{ dg. } 2 \text{ cg. } 5 \text{ mg.} \times 100 \\ &= 3 \text{ Dg. } 2 \text{ g. } 5 \text{ dg.} \\ &= 32{\cdot}5 \text{ g.} \end{aligned}$$

In the case of abstract decimals, the effect of multiplying by 10, 100, ... is best seen by first expressing the decimal as the sum of a number of vulgar fractions. Thus:

$$\cdot 768 = \tfrac{7}{10} + \tfrac{6}{100} + \tfrac{8}{1000},$$

$$\therefore\ 10 \times \cdot 768 = 7 + \tfrac{6}{10} + \tfrac{8}{100} = 7{\cdot}68,$$

and

$$100 \times \cdot 768 = 70 + 6 + \tfrac{8}{10} = 76{\cdot}8.$$

Again

$$\cdot 0638 = \tfrac{0}{10} + \tfrac{6}{100} + \tfrac{3}{1000} + \tfrac{8}{10000},$$

$$\therefore\ 10 \times \cdot 0638 = \tfrac{6}{10} + \tfrac{3}{1000} + \tfrac{8}{10000} = \cdot 638,$$

and

$$100 \times \cdot 0638 = 6 + \tfrac{3}{10} + \tfrac{8}{100} = 6{\cdot}38.$$

Hence, *to multiply a decimal by* 10, 100, 1000, ..., *we move the decimal point* 1, 2, 3, ... *places respectively to the right.*

EXAMPLES. 79. *Oral.*

Multiply each of the following numbers and quantities successively by 10, 100, and 1000:

1.	4·6.	**2.**	7·7.	**3.**	·5.	**4.**	·8.	**5.**	4·19.
6.	15·79.	**7.**	·87.	**8.**	·15.	**9.**	1·05.	**10.**	·04.
11.	·09.	**12.**	5·345.	**13.**	·628.	**14.**	·015.	**15.**	·005.
16.	·5283.			**17.**	2·006 m.			**18.**	·1579 Km.
19.	14·003 g.			**20.**	·0528 Kg.			**21.**	·0005 Km.
22.	·005781 Kg.			**23.**	·000057 m.			**24.**	·000001 Km.

XIX. (c). TO DIVIDE A DECIMAL BY 10, 100, 1000, etc.

203. We have $4{\cdot}54 \text{ m.} \div 10 = 4 \text{ m. } 5 \text{ dm. } 4 \text{ cm.} \div 10$

$= \frac{4}{10} \text{ m. } \frac{5}{10} \text{ dm. } \frac{4}{10} \text{ cm.}$

$= 4 \text{ dm. } 5 \text{ cm. } 4 \text{ mm.}$

$= {\cdot}454 \text{ m.}$

Again, $5{\cdot}345 \text{ Kg.} \div 100 = 5 \text{ Kg. } 3 \text{ Hg. } 4 \text{ Dg } 5 \text{ g.} \div 100$

$= \frac{5}{100} \text{ Kg. } \frac{3}{100} \text{ Hg. } \frac{4}{100} \text{ Dg. } \frac{5}{100} \text{ g.}$

$= 5 \text{ Dg. } 3 \text{ g. } 4 \text{ dg. } 5 \text{ cg.}$

$= 53{\cdot}45 \text{ g.}$

$= {\cdot}05345 \text{ Kg.}$

In the case of abstract decimals we will express the decimal as the sum of a number of vulgar fractions. Thus:

$$54{\cdot}321 = 50 + 4 + \tfrac{3}{10} + \tfrac{2}{100} + \tfrac{1}{1000},$$

$$\therefore\ 54{\cdot}321 \div 10 = 5 + \tfrac{4}{10} + \tfrac{3}{100} + \tfrac{2}{1000} + \tfrac{1}{10000} = 5{\cdot}4321,$$

and $$54{\cdot}321 \div 100 = \tfrac{5}{10} + \tfrac{4}{100} + \tfrac{3}{1000} + \tfrac{2}{10000} + \tfrac{1}{100000} = {\cdot}54321.$$

Again, $${\cdot}543 = \tfrac{5}{10} + \tfrac{4}{100} + \tfrac{3}{1000},$$

$$\therefore\ {\cdot}543 \div 10 = \tfrac{5}{100} + \tfrac{4}{1000} + \tfrac{3}{10000} = {\cdot}0543,$$

and $${\cdot}543 \div 100 = \tfrac{5}{1000} + \tfrac{4}{10000} + \tfrac{3}{100000} = {\cdot}00543.$$

Hence we see that, *to divide a decimal by* 10, 100, 1000, ..., *we move the decimal point* 1, 2, 3, ... *places respectively to the left · and that, if in any case there should be no digits to the left of the decimal point, we suppose* 0's *to be there.*

EXAMPLES. 80. *Oral.*

Divide each of the following numbers and quantities successively by 10, 100, and 1000.

1.	104·6.	2.	167·7.	3.	157·9.	4.	361·1.
5.	15·79.	6.	14·0003.	7.	37·0007.	8.	28·03205.
9.	17·05706.	10.	43·76201.	11.	30·0012.	12.	83·00156.
13.	4·19 m.	14.	1·05 g.	15.	5·345 m.	16.	2·006 g.
17.	1·1003 Kg.	18.	5·372 m.	19.	·5 Km.	20.	·87 Kg.
21.	·628 Km.	22.	·04 g.	23.	·015 Kg.	24.	·005 Km.

XIX. (d). ADDITION OF DECIMALS.

204. For addition, decimals must be arranged as whole numbers are, so that digits of the same order may be one under another in a column. This will happen if the *decimal points* are placed one under another.

For example, to add 32·7543, 3·0501, ·051, 1·003, 17·1 and 126·0012, we arrange them as in the margin. We have placed zeros at the end of some of the decimals so as to have the same number of places in all. This may be done (Art. 201) without affecting the value of a decimal, and the student will at first find it convenient.

32·7543
3·0501
·0510
1·0030
17·1000
126·0012
179·9596

It follows, from the meaning of the notation, that we may now add the decimals as if they were whole numbers, carrying forward multiples of 10 and setting down remainders, and inserting the decimal point in the result at the foot of the column of points. Thus the result is 179·9596.

The method may be perhaps more easily understood if we consider the following examples.

Example i. *Add together* 35 *m.* 15 *cm.*; 25 *mm.*; 5 *Dm.* 3 *m.* 5 *cm.*; 7 *dm.* 3 *cm.* 4 *mm.*; and 75 *cm.*

	m.	dm.	cm.	mm.
(a)	35	1	5	0
			2	5
	53	0	5	0
		7	3	4
		7	5	0
	89	7	0	9

	m.
(b)	35·15
	·025
	53·05
	·734
	·75
	89·709

The only difference between (a) and (b) is that in (a) we have expressed the quantities in terms of the separate successive denominations, while in (b) we have expressed each in terms of a common unit. The student should compare these arrangements with those of Art. 125.

Example ii. *Add together* ·08 *g.*, 26·3 *g.*, 9 *g.*, ·803 *g.*, ·01 *g.*

	g.	dg.	cg.	mg.
(a)			8	0
	26	3	0	0
	9	0	0	0
		8	0	3
			1	0
	36	1	9	3

	g.
(b)	·08
	26·3
	9·0
	·803
	·01
	36·193

EXAMPLES. 81.

Add together

1. 3·76, 82·54.
2. 2·05, 4·55.
3. 3·12, 4·56.
4. 17·01, 16·00.
5. 25 036, 36·123.
6. 48·135, 6·241, 8·005.
7. 65·413, 2·001, ·02.
8. 47·351, 176·37, 3·104.
9. ·001, ·05, ·41.
10. 56·37, 2·451, 8·56.
11. 23·3045, 136·489, 14·0364.
12. 361·1264, 8·351, 10·0004.
13. 10·164, ·232, ·00323, ·0001.
14. ·0002, ·001, ·0001, ·01.
15. 1, ·1, ·01, ·001, ·0001.
16. 10, ·01, 9·00101, 1·101.
17. 15·15, 1·515, ·151, ·01515.
18. 2·0002, 2·002, 1·001, ·02.
19. 349, 31·36, 365·005, 2·07, ·76.
20. ·05, ·0051, ·0001, 365·00009, ·000001.
21. 16·01, 1884, ·001884, 86·00080801, 2·6.
22. 350 f. 10 c., 541 f. 2 c.
23. 68 f. 1 c., 5 f. 40 c., 2 c.
24. 3 c., 5 f. 48 c., 98 f. 56 c.
25. 16 f. 48 c., 48 f. 10 c., 96 f. 2 c.

Express as decimals of the measure of highest denomination mentioned, and add together

26. 3·025 m., 5 cm. 2 mm., 36·035 cm., 35·81 m.
27. ·3245 Km., 8 Km. 125 m., 50·347 Dm., 27·815 m.
28. ·0005 Kg., 3·124 Kg., 84·5 g., 97·326 Dg.
29. 13·75 g., ·0643 g., 64·76 cg., 8·42 dg.
30. ·000578 Km., ·842 Hm., 15·3201 m., ·03 Km., ·0175 Dm.

31. From a piece of wire, lengths 2·325 m., 5·07 m., 15·841 m., 101·5 m. and 37·6 cm. are cut off. If it is found that 7·521 Dm. remains, what was the original length of the wire in metres?

32. A weighs 67·5 Kg.; B weighs ·851 Kg. more than A; C weighs 1·025 Kg. more than B; and D weighs 5·75 Kg. more than A; what is the total weight of A, B, C and D expressed in kilograms?

33. A man buys 14·85 m. of cloth for 32·74 f., 8·05 m. for 18·35 f., 25·70 m. for 56·15 f., and 10·08 m. for 23·05 f. How many metres does he buy in all, and what money does he spend?

34. The height of A is 1·656 m., B is 1·25 cm. higher than A, and C is 3·5 mm. higher than B. Find the height of C in metres.

35. The distance by railway from A to B is 15 Km. 286 m., from B to C, 35 Km. 85 m., from C to D, 41 Km. 5 m., and from D to E, 5 Km. 105 m. Find in kilometres the distance by railway from A to E.

36. A rod is 1 metre long; ·256 of it is coloured red, ·024 of it blue, ·127 of it yellow, and ·085 of it green; the rest is white. Find in centimetres the total length of the coloured portion.

37. A number of parcels, whose weights are 15·867 Kg., 21·345 Kg., 10·05 Kg., 45·126 Kg., and 30·03 Kg., are placed in a cart. What is the total load?

38. A tree is 10 metres high; the next is higher by ·03575 of the first; and the third is 2·75 m. higher than the second. Express the height of each in metres, and find their sum.

XIX. (e). SUBTRACTION OF DECIMALS.

205. To subtract one decimal from another we place the one below the other so that the decimal points may be together.

For example, to subtract 5·0019 from 12·001, we arrange them as in the margin, the 0 being put at the end of the first decimal to make the number of places the same in both. We subtract now as if they were whole numbers, and we put a decimal point before the fourth figure from the end.

```
12·0010
 5·0019
--------
 6·9991
```

Example i. *Subtract* 1·05364 *from* 3·1354.

We will *imagine* a 0 placed at the end of the minuend. It need not be written down.

```
3·1354
1·05364
--------
2·08176
```

Example ii. *Subtract* ·032 *m. from* 36 *m.*

(a)

m.	dm.	cm.	mm.
36	. 0	. 0	. 0
		3	. 2
35	. 9	. 6	. 8

(b)

m.
36·0
·032
35·968

EXAMPLES. 82.

Subtract

1. 4·75 from 6·825.
2. ·931 from 1·002.
3. ·001 from 1.
4. 3·01 from 4.
5. 1·98 from 8·471.
6. 1·875 from 11·2.

Subtract

7. 8·45 from 19·336.
8. ·4761 from 3.
9. 19·003 from 20·1.
10. ·127 from 86·4.
11. ·0864 from ·183.
12. ·076 from 6·2.
13. ·075 from 3·5.
14. 20·476532 from 22.
15. 20·9 from 32·17654.
16. 1·00005 from 2·14.
17. 3·001004 from 4·62.
18. 1·0015 from 56·2.
19. ·29 from 864.
20. ·0025 from ·01.

Express as francs, dollars, or marks, and subtract

21. 15 f. 34 c. from 16 f.
22. 401 f. 3 c. from 500 f.
23. 4 c. from 3 f.
24. 115 c. from 3 f. 50 c.
25. 5 dol. 14 c. from 6 dol. 2 c.
26. 2 c. from 2 dol.
27. 203 c. from 5 dol. 50 c.
28. 12 pf. from 12 m.
29. 1 m. 12 pf. from 3 m.
30. 412 pf. from 6 m. 1 pf.

Express as decimals of the measure of highest denomination mentioned, and subtract

31. 5 m. 37 mm. from 15·003 m.
32. 17 Hm. 86 m. from 18 Km.
33. 8 g. 5 cg. 4 mg. from 15·75 Dg.
34. 7·685 Hg. from 10 Kg.
35. ·0459 m. from 55 cm. 7·5 mm.
36. A road is 6 Km. long and 4689 m. of it are paved; how much is unpaved?
37. From a piece of cloth, 20 m. long, three pieces measuring 5·17 m., 6·075 m. and 4·95 m. are cut off. What length remains?
38. Of 500 dollars, A gets 50·75 dol., B 25·15 dol. more than A, and C 20·50 dol. more than B. After D has received his share, there is 175·35 dol. left. How much does D get?
39. The length of a rod was measured and found to be 1·385 metres, but when 75 cm. had been cut off it was found that only 62 cm. remained. By how much was the first measurement wrong?
40. The distance from A to B is 25·384 Km. The distance of one inn from A is 12·625 Km., and the distance of another from B is 17·249 Km. How far apart are the inns, if both of them lie between A and B?
41. A kilogram = 2·2046213 lb., but it is generally sufficient to take 1 Kg = $2\frac{1}{5}$ lb.; by how much is this wrong?
42. 3·576 Kg. of tea is put up in packets weighing respectively 1·205 Kg., ·968 Kg. and 1·128 Kg. How much is wasted?

XIX. (f). TO MULTIPLY A DECIMAL BY A WHOLE NUMBER.

206. To multiply a decimal by an integer we simply extend the method commonly applied to whole numbers; that is, we carry forward multiples of 10, and set down remainders, the units' digit in the result, and in each partial product, being marked out as before by the position of the decimal point. Thus if we multiply 15·253 by 5, we get 76·265.

To multiply 15·253 by 50, we multiply it by 5 and the result by 10. From multiplication by 5 we get 76·265; this when multiplied by 10 becomes 762·65, since multiplication by 10 alters only the place-values of the digits, *i.e.* the position of the decimal point.

To multiply 15·253 by 500 is equivalent to multiplying it by 5 and the result by 100; *i.e.* it is the same as the multiplication of 76·265 by 100. The result is 7626·5.

207. We will next consider the multiplication of 15·253 by 475. This is equivalent to multiplying it by 400, 70 and 5 in turn, and adding these partial products to one another. For the purposes of addition it is of the utmost importance that the partial products shall be so placed that digits of the same order may be in the same column. A little consideration will show that this will be done if we arrange the work thus: *Place the* **units'** *digit of the multiplier under the* **last** *digit of the multiplicand, and then proceed as in ordinary multiplication, placing the right-hand digit of each partial product* **underneath that digit of the multiplier to the use of which it is due.** *The decimal point in each partial product will then fall underneath the decimal point in the multiplicand.*

We arrange the multiplication of 15·253 by 475 in this way in the margin.

We multiply first by the 4, *i.e.* by 400, placing the right-hand digit 2 of the partial product under the multiplying digit 4, and the decimal point under the decimal point of the multiplicand.

The right-hand digit of the partial product is thus two places more to the left than the right-hand digit of the multiplicand. This means that we have multiplied not merely by the digit 4, but by its complete value, namely, 400.

We next multiply by the digit 7 of the multiplier, *i.e.* by 70. We place the right-hand digit 1 of the partial product under the

```
 15·253
    475
------
6101·2
1067·71
 76·265
------
7245·175
```

multiplying digit 7 of the multiplier, and a decimal point under the other decimal points; and we see as before that by this arrangement we have obtained a product which is due, not merely to the digit 7, but to its complete value, namely, 70.

We lastly multiply by the digit 5 of the multiplier, and we place the right-hand digit 5 of the partial product underneath the multiplying digit 5.

Digits of the same order are thus in the same column, and by addition we find the complete product to be 7245·175.

The following examples will illustrate the process further.

Example i. *Multiply* 6·834 *by* 45.

We may multiply either (a) by the factors 5 and 9 in succession, or (b) by 45 as above.

(a)	6·834	(b)	6·834
	5		45
	34·170		273·36
	9		34·170
	307·53		307·53

We omit the final 0 in the result.

Example ii. *Multiply* ·00356 *metres by* 1255.

Rough approximation.

The result must lie between 1255 × ·003 metres and 1255 × ·004 metres; *i.e.* between 3·765 and 5·02.

The required product = 4·4678 metres.

metres.
·00356
1255
3·56
·712
·1780
·01780
4·4678

Example iii. *Multiply* ·000251 *by* 35.

Rough approximation.

The result must lie between 35 × ·0002 and 35 × ·0003, *i.e.* between ·007 and ·0105.

The required product = ·008785.

·000251
35
·00753
·001255
·008785

208. It is important to find a rough approximation to a result, in order to detect any error in the position of the decimal point.

EXAMPLES. 83.

Multiply

1. ·5 by 2, 3, 5, 9, 10.
2. ·125 by 2, 3, 4, 8, 5, 25, 125.
3. ·15 by 3, 4, 5, 6, 8, 9, 11.
4. ·05 by 8, 14, 18, 21, 25.
5. 3·24 by 4, 7, 56, 48, 60.
6. 3·1678 by 3, 45, 63, 112, 108.

7. 83·7325 by 1545, 76321. 8. ·0000025436 by 17, 51, 49.
9. 5 f. 25 c. by 4, 53, 64, 85, 96. 10. 15 dol. 5 c. by 3, 6, 48, 90, 108.
11. 5·756 m. by 7, 63, 504.
12. 7 Km. 8 Dm. 5 m. 3 cm. by 9, 108, 648.
13. 27·875 g. by 11, 19, 836. 14. 25 Kg. 27 g. by 13, 156, 697.
15. 2 Kg. 3 Hg. 7 g. 5 cg. by 15, 225, 1325.

XIX. (g). TO MULTIPLY BY A DECIMAL.

209. The product of 5·242 and 3 is 15·726.

To multiply 5·242 by ·3, we multiply by 3 and then divide the product by 10. The result will therefore be one-tenth of the former result, *i.e.* 1·5726.

To multiply 5·242 by ·03, we multiply by 3 and then divide the product by 100. Multiplying by 3 we get 15·726; hence the required result is ·15726.

Similarly, $5{\cdot}242 \times {\cdot}003 = {\cdot}015726,$
$${\cdot}5242 \times {\cdot}003 = {\cdot}0015726,$$
and $${\cdot}05242 \times {\cdot}003 = {\cdot}00015726.$$

If we examine these products, we shall see that to multiply by any digit to the right of the decimal point, we proceed as if that digit were in the units' place; *and we then move the decimal point in the product as many places to the* **left** *as the multiplying digit is to the* **right** *of the decimal point in the multiplier.*

210. We will next consider the multiplication of 5·242 by 25·314. It will be clear that to multiply by 25·314, we multiply by

20, 5, ·3, ·01, ·004

in succession, and that we add these partial products together to obtain the complete product. We see too that when we multiply by the digits 2, 5, 3, 1, 4 in succession, the decimal point in each partial product will be one place more to the left than in the partial product immediately preceding it; hence the decimal points will be in a vertical column, and digits which have the same place value will be one under another, if we adopt the following arrangement:—*Place the digit in the* **units'** *place of the multiplier under the* **last** *digit of the multiplicand, and then proceed as in ordinary multiplication,* **placing the right-hand digit of each partial product under that digit of the multiplier to the use of which it is due.** *The decimal point in each partial product will then fall underneath the decimal point of the multiplicand.*

The product obtained from the multiplier 2 consists of the digits 1, 0, 4, 8, 4; we place the right-hand digit 4 under the multiplier 2, and we place a decimal point under the decimal point of the multiplicand.

The right-hand digits of the other partial products are successively 0, 6, 2, 8, obtained respectively from the multiplying digits 5, 3, 1, 4. We place these partial products so that the 0 falls below the multiplier 5, the 6 below the multiplier 3, the 2 below the multiplier 1, and the 8 below the multiplier 4; and in each case we place a decimal point under the decimal point of the multiplicand. After addition we find the product to be 132·695988.

```
  5·242
    25·314
 ---------
 104·84
  26·210
   1·5726
    ·05242
    ·020968
 ---------
 132·695988
```

Rough approximation. The product must lie between 5×25 and 6×26; *i.e.* between 125 and 156.

Example i. *Multiply* ·0125 *by* ·00035.

We insert the digit **0** in the units place of the multiplier, and we place the multiplier so that that digit **0** falls under the right-hand digit **5** of the multiplicand.

The place-values of the digits and the positions of the decimal points being clearly understood, it is unnecessary to insert the 0's in the partial products, but of course they must be inserted wherever needed in the complete product.

```
   ·0125
     0·00035
 ----------
      375
       625
 ----------
 ·0000 04375
```

Rough approximation.

$·01 \times ·0003 = ·000003$,
and $·02 \times ·0004 = ·000008$.

Example ii. *Find the value of* 18 *m.* 75 *cm. at* 8 *f.* 24 *c. per metre.*

Since 18 m. 75 cm. = 18·75 m. and 8 f. 24 c. = 8·24 f., it will be clear that we have only to multiply 8·24 f. by 18·75.

Rough approximation.
The result must lie between
8 f. × 18 and 9 f. × 19,
i.e. between 144 f. and 171 f.

The required value = 154 f. 50 c.

```
   8·24 f.
     18·75
 ----------
 82·4
 65·92
  5·76 8
   ·41 20
 ----------
 154·5 francs.
```

EXAMPLES. 84.

Multiply

1. 19·75 by 7·2.
2. 13·13 by 11·17.
3. 692 by ·305.
4. 1672 by ·014.
5. 679·85 by ·00056.
6. 150·3205 by ·087218.

7. ·0076 by ·017.
8. ·3785 by ·0003.
9. ·0703 by ·00055.
10. ·00001 by ·00036.
11. 83·75 by 15·45, ·01625, 762·1.
12. ·125 by ·2, ·8, ·32, ·5, ·15, ·25.
13. 3·1678 by ·03, ·045, ·63, ·0112, ·108.
14. ·00000256 by ·017, ·51, 1·53.
15. 12·375 by 1·25, ·125, ·0125, ·00125.
16. 6·5 by 15·01, 1·501, 150·1, ·001501.
17. ·00167 by 6·541, 654·1, ·00006541.
18. ·0505 by ·001, ·00001.

Simplify each of the following expressions:

19. $\cdot1 \times \cdot1 \times \cdot1 \times \cdot1 \times \cdot1 \times \cdot1$.
20. $\cdot01 \times \cdot001 \times \cdot1 \times 1\cdot1 \times 10\cdot1$.
21. $\cdot1 \times 10\cdot2 \times \cdot3 \times \cdot5 \times \cdot2$.
22. $\cdot5 \times \cdot5 \times 1\cdot5 \times 5\cdot5 \times \cdot05$.
23. $\cdot834 \times \cdot12 \times 8\cdot012 \times 50\cdot0001$.

24. Find the cost of 148·8 Kg. at 15·20 f. per Kg.
25. What is the height of a flight of steps composed of 24 steps, each 19·5 cm. high?
26. How many Kilograms are contained in the weight of 189 chests, each weighing 29·73 Kg.?
27. A person owing 6750 f. paid ·544 of that amount and then ·5 of the remainder. How much was still owing?
28. Find the cost of 54·135 Kg. of butter at 3 f. 84 c. per Kg.
29. Find the weight of 75·25 m. of wire, if 1 m. weighs 5·64 g.
30. Find the value of 469 marks 75 pf. × 13·64.
31. If £1 = 24·75 f., find the value of £347·624 in francs.
32. A man buys 85·74 m. at 5·25 f. per metre, 73·24 m. at 6 f. 15 c. per metre, and 65·23 m. at 7 f. 50 c. per metre. He sells the whole for 1400 f.; what does he gain or lose?
33. A rod is 15 metres long; ·625 of it is cut off, and then ·175 of the remainder. Find the length of what is left.
34. If 1 mile = 1·6093 Km., find in Km., etc., the value of 5·2625 mi.
35. The rent of a flat on the 1st floor is £80; on the 2nd floor it is ·725 of this; on the 3rd floor it is ·725 of the rent on the 2nd floor; and the rent of a flat on the ground floor is 1·04 of that on the 3rd floor. Find the rent of each.

XIX. (h). TO DIVIDE A DECIMAL BY A WHOLE NUMBER.

211. The method is similar to that of dividing one whole number by another, the process being extended to digits to the right of the units' place. For example, the division of 6376·1275 by 5 may be stated thus:

$$5 \,|\, 6376{\cdot}1275$$
$$\underline{\quad 1275{\cdot}2255}$$

It is important to notice that the units' digit in the quotient is that which we get by using the units' digit in the dividend, so that there should be no difficulty as to the position of the point.

212. The extension of the decimal notation aids us in representing the quotient, when we divide one number by another which will not go into it an exact integral number of times. When we have used the last digit of the dividend in the ordinary way, we annex a decimal point and zeros, and we continue the operation as long as we please, or until it terminates.

For example, to divide 579 by 12, we represent 579 by 579·0000... ; and we get 48·25 for the correct and complete quotient *without remainder*.

$$12 \,|\, 579{\cdot}00$$
$$\underline{\quad 48{\cdot}25}$$

Ex. i. *Divide* 56·787 *by* 12.

$$12 \,|\, 56{\cdot}78700$$
$$\underline{\quad 4{\cdot}73225}$$

Ex. ii. *Divide* 725·5515 *by* 15.

$$3 \,|\, 725{\cdot}5515$$
$$5 \,|\, 241{\cdot}8505$$
$$\underline{\quad 48{\cdot}3701}$$

Ex. iii. *Divide* 1·0076 *by* 11.

$$11 \,|\, 1{\cdot}0076$$
$$\underline{\quad 0{\cdot}0916}$$

Ex. iv. *Divide* 8 *by* 3.

$$3 \,|\, 8{\cdot}00000.....$$
$$\underline{\quad 2{\cdot}6666666...}$$

In Ex. iv. the quotient is 2·66666.... We may carry on the decimal to as many places as we please, but it will never end. We shall presently consider examples of this kind more fully.

It should be noticed very carefully that **for each digit in the decimal part of a dividend there must be a digit in the decimal part of the quotient.**

213. If the divisor can easily be separated into small prime factors, the division should be performed by 'short' division; if not, the usual 'long' division arrangement may be adopted.

Example i. *Divide* 213·435 *by* 85.

We see that 85 will go into 213 twice; hence the first digit of the quotient is 2, and this is the whole of the integral part of the quotient.

As soon as we bring down the digit after the decimal point in the dividend, we insert a point in the quotient, and we continue the division just as if the divisor and the dividend were whole numbers.

To ensure the correct position of the point in the quotient it will be found convenient to **write each digit of the quotient immediately over that digit of the dividend which produces it**, as in this and the following examples.

Quot. = 2·511

85) 213·435

170

434

425

93

85

85

85

Example ii. *Divide* ·00213435 *by* 85.

The integral part of the dividend is 0; hence the integral part of the quotient is 0. Then comes the decimal point in the quotient. Then follow four zeros in the quotient, produced by the digits 0, 0, 2, 1 of the dividend, for we have to bring down *five* digits before we get a number 213 which is greater than 85. The fifth digit of the quotient is 2. The operation then goes on as in the preceding example.

Quot. = ·00002511

85) ·00213435

434

93

85

Example iii. *Divide* 2·13435 *by* 85.

The integral part of the quotient is zero, and there must be one zero after the point.

Quot. = 0·02511

85) 2·13435

434

93

85

EXAMPLES. 85.

Divide

1. 50 by 4, 8, 16, 32, 64.
2. 72 by 40, 45, 48, 50, 144.
3. 289 by 34, 85, 170, 680.
4. 783·5 by 5, 25, 125, 625, 6250.
5. ·01001 by 5, 25, 125, 625.
6. 25·5 by 15, 150, 170, 510.
7. ·255 by 3, 34, 102, 255.
8. 3·6288 by 3, 63, 126, 189.
9. 12·45 by 5, 166, 332, 415.
10. 773·682 by 13, 169, 1014.
11. ·0289 by 17, 34, 680.
12. 53·018 by 4, 196, 392.
13. 932·463 by 287, 861, 574.
14. ·00750116 by 1354, 2708.
15. 8·56406 by 22537, 112685.

XIX. (i). TO DIVIDE BY A DECIMAL.

214. The quotient obtained when we divide one number by another is the same as the quotient obtained when we divide any

multiple of the first number by the same multiple of the second. When the divisor is a decimal we may therefore multiply both divisor and dividend by such a power of 10 as will *make the divisor a whole number.*

But multiplication by 10 is equivalent to moving the decimal point one place to the right. Hence the operation just stated is equivalent to moving the decimal point, in both divisor and dividend, as many places to the right as there are decimal places in the divisor. The divisor then becomes a whole number, and we proceed as in Art. 214. The method stated briefly is: **Make the divisor a whole number.**

Example i. *Divide* ·0011355 *by* ·003.

Following the rule, we get 3 for our new divisor, and 1·1355 for our dividend. Then proceeding as before, the required quotient is ·3785.

$$3 \overline{)\, 1{\cdot}1355} \quad {\cdot}3785$$

Ex. ii. *Divide* 15·46875 *by* 12·375.

This is equivalent to dividing 15468·75 by 12375.

$$\begin{array}{r} \text{Quot.} = 1{\cdot}25 \\ 12375 \overline{)\, 15468{\cdot}75} \\ 3093\ 7 \\ \hline 618\ 75 \end{array}$$

Ex. iii. *Divide* 12·5 *by* ·000025.

This is equivalent to dividing 12500000 by 25. The quotient is 500000.

$$25 \overline{)\, 12500000} \quad 500000$$

Remainder after Division of a Decimal.

215. In the case of a whole number we have shown that the remainder after division is in terms of the same unit as the dividend, but in the case of a decimal we frequently begin by multiplying the divisor and the dividend by some power of 10. The following example will illustrate what we must do in such a case, in order to obtain the *true remainder* after division.

Example. *How many lengths of* ·023 *ft. can be cut from a rod whose length is* 3·14159 *ft.? What length will remain over?*

Dividing 3·14159 by ·023 is equivalent to dividing 3141·59 by 23. We obtain 136 as the integral part of the quotient, and we do not proceed further. The remainder after this step is 13·59, *if we take the dividend to be* 3141·59. But this number is 10^3 times the real dividend, and consequently 13·59 is 10^3 times the real remainder. Hence the true remainder is ·01359.

$$\begin{array}{r} 136 \\ 23 \overline{)\, 3141{\cdot}59} \\ 23 \\ \hline 84 \\ 69 \\ \hline 151 \\ 138 \\ \hline 13{\cdot}59 \end{array}$$

The pupil should test this by proving that

$$136 \times {\cdot}023 + {\cdot}01359 = 3{\cdot}14159.$$

EXAMPLES. 86.

Divide

1. 384·006 by ·123.
2. 7·808 by ·128.
3. 7·4339 by ·0079.
4. 1·6 by ·0000004.
5. 815·804 by ·00011.
6. 198·198 by ·00066.
7. ·00071145 by ·00000837.
8. ·08748 by ·00108, ·0108.
9. 614·50824 by ·010201.
10. 785·3586 by ·04173.
11. 145·0209 by ·00785.
12. 614·50824 by ·0010201.
13. 155806·56 by ·006552.
14. 25·5 by ·5, ·3, ·015, 1·7.
15. 72 by ·018, 1·8, ·0018.
16. 72·5026 by 7·85.
17. ·065341 by ·00475.
18. 2560·0160001 by 1·59601.
19. ·00112608 by ·01700.
20. ·055757592 by ·009207.
21. 46·4856 by ·4, ·0000004.
22. 1·18596067 by ·0004673.
23. 7·835 by 12·5, 6·25, ·625.
24. 3·762505 by ·0785.
25. ·003125 × ·48 by ·0000125.
26. 3·995418 by ·00753.
27. ·282892 by ·394.
28. 1·365 by 1·25, 12·5, ·00125.
29. ·16016 by 1·43, 14·3, 143, ·0000143.
30. ·0213419596 by 1·00103.
31. ·00307326 by 3·013.
32. ·2332728 by 32·58.
33. ·2107206 by 42·06.
34. ·0189 by 2·5, ·0125, 8·75.
35. ·0598635 by 112·95.
36. ·000133 by 8·75.

37. What will be the weight of 1·25 m. of cloth, if 100 m. weigh 20 Kg.?
38. How many steps, 65 cm. long, does a French soldier take in marching 273 Km.?
39. A piece of stuff, at 2·75 f. per metre, costs 231 f.; how many metres are there?
40. The top of a tower, 48·75 m. high, is reached by 325 steps. What is the height of each step?
41. A man rides a bicycle at the rate of 20·7 Km. per hour, and he rides for 8 hrs. per day. How many days will he take to ride a distance of 828 Km.?
42. The diameter of a 5-centime piece is 2·5 cm. How many placed in a straight line, touching one another, will extend to a distance of 15·75 m.?
43. Into how many parcels, each weighing 2·35 Kg., can 338·4 Kg. be divided?

44. In a waggon are a certain number of parcels, each weighing 5·325 Kg., 7 times as many, each weighing 10·34 Kg, 12 times as many, each weighing 15·175 Kg, and 25 times as many, each weighing 17·365 Kg. If the total weight be 10 t. 408·95 Kg., find the number of parcels.

45. How many times is ·0135 in. contained in 1 inch? What is the remainder?

46. How many lengths of 15·35 cm. can be cut from a piece of cloth 5·68 m. long? What length remains?

XIX. (j). TO EXPRESS A DECIMAL BY AN EQUIVALENT VULGAR FRACTION.

216. Since $\cdot 76 = \frac{7}{10} + \frac{6}{100} = \frac{70+6}{100} = \frac{76}{100}$,

and $\cdot 025 = \frac{0}{10} + \frac{2}{100} + \frac{5}{1000} = \frac{20+5}{1000} = \frac{25}{1000}$,

and $\cdot 0063 = \frac{0}{10} + \frac{0}{100} + \frac{6}{1000} + \frac{3}{10000} = \frac{6}{1000} + \frac{3}{10000} = \frac{63}{10000}$,

we see that *the vulgar fraction equivalent to a decimal has, for its denominator, unity followed by as many zeros as there are digits in the decimal; and, for its numerator, the number formed by the digits of the decimal, disregarding any zeros that there may be immediately after the decimal point.*

Thus $\cdot 762 = \frac{762}{1000}$; $\cdot 0762 = \frac{762}{10000}$; $\cdot 00123 = \frac{123}{100000}$.

The vulgar fractions must then be reduced to their lowest terms.

217. Mixed numbers in the decimal form may be treated in a similar way. For example, 50·0309 is equivalent to the mixed number $50\frac{309}{10000}$, or to the improper fraction $\frac{500309}{10000}$.

EXAMPLES. 87.

Determine vulgar fractions equivalent to the following decimals:

1.	·2.	2.	·4.	3.	·5.	4.	·6.	5.	·8.
6.	·14.	7.	·12.	8.	·15.	9.	·16.	10.	·25.
11.	·35.	12.	·45.	13.	·55.	14.	·64.	15.	·65.
16.	·575.	17.	·01.	18.	·04.	19.	·06.	20.	·05.
21.	·08.	22.	·125.	23.	·155.	24.	·128.	25.	·256.
26.	·305.	27.	·345.	28.	·372.	29.	·628.	30.	·625.
31.	·025.	32.	·015.	33.	·075.	34.	·016.	35.	·064.

36.	·002.	**37.**	·005.	**38.**	·006.	**39.**	·6125.
40.	·0125.	**41.**	·0625.	**42.**	·0225.	**43.**	·0052.
44.	6·0005.	**45.**	13·01125.			**46.**	29·03205.
47.	71·00625.	**48.**	93·3125625.			**49.**	108·625125.

XIX. (k). TO EXPRESS CERTAIN VULGAR FRACTIONS AS DECIMALS.

218. It follows from the meaning of the notation that vulgar fractions, which have for their denominators 10, 100, 1000, etc., can be expressed at once in the decimal form. For example,

$$\tfrac{3}{10} = \cdot 3, \quad \tfrac{29}{10} = 2\tfrac{9}{10} = 2\cdot 9, \quad \tfrac{3}{100} = \cdot 03,$$

$$\tfrac{57}{100} = \tfrac{50}{100} + \tfrac{7}{100} = \tfrac{5}{10} + \tfrac{7}{100} = \cdot 57,$$

and $$\tfrac{349}{100} = 3\tfrac{49}{100} = 3\cdot 49.$$

Similarly, $\tfrac{9}{1000} = \cdot 009$, $\tfrac{49}{1000} = \cdot 049$, $\tfrac{678}{1000} = \cdot 678$,

and $$\tfrac{9241}{1000} = 9\cdot 241.$$

Hence, *when the denominator of a vulgar fraction is* 10, *or any power of* 10, *the fraction is equivalent to a decimal the significant figures of which are the digits of the numerator, and which has as many digits after the point as there are zeros in the denominator.*

If there are **more** *digits in the numerator than there are zeros in the denominator, we can at once put the point in its right place; but, if there are* **fewer**, *we prefix zeros until there are as many, and we then put the decimal point in front of all.*

We will consider presently how *any* vulgar fraction may be expressed as a decimal.

EXAMPLES. 88. *Oral.*

Express in the decimal form the following vulgar fractions and mixed numbers:

1.	$\tfrac{3}{10}$.	**2.**	$\tfrac{9}{10}$.	**3.**	$\tfrac{23}{10}$.	**4.**	$\tfrac{37}{10}$.	**5.**	$\tfrac{113}{10}$.
6.	$\tfrac{259}{10}$.	**7.**	$17\tfrac{9}{10}$.	**8.**	$\tfrac{97}{100}$.	**9.**	$\tfrac{79}{100}$.	**10.**	$\tfrac{53}{100}$.
11.	$\tfrac{13}{100}$.	**12.**	$\tfrac{9}{100}$.	**13.**	$\tfrac{4}{100}$.	**14.**	$\tfrac{3}{100}$.	**15.**	$\tfrac{1}{100}$.
16.	$20\tfrac{1}{100}$.	**17.**	$805\tfrac{33}{100}$.	**18.**	$\tfrac{109}{1000}$.	**19.**	$\tfrac{565}{1000}$.		

Express in the decimal form the following vulgar fractions and mixed numbers:

20. $\frac{59}{1000}$. 21. $\frac{13}{1000}$. 22. $\frac{9}{1000}$. 23. $\frac{5}{1000}$.

24. $\frac{3}{1000}$. 25. $\frac{1}{1000}$. 26. $20\frac{1}{1000}$. 27. $805\frac{23}{1000}$.

28. $79\frac{264}{1000}$. 29. $\frac{1092}{10000}$. 30. $\frac{5651}{10000}$. 31. $\frac{593}{10000}$.

32. $\frac{59}{10000}$. 33. $\frac{9}{10000}$. 34. $\frac{3}{10000}$. 35. $\frac{1}{10000}$.

36. $\frac{1}{100000}$. 37. $\frac{1}{1000000}$. 38. $\frac{1}{10000000}$.

39. $\frac{1689101}{1000000}$. 40. $\frac{1001}{1000000}$. 41. $100\frac{301}{100000}$.

42. $18462\frac{2}{10000}$. 43. $2547\frac{13}{1000000}$.

XIX. (1). TO EXPRESS ANY VULGAR FRACTION IN THE FORM OF A DECIMAL.

219. When a vulgar fraction has for its denominator one of the numbers 10, 100, 1000, ..., or any power of 10, we have seen that it can be expressed as a decimal at sight.

When the fraction is in its lowest terms and the denominator is not one of these numbers, the number forming the denominator must be one of two kinds: (i) its only prime factors may be 2, 5, or 2 *and* 5, these being the only prime factors of 10; or (ii) it may have other prime factors than 2 and 5.

220. Any number whose only prime factors are 2 and 5 can, by multiplication, be converted into a power of 10. For example, the number 40, which is equivalent to 5×8, or 5×2^3, becomes a power of 10 when it is multiplied by 5^2, since

$$40 \times 5^2 = 5 \times 2^3 \times 5^2 = 5^3 \times 2^3 = 10^3.$$

It should be noticed that the product of a number of 2's and the *same* number of 5's is always equal to a power of 10;

e.g. $5^4 \times 2^4 = 10^4$, and generally $5^n \times 2^n = 10^n$.

The denominators which we are now considering are of the form $5^m \times 2^n$, where m and n are different. If m be less than n, the number can be converted into a power of 10, by multiplying it by as many 5's in succession as will make the number of 5's equal to the number of 2's; if n be less than m, we should multiply by as many 2's in succession as would make the number of 2's equal to the number of 5's. It will be remembered

of course that, if we multiply the denominator of a vulgar fraction, we must multiply the numerator also by the same number; otherwise the value of the fraction would be altered.

A few examples will make the process easily understood. Thus:

$$\frac{1}{4}=\frac{1}{2^2}=\frac{5^2}{2^2 5^2}=\frac{5^2}{10^2}=\frac{25}{100}=\cdot 25,$$

$$\frac{3}{8}=\frac{3}{2^3}=\frac{3\times 5^3}{10^3}=\frac{3\times 125}{1000}=\frac{375}{1000}=\cdot 375,$$

and

$$\frac{7}{40}=\frac{7}{5\times 8}=\frac{7}{5\times 2^3}=\frac{7\times 5^2}{10^3}=\frac{7\times 25}{1000}=\frac{175}{1000}=\cdot 175.$$

221. When the denominator of a vulgar fraction is obviously of the form $2^m \times 5^n$, we may in practice shorten the process by multiplying the numerator and the denominator by 5's or 2's, in succession, in such a way as the form of the denominator may suggest. For example,

$$\tfrac{1}{4}=\tfrac{5}{20}=\tfrac{25}{100}=\cdot 25,$$

$$\tfrac{7}{80}=\tfrac{35}{400}=\tfrac{175}{2000}=\tfrac{875}{10000}=\cdot 0875,$$

and

$$\tfrac{23}{125}=\tfrac{184}{1000}=\cdot 184.$$

222. In all cases in which 2, 5, or 2 and 5 are the only prime factors of a denominator the equivalent decimal will terminate. In all other cases it will **not** terminate, but may be obtained to as many decimal places as we please.

We have stated in Art. 140 that a vulgar fraction represents a division, of which the numerator is the dividend and the denominator is the divisor. In cases to which the method of Art. 220 will not apply, the required decimal can be obtained by simple division. For example:

$$\tfrac{2}{3}=\cdot 666666\ldots,\ \tfrac{4}{11}=\cdot 363636\ldots,\ \tfrac{5}{7}=\cdot 714285\ldots,\ \text{etc.}$$

Example 1. *Express* $\frac{3}{32}$ *in the decimal form.*

In this case we can obtain the decimal by the method of Art. 221. We introduce it here in order to shew that the two methods, or a combination of the two methods, lead to the same result.

(a) $\frac{3}{32}=\frac{15}{160}=\frac{75}{800}=\frac{\cdot 75}{8}=\cdot 09375.$

(b)
$$32\left\{\begin{array}{l|l} 4 & 3\cdot 00000 \\ 8 & \cdot 75000 \\ \hline & \cdot 09375 \end{array}\right.$$

Example ii. *Express* $\frac{18}{19}$ *as a decimal.*

Proceeding by long division as far as five decimal places, we see that $\frac{18}{19} = \cdot94736\ldots$. The quotient may however be represented *completely* by $\cdot94736\frac{16}{19}$, where $\frac{16}{19}$ represents *not* $\frac{16}{19}$ of unity, but $\frac{16}{19}$ of the unit to which the preceding digit 6 refers; that is, it denotes $\frac{16}{19}$ of $\frac{1}{100000}$ of unity.

Quot. = 0·94736
19) 18·00000
90
140
70
130
16

NOTE.—The following method of obtaining a large number of decimal places is deserving of notice. We carry on the division until we get a small remainder, and then we proceed as below:

$\frac{1}{17} = \cdot0588235\frac{5}{17}$; .....(i)

$\therefore$ multiplying (i) by 5, $\frac{5}{17} = \cdot2941176\frac{8}{17}$;

$\therefore$ substituting in (i), $\frac{1}{17} = \cdot05882352941176\frac{8}{17}$; .....(ii)

and multiplying (ii) by 8, $\frac{8}{17} = \cdot47058823529411\frac{13}{17}$;

$\therefore$ substituting in (ii), $\frac{1}{17} = \cdot0588235294117647058823529411\frac{13}{17}$.

Hence we may deduce $\frac{2}{17}$, $\frac{3}{17}$, etc.

EXAMPLES. 89. *Oral.*

Examine each of the following fractions, and state whether it can be expressed as a terminating decimal or not:

1.	$\frac{1}{2}$.	2.	$\frac{1}{3}$.	3.	$\frac{1}{5}$.	4.	$\frac{1}{6}$.	5.	$\frac{1}{8}$.
6.	$\frac{3}{4}$.	7.	$\frac{17}{25}$.	8.	$\frac{18}{24}$.	9.	$\frac{17}{16}$.	10.	$\frac{13}{20}$.
11.	$\frac{8}{23}$.	12.	$\frac{7}{32}$.	13.	$\frac{8}{125}$.	14.	$\frac{13}{64}$.	15.	$\frac{17}{80}$.
16.	$\frac{213}{625}$.	17.	$\frac{13}{128}$.	18.	$\frac{157}{256}$.	19.	$\frac{117}{264}$.	20.	$\frac{717}{3125}$.
21.	$\frac{231}{512}$.	22.	$\frac{94}{400}$.	23.	$\frac{117}{350}$.	24.	$\frac{208}{1200}$.	25.	$\frac{91}{264000}$.

EXAMPLES. 90.

Express each of the following vulgar fractions as a terminating decimal:

1.	$\frac{3}{20}$.	2.	$\frac{7}{20}$.	3.	$\frac{9}{20}$.	4.	$\frac{18}{25}$.	5.	$\frac{19}{250}$.
6.	$\frac{111}{125}$.	7.	$\frac{560}{125}$.	8.	$\frac{17}{40}$.	9.	$\frac{5}{16}$.	10.	$\frac{9}{16}$.
11.	$\frac{413}{625}$.	12.	$\frac{71}{625}$.	13.	$\frac{8114}{1250}$.	14.	$\frac{5}{64}$.	15.	$\frac{119}{256}$.
16.	$\frac{3}{5120}$.	17.	$9\frac{3}{16}$.	18.	$2\frac{11}{64}$.	19.	$\frac{115}{32}$.	20.	$105\frac{23}{320}$.

In the following cases find the quotient as far as the 6th figure after the decimal point:

21. $\frac{1}{3}$. 22. $\frac{1}{6}$. 23. $\frac{3}{7}$. 24. $\frac{5}{9}$. 25. $\frac{3}{11}$.

26. $\frac{15}{22}$. 27. $\frac{7}{26}$. 28. $\frac{13}{42}$. 29 $21\frac{11}{12}$. 30. $241\frac{11}{13}$.

31. $8\frac{9}{14}$. 32. $1\frac{1}{15}$. 33. $24\frac{6}{17}$. 34. $41\frac{11}{21}$. 35. $2\frac{1}{27}$.

36. $81\frac{15}{34}$. 37. $3\frac{4}{45}$. 38. $71\frac{15}{46}$. 39. $33\frac{7}{96}$ 40. $9\frac{5}{108}$.

XX. APPROXIMATION.

223. For many purposes a sum of money is expressed with sufficient accuracy, if it is correct to the nearest penny; in no case can we give, or receive, a fraction of a *farthing*. When we ask the time, we are usually content if we are told the number of minutes past, or to, the hour; we do not generally care to know the number of seconds And in stating the distance of a remote place, we are generally accurate enough if we state the number of miles as nearly as possible, disregarding yards, feet and inches. In the case of a star, even a million miles might be considered so small a portion of the whole distance as to be negligible.

224. A number, or a measure, is said to be **approximately** correct, when it is sufficiently near the true value for the purpose to be served.

Suppose, for example, that a man sells cloth at the rate of 3 metres for 5 francs. If we buy exactly 1 metre of it, what must we pay? The exact value of 1 metre would be $1\frac{2}{3}$ francs, or 1·6666... francs, or 1 f. 66·66... c. But there are no coins by which we can pay exactly $\frac{2}{3}$ of a franc, or 66·666... centimes. The nearest we can get to it is 1 f. 67 c., which is therefore the *approximate* value of 1 metre of cloth. It is useless to find the value more accurately than to $\frac{1}{100}$ of a franc, *i.e.* to 2 decimal places.

Suppose, again, that we are told that the height of the Matterhorn is 4432 metres, and that a metre is equivalent to 3·280899 feet; and suppose that we wish to know the number of *feet* in the height. The result would be obtained by multiplying 3·280899 ft. by 4432. In the complete product we should have 6 decimal places, so that the height would be obtained apparently to the *millionth*-part of a foot. But for all practical purposes it would be sufficient to determine a measure of this magnitude to the nearest foot. Moreover, we cannot regard the original measurement of 4432 metres as absolutely exact; we can regard it as accurate only to the nearest metre. The actual height may

therefore be anything between 4431½ m. and 4432½ m., and the original error may thus be as much as ½ of a metre, or 1 foot 7 inches. This being so, it will obviously be absurd to pretend that, merely by an arithmetical operation, a result can be obtained from it which is accurate as far as the *millionth*-part of a foot. In fact, a result correct to the nearest foot is not only all that is necessary, but is also all that can be relied upon.

225. The pupil should understand that when we *approximate* to a result by applying the methods of this chapter, we obtain results as accurate as is practically possible or necessary, and that we avoid the labour which would be involved in useless calculations, which, if completed, would often be wrong.

226. If a weight is known to be exactly 9·45843 grains, and if, for a particular purpose, we need not trouble ourselves with anything less than *one-thousandth* part of a grain, we may for this purpose consider the weight simply as 9·458 grains.

When, however, we omit all the figures after a certain decimal place, we must not ignore entirely the *first* of the omitted figures. The approximation in the case of 9·45843 is, as we have said, 9·458; but in the case of 9·45853 it would be 9·459. In the latter case 9·459 is ·00047 too much, but 9·458 would be ·00053 too little. The *error* involved in taking 9·459 would therefore be *less* than that involved in taking 9·458, and 9·459 is therefore the more accurate approximation. In fact an approximation, as far as a certain number of decimal places, does not mean that the figures which follow the decimal point are identical with the figures in the corresponding places of the true value, but that the figures in the approximation give a result nearer the true value than the same number of any other figures would give. For example, a correct approximation to 62·19999642, as far as 5 decimal places, would be 62·20000, although no figure in ·20000 is the same as the corresponding figure in ·19999. We may state this in the form of a rule thus:—If the first of the omitted figures be 9, 8, 7, 6 or 5, we increase the last figure retained by 1; and if the first omitted figure be 4, 3, 2, 1 or 0, we make no such change, but we leave the last retained figure unaltered. Thus, if we retain 4 decimal places only, we see that

3·4562499 becomes 3·4562,

but 3·4562501 becomes 3·4563.

NOTE.—If a result is true to *one* decimal place, the error cannot be more than ·05 of the unit; if to two decimal places, the error cannot be greater than ·005 of the unit, and so on.

227. Whole numbers may be treated in a similar way. In tables of statistics, for example, we may sometimes ignore the hundreds, tens and units. In such a case the nearest approximation to 37624392 would be 37624000, and that to 84304663 would be 84305000, and these approximations would be the numbers recorded in the tables

EXAMPLES. **90.** *Oral.*

State the integers next above and next below the following numbers; and state the values of the numbers to the *nearest* integer:

1	6·75,	**2.**	11·01,	**3.**	18·54,	**4.**	17·394,
5.	220·09,	**6.**	567·61,	**7.**	85·91,	**8.**	745·831,
9.	1001·5004,	**10.**	10·924,	**11.**	999·51.		

State the values of the following lengths correct to the nearest centimetre:

12	3·456 m.,	**13.**	14·84 cm.,	**14.**	18·432 m.,
15.	24·376 m.,	**16.**	218·3 cm.,	**17.**	5·673 cm.,
18.	3·8952 m.,	**19.**	917 mm.,	**20**	18764 mm.

State the values of the following measures correct to two decimal places:

21.	37·4132 cm.,	**22.**	13·2051 Kg.,	**23.**	24·3643 Km.,
24.	8·417 g.,	**25.**	4·5738 m.,	**26.**	21·3626 Km.,
27.	57·7112 mm,	**28.**	19·8939 Kg.,	**29.**	7·6381 ft.,
30	23·4235 yds.,	**31.**	47·3843 in.,	**32.**	10·2957 ft.,
33.	3·5968 yds.,	**34.**	19·9991 in.,	**35.**	399·9997 yds.

Write down the following numbers correct to the nearest thousand:

36.	36325,	**37.**	134369,	**38.**	91638,	**39.**	78869,
40.	45234,	**41.**	273724,	**42.**	88315,	**43.**	147471,
44.	56943,	**45.**	239968,	**46.**	189924,	**47.**	197572,
48.	30791,	**49.**	125441,	**50.**	374814,	**51.**	399903.

(i) Addition and Subtraction.

228. We will now consider how to find the approximate *sum* of two or more numbers, and the approximate *difference* of two numbers, *i.e.* the sum, or the difference, correct as far as a certain decimal place.

Suppose, for example, that we require the sum and the difference of 3·79841159321 and 5·62018078024, *correct* to *four* decimal places. In each case we retain *six* decimal places (two more than the *four* which must be correct), and we write the numbers in column as in the margin. In neither case do we write any figure in the 5th and 6th decimal places of the result. The figure in the 5th place of the sum would be 9, and we take account of it by writing 6 instead of 5 in the 4th place; and for a similar reason we write 8 instead of 7 in the 4th place of the difference.

5·620181
3·798412
sum = 9·4186
difference = 1·8218

NOTE.—We retain in each number two decimal places more than we want in the result, because we cannot be sure of having the *fourth* figure correct unless we know whether the *fifth* figure is one of the numbers, 0, 1, 2, 3 and 4, or one of the numbers 5, 6, 7, 8 and 9.

EXAMPLES. 91.

Find the sum, correct to 2 decimal places, of

1. 3·45625, ·008458, 17·859249, 8·632454.
2. 15 67301, 4·370956, 5·210562, 18·86483.
3. 6·43259, 7·520187, 4·541894, ·372687.
4. ·82194, ·335, 1·864359, 2·346724.
5. 3·15792, ·314157, 6·354671, ·637205.
6. ·94621, 5·724153, 8·941263, 1·821738.

Find the difference, to the nearest hundredth, of

7. 5·6723 and 18·9421.
8. 10·9843 and 8·6734.
9. 3·6725 and ·5697.
10. 2·6546 and 3·7829.
11. 8·6201 and 5·4983.
12. ·3792 and 3·8463
13. 7·8629 and ·5432.
14. 1·5249 and 3·4795.

(ii) Multiplication.

229. We have seen that in the addition of decimals we retain in each number 2 decimal places more than are to be correct in the result. In the process of multiplication we determine a number of partial products, and we then find their sum; consequently, in each of these partial products, we must determine 2 decimal places more than we require correct in the final result of the process. If we require the result to be correct to the nearest

unit, we must determine each partial product as far as *two decimal places*. As an example, we will determine the product of 123·76592 and 324·786, correct to the units' place.

In each partial product we retain **two** decimal places; we therefore place the **units' digit 4** of the multiplier beneath the digit **6** in the **second** decimal place of the multiplicand, as explained in Art. 210. We then draw a vertical line to the *right* of the digits 6 and 4, and it follows that the right-hand digit of each partial product must be placed immediately to the left of this vertical line.

123·76	592
3 24	·786
37129·78	
2475·32	
495·06	
86·63	
9·90	
·74	
40197·43	

= 40197 to nearest integer.

We multiply first by the digit 3, which means 300, and we therefore begin upon the digit 9 in the 4th decimal place of the multiplicand, in order to obtain a digit in the 2nd decimal place of the partial product; and, when we multiply the 9 by the 3, we must for greater accuracy determine what would have been carried from the multiplication of the digit 2, which is next after the 9. Since $3 \times 2 = 6$, we see that the number carried would be 1; hence 3 times 9, or 27, is increased by 1 and becomes 28. The first partial product is thus found to be 37129·78.

We next multiply by the digit 2 of the multiplier, which means that we multiply by 20. We therefore begin with it on the digit 5 in the 3rd decimal place of the multiplicand, reckoning and adding what would be carried from the multiplication of the next digit 9. Since $2 \times 9 = 18$, we carry 2, and the second partial product becomes 2475·32.

We next multiply by the third digit 4, and we begin upon the figure 6 of the multiplicand, reckoning as before, and adding, the number 2 which would have been carried from the multiplication of the digit 5 to the right of the 6. We thus obtain 495·06 for the third partial product.

Proceeding in this way, when we multiply by the digit 7 of the multiplier, we begin upon the digit 7 of the multiplicand; with the digit 8 we begin upon the digit 3; with the digit 6, upon the digit 2; and the operation then terminates. Adding the partial products together we get 40197·43, which gives a result 40197, correct to the nearest unit.

230. The pupil should not pass over what has been said about the placing of the decimal points in the partial products, without clearly understanding the principle upon which it is based. Thus the place-value of the digit 3 of the multiplier

is 10^2, and the place-value of the digit 9 of the multiplicand is $\frac{1}{10^4}$; hence the place-value of their product is $10^2 \times \frac{1}{10^4}$, or $\frac{1}{10^2}$, and the resulting digit will consequently fall into the *second* decimal place of the product.

In the example considered it will be seen, too, that as we moved from left to right when using the digits of the multiplier in turn, so we moved from right to left as we selected the digit upon which to begin the operation. The pupil should not be content always to rely upon any rule such as this, but should see from time to time that he is carrying out the principle upon which the operation is founded.

In these cases it is also very important that a rough approximation test should be applied, as the most common mistakes in working with decimals arise from the misplacing of the decimal point. This is an error of a kind which a rough test will nearly always detect.

231. We may summarize the operations as follows:

(i) Determine the number of decimal places to be retained (two more than must be correct in the result), and draw a vertical line to the right of the digit in the last of these decimal places in the multiplicand.

(ii) Place the multiplier under the multiplicand, so that the digit in its *units'* place is under that digit of the multiplicand which is next to, and to the left of, the vertical line If there is no other digit in the units' place of the multiplier, put the digit 0 in it.

(iii) Place each partial product, as it is formed, so that its right-hand digit is under the units' digit of the multiplier, and therefore next to the vertical line The decimal points of the partial products, and of the multiplicand, will then be in column.

(iv) The partial product due to the units' digit of the multiplier is formed by beginning the multiplication at that digit of the multiplicand which is vertically above it.

(v) To form the partial product due to a digit of the multiplier which is 2, 3, 4, ... places to the *left* of the vertical line, we begin the multiplication at a digit of the multiplicand which is 1, 2, 3, ... places, respectively, to the *right* of the vertical line.

(vi) To form the partial product due to a digit of the multiplier which is 1, 2, 3, ... places to the *right* of the vertical line, we begin the multiplication at a digit of the multiplicand which is 2, 3, 4, ... places, respectively, to the *left* of the vertical line.

(vii) To ensure accuracy, we take account in each case of what would be carried from the use of the digit of the multiplicand to the right of that upon which the multiplication begins.

Example i. *If* 1 *metre* = 3·280899 *feet, find to the nearest foot the value of* 4432 *metres.*

We place the multiplier so that the digit **2** in the units' place is under the digit 8 in the 2nd decimal place of the multiplicand.

In multiplying by the left-hand digit **4** of the multiplier, we begin upon the digit **9** in the 3rd place of the multiplicand to the right of the vertical line, carrying 4 from $4 \times 9 (= 36)$. This is in accordance with Rules (v) and (vii).

In adding the partial products we need not, in the sum, *write down* any digits to the *right* of the decimal point.

We add the last two columns *mentally*, and we use their sums only to obtain the carrying figure. The required result is 14541 feet.

3·28	0899
44 32	
13123·60	
1312·36	
98·43	
6·56	
14541	

Rough approximation.

The result must lie between 3×4000 ft. and 4×5000 ft., *i.e.* between 12,000 ft. and 20,000 ft.

Example ii. *Find the value of* 56·876 × ·00144 *as far as* 4 *decimal places.*

We affix three 0's to the multiplicand in order to have six (*i.e.* 4+2) decimal places in it, and we put a digit 0 in the units' place of the multiplier.

The 0's in front of the second and third partial products may be omitted, as the place-values of the other figures are already determined by the positions in which they are placed.

The result is ·0819.

56·876000	
0	·00144
·056876	
22750	
2275	
·0819	

Rough approximation.

The result must lie between $50 \times \cdot001$ and $60 \times \cdot002$, *i.e.* between ·05 and ·12.

Example iii. *Multiply* ·00040635 *by* 241·6358, *so that the result may be correct to within* ·0000005.

We place two 0's to the right of the vertical line in the multiplicand, because the first digit of the multiplier denotes 200, and we must have the digits upon which the multiplying figures 2 and 4 will begin.

We have omitted the initial 0's in all the partial products except the first, for the reason given in Example ii.

The result is ·098189.

Rough approximation.

The result must lie between $200 \times \cdot0004$ and $300 \times \cdot0005$, *i.e.* between ·08 and ·15.

·00040635	00
241	·6358
·08127000	
1625400	
40635	
24381	
1219	
203	
32	
·098189	

EXAMPLES. 92.

Determine the following products, correct to two decimal places.

1. $3{\cdot}207 \times {\cdot}321$.
2. $4{\cdot}367 \times {\cdot}208$.
3. $2{\cdot}563 \times {\cdot}672$.
4. $6{\cdot}825 \times {\cdot}623$.
5. $29{\cdot}582 \times {\cdot}476$.
6. $21{\cdot}347 \times 10{\cdot}365$.
7. $14{\cdot}287 \times 25{\cdot}894$.
8. $35{\cdot}478 \times 168\,825$.
9. $18{\cdot}291 \times 223{\cdot}643$.
10. $12{\cdot}847 \times 352{\cdot}457$.

(iii) Division.

232. If we divide 16·80437921 by 3·142 as far as 6 places, the work appears as in the margin.

Quot. = 5·348306

3142) 16804·37921
15710
10943
9426
15177
12568
26099
25136
9632
9426
2060

To shorten the work we omit the digits to the right of the vertical line. The following rules, by which we may dispense altogether with these digits, will be easily understood after what we have said about multiplication, with the help of the examples that follow:

(i) *Determine one decimal place more than must be correct.*

(ii) *Find the first digit of the quotient in the ordinary way, or by inspection, so that we may know how many digits there will be in the whole of the quotient wanted—integral part as well as decimal.*

(iii) *Proceed in the ordinary way until the number of quotient-digits still to be obtained is two less than the number of digits in the divisor.*

(iv) *As soon as this happens, instead of bringing down the next digit of the dividend, strike off a digit from the end of the divisor, and proceed one step with this curtailed divisor; remembering however, in the multiplication, to add in the carrying figure from the digit just struck off. Repeat this process, striking off the digits of the divisor in succession.*

(v) *If, at the beginning, there are more digits in the divisor than there are to be in the quotient, retain one more, and strike out the rest.*

Example 1. *Divide* 673·1489 *by* ·41432 *to the unit's figure.*

After determining the quotient-digits 1, 6, there will be only three more to be determined, which is two less than the number of digits in the divisor; we therefore begin then to curtail the divisor.

1624·7
41432) 67314890
41432
258828
248592
10236

41,4,3,2 | 10237
8286
1951
1657
294
290
4

Since the first rejected digit of the dividend is 9, we change the final digit of the second remainder into 7. The required quotient is 1625.

Example ii. *Divide* 2·7182818 *by* 3·1415927 *to* 6 *places.*

Making the divisor a whole number it becomes 31415927, and the dividend becomes 27182818.

The first digit of the quotient is that which follows the decimal point. We have therefore 7 digits to determine, and we have 8 digits in our divisor. Consequently we determine one digit in the ordinary way. There will then remain 6 digits to be determined, which is two less than the number of digits in the divisor. We may then begin to curtail the divisor.

The required quotient is ·865256.

$$
\begin{array}{r}
\cdot 8652559 \\
31,4,1,5,9,2,7\,)\,27182818\cdot 0 \\
\underline{20500764} \\
\underline{1651208} \\
\underline{80412} \\
\underline{17580} \\
\underline{1872} \\
\underline{301} \\
18
\end{array}
$$

EXAMPLES. 93.

Obtain approximately the results of the following divisions, so that the error may be less than ·005.

1. ·352073 ÷ ·321562.
2. ·463672 ÷ ·208431.
3. ·245638 ÷ ·672188.
4. ·688256 ÷ ·623054.
5. ·2995824 ÷ ·476126.
6. 6·1365 ÷ 2·12347.
7. 5·894 ÷ 14·2873.
8. 88·25 ÷ 3·54783.
9. 364·3 ÷ 18·2912.
10. 2·457 ÷ 1·28474.

EXAMPLES. 94.

Multiply, correct to 5 decimal places,

1. 32·1598 by ·562089.
2. 3·72 by ·0005962.
3. ·005762 by 3·20876.
4. ·0006284 by 840·32105.
5. 28·5631 by ·37642567.
6. 3·208943621 by 8·4230156201.
7. 4·320972418 by 5·630142197.
8. ·0087624 by itself.

Divide, correct to 4 decimal places,

9. 3476·321 by 1248·4176.
10. 56·9812 by 47·87631.
11. 37·20518 by ·1462304.
12. 185·37612 by ·08764032.
13. ·13751432 by 1·8763015.
14. 17·1890865 by ·13249672.
15. 23·2843612 by ·03205617.
16. Find, to the nearest integer, the product of 7·3129 and 115·73.
17. Find, to the nearest gram, the weight of 15 m. 35 cm. of wire, if 1 m. weighs 6·735 g.
18. Find, to the nearest franc, the cost of 57·315 Kg. at 5·78 f. per Kg.

19. If the time taken by light to travel from the sun to the earth be 499 seconds, and the distance of the earth from the sun be 93,000,000 miles, find the velocity of light in miles per hour, to the nearest thousand.
20. A man buys 576 m. 73 cm. of cloth for 2345 f. 35 c.; find, to the nearest centime, the cost per metre.
21. The value of a metre is 39·37079 in. Find, correct to ·0005 of an inch, the error per metre in the following approximations:—1 cm. $= \frac{2}{5}$ in., 1 m. $= 39\frac{1}{3}$ in, and 1 Km. $= \frac{5}{8}$ mile.
22. If 34 oz. 13 dwt. 10 grs. of gold be coined into 135 sovereigns, find in grains, to 2 decimal places, the weight of a sovereign.
23. If 9 Km. 781 m. of wire weighs 38 Kg. 7 Hg. 9 Dg., find, to to the nearest milligram, the weight of 1 metre of wire.
24. A brass rod, 12·35 m. long at 10° C., increases in length by ·0000189 of this length for each 1° C. by which its temperature increases. Find, to the nearest millimetre, how much its length has increased when the temperature is 20° C.
25. Taking the area of the United Kingdom (121,481 sq. miles) as unit, express in terms of this unit, to 3 decimal places, the areas of the following countries: France, 204,092 sq. miles; Portugal, 34,038 sq. miles; India, 1,987,427 sq. miles; Cape Colony, 288,171 sq. miles; Canada, 3,165,390 sq. miles.
26. If 1 ft. = ·3047945 m., express the following heights in metres (to nearest metre): Mont Blanc, 15,781 ft.; Etna, 10,840 ft.; Chimborazo, 20,608 ft.; Everest, 29,002 ft., Ararat, 16,964 ft.
27. Multiply 567·84560259 by 27·00384562, correct to 6 places.
28. Find the value of $(3{\cdot}8456)^2 - (3{\cdot}7206)^2$.

XXI. DECIMALIZATION OF MONEY.

233. The process by which we express a sum of money in the form of the decimal of a pound depends, in the first place, upon the fact that 2*s.* $= \frac{1}{10}$ of £1. Hence

$$2\text{s.} = £{\cdot}1, \quad 1\text{s.} = £{\cdot}05, \quad \text{and} \quad 6\text{d.} = £{\cdot}025.$$

These decimals are *exact*, and by means of them we can at once express, as an *exact* decimal of £1, any sum of money which consists only of shillings and a sixpence. For example 4*s.* = £·2, 7*s.* = £·35, 8*s.* 6*d.* = £·425, etc. We can obtain these results from the above relations by simple multiplication and addition.

EXAMPLES. 95. *Oral.*

Express each of the following sums of money as the decimal of a pound, and each decimal as a sum of money:

1. 2*s.*, 4*s.*, 6*s.*, 8*s.*, 10*s.*, 12*s.*, 14*s.*, 16*s.*, 18*s.*, £·4, £·6, £·9, £·7.
2. 3*s.*, 5*s.*, 7*s.*, 9*s.*, 11*s.*, 13*s.*, 15*s.*, 17*s.*, 19*s.*, £·25, £·95, £·55.
3. 2*s.* 6*d.*, 4*s.* 6*d.*, 6*s.* 6*d*, 8*s.* 6*d.*, 10*s.* 6*d.*, 12*s.* 6*d.*, 14*s.* 6*d.*, 16*s.* 6*d.*, 18*s.* 6*d*, £·225, £·625, £·925, £·725, £·525, £·425.
4. 1*s.* 6*d.*, 3*s.* 6*d.*, 5*s.* 6*d.*, 7*s.* 6*d.*, 9*s.* 6*d.*, 11*s.* 6*d.*, 13*s.* 6*d.*, 15*s.* 6*d.*, 17*s.* 6*d.*, 19*s.* 6*d.*, £·375, £·775, £·975, £·575, £·675, £·875.

234. From the relations in the preceding article we see that

$$6\text{d.} = £\cdot 025\,;$$
$$\therefore\ 1\text{d.} = £\cdot 004\tfrac{1}{6} \quad \text{or} \quad 4\tfrac{1}{6} \times £\cdot 001\,;$$
$$\therefore\ \tfrac{1}{4}\text{d.} = £\cdot 001\tfrac{1}{24} \quad \text{or} \quad 1\tfrac{1}{24} \times £\cdot 001.$$

If, in the expression for $\frac{1}{4}d.$, we may neglect the fraction $\frac{1}{24}$, we get $\frac{1}{4}d. = £\cdot 001$, and a number of farthings would then be expressed approximately as a decimal of £1 by multiplying £·001 by the number of farthings. This of course would not be quite accurate, and the third figure of the decimal might not be correct, but it may be sufficiently accurate for some purposes.

As the shillings can be expressed as the decimal of £1 by multiplying the number of shillings by ·05, *i.e.* by multiplying the number of shillings by 5 and placing a decimal point two places to the left, we may state the process briefly thus:—To decimalize a sum of money, **correct to 2 decimal places**, *we multiply the number of pence by* 4, *and the number of shillings by* 5, *adding to the latter product the carrying figure from the former; we write down the figures as we obtain them, and we place the decimal point so as to have three figures to the right of it.*

Example. *Express* 13*s.* $5\frac{3}{4}d.$ *as the decimal of* £1, *correct to* 2 *places.*

We multiply $5\frac{3}{4}$ by 4, and we get 23. We write down 3 in the 3rd decimal place, and we carry the 2.

We next multiply 13 by 5, and we get 65; adding the 2 carried, we get 67; these are the first and second figures of the decimal. We thus get

$$13s.\ 5\tfrac{3}{4}d. = £\cdot 673.$$

Similarly, $$5s.\ 3\tfrac{1}{4}d. = £\cdot 263.$$

The figure in the *third* decimal place cannot be relied upon. Otherwise, we might proceed as follows. In the first case, $13s. = £\cdot 65$, and $5\frac{3}{4}d. = 23\ f = £\cdot 023$; $\therefore\ 13s.\ 5\frac{3}{4}d. = £\cdot 673$, approximately.

235. Conversely, to convert the decimal of a pound into shillings and pence, we reverse the process explained above :—*We divide by* 5 *the number made up of the first and second figures of the decimal; the quotient gives the number of shillings. We then prefix the remainder, if any, to the third figure of the decimal, and we divide by* 4*; this quotient gives the number of pence.*

Example. *Express* £·263 *in shillings and pence.*

We divide 26 by 5 ; the quotient is **5**, and the remainder is **1**.

We now divide 13 by 4 ; the quotient is $3\frac{1}{4}$.

Thus £·263 = 5*s.* $3\frac{1}{4}d.$

Similarly, £·673 = 13*s.* $5\frac{3}{4}d.$

Otherwise, we might proceed as follows. In the first case, £·250 = 5*s.*, and £·013 = $3\frac{1}{4}d.$; and in the second case, £·650 = 13*s.*, and £·023 = $5\frac{3}{4}d.$

236. It is necessary to consider the possible error involved in this process. We have taken $\frac{1}{4}d.$ to be £·001, instead of $1\frac{1}{2\,4}$ of £·001. Thus the error in the digit in the third place = no. of farthings $\times \frac{1}{2\,4}$. This can never be greater than $48 \times \frac{1}{2\,4}$, or 2. But an error of 2 in the third decimal place means an error of £·002 in the money, *i.e.* an error of $£\frac{1}{5\,0\,0}$. This is less than $\frac{1}{2}d.$

237. The error which may occur, when we take £·001 instead of $1\frac{1}{2\,4}$ of £·001 to represent $\frac{1}{4}d.$, can easily be corrected, and we can thus obtain a decimal **correct to three places.**

In the case of 3*d.*, or 12*f.*, the error = $\frac{1\,2}{2\,4}$ of £·001 = $\frac{1}{2}$ of £·001, and in the case of 6*d.*, or 24*f.*, the error = $\frac{2\,4}{2\,4}$ of £·001 = £·001.

Thus for any sum less than 3*d.* the error is less than $\frac{1}{2}$ of £·001, and may therefore be neglected as regards the third decimal place ; but for 3*d.*, or any sum *greater than* 3*d.* and *less than* 6*d.*, the error must be allowed for by increasing by 1 the figure in the third decimal place.

Thus $2\frac{3}{4}d. = 11f. =$ £·011, but $3d. = 12f. =$ £·013.

We need not consider any sum greater than 6*d.* To decimalize $10\frac{1}{2}d.$, for example, we should put £·025 for the 6*d.*, and treat only the $4\frac{1}{2}d.$ by the method we are considering. The partial decimals should be added mentally ; thus

£4. 13*s.* $7\frac{3}{4}d.$ = £4. 13*s.* 6*d.* + 7*f.* = £(4·675 + ·007) = £4·682,

and £4. 17*s.* $4\frac{1}{2}d.$ = £4. 17*s.* 0*d.* + 18*f.* = £(4·85 + ·019) = £4·869.

NOTE.—In a decimal of a pound to 3 places, the error is less than $\frac{1}{4}d.$, and is therefore a smaller amount than we can take practically into account.

238. In converting to shillings and pence a decimal of a pound correct to 3 decimal places, it must be noticed that if the remainder, after the shillings and the sixpence (if any) are disposed of, is greater than £·012, we convert this remainder to farthings by **subtracting** 1 from the number represented by the last two figures. Thus £·014 = 13*f.* = 3¼*d.*, and £·020 = 19*f.* = 4¾*d.* Similarly,

£·367 = £·35 + £·017 = 7*s.* + (17 − 1)*f.* = 7*s.* 4*d.*,

and £·789 = £·775 + £·014 = 15*s.* 6*d.* + 13*f.* = 15*s.* 9¼*d.*

EXAMPLES. 96. *Oral.*

Express as decimals of £1, correct to 2 places:

1. 3*s* 2½*d.* 2. 5½*d.* 3. 1*s.* 7¼*d* 4. 3¾*d.*
5. 9*s.* 7½*d.* 6. 10*s.* 9¼*d.* 7. 12*s* 5½*d.* 8. 13*s.* 8¼*d.*
9. 15*s.* 4½*d.* 10. 17*s.* 5*d.*

Express as decimals of £1, correct to 3 places.

11. 2*s.* 6*d.* 12. 7*s.* 6*d.* 13. 8*s.* 4*d.* 14. 9*s.* 7¼*d.*
15. 12*s.* 8*d.* 16. 15*s.* 9¾*d.* 17. 14*s.* 5½*d.* 18. 19*s.* 6*d.*
19. 13*s.* 2¾*d.* 20. 18*s.* 2¼*d.* 21. 9*s.* 3½*d.* 22. 17*s.* 4¾*d.*

Express in pounds, shillings and pence, to the nearest farthing:

23. £·156321. 24. £·359462. 25. £·258295.
26. £·654846. 27. £·752983. 28. £·556537.
29. £3·368244. 30. £2·417376. 31. £4·672468.
32. £8·734729. 33. £6·543564. 34. £2·827541.
35. £2·983472. 36. £3·662538. 37. £5·509569.

239. When we wish to obtain a result **correct to 5 decimal places**, or to obtain an exact result, we treat the shillings and the sixpence (if there is one) as before, since the decimals which they produce are already *exact*. For the pence we use the relation

$$1d. = 4\tfrac{1}{6} \times £\cdot001 = £\cdot004 + \tfrac{1}{6} \text{ of } £\cdot001\,;$$

i.e. in the 4th and 5th decimal places we put the digits obtained when we divide the number of pence (less than 6) by 6. For this purpose it will be convenient to have the farthings expressed as the decimal of 1*d.*, *i.e.* to put 2·25*d.* for 2¼*d.*, 3·75*d.* for 3¾*d.*, etc.

Example i. *Express* 12*s.* 4¼*d.* *as the decimal of a pound, to* 5 *places.*

$$\begin{aligned} 12s.\ 4\tfrac{1}{4}d. &= £(\cdot600 + \cdot017 + \tfrac{1}{6} \times 4\cdot25 \times \cdot001) \\ &= £(\cdot617 + \cdot00070\tfrac{5}{6}) \\ &= £\cdot61770\tfrac{5}{6} \text{ exactly} \\ &= £\cdot61771 \text{ to 5 places.} \end{aligned}$$

Example ii. *Express* 7*s.* $2\frac{1}{2}d.$ *as the decimal of* £1.

$$7s.\ 2\tfrac{1}{2}d. = £(\cdot360 + \tfrac{1}{6} \times 2\cdot5 \times \cdot001)$$
$$= £(\cdot360 + \cdot00041\tfrac{2}{3})$$
$$= £\cdot36041\tfrac{2}{3}.$$

Example iii. *Express* 14*s.* $5\frac{1}{4}d.$ *as an exact decimal of* £1.

$$14s.\ 5\tfrac{1}{4}d. = £(\cdot700 + \cdot021 + \tfrac{1}{4} \times 5\cdot25 \times \cdot001)$$
$$= £(\cdot721 + \cdot000875)$$
$$= £\cdot721875.$$

NOTE.—If the number of farthings in the pence and farthings is divisible by 3, an exact decimal can always be obtained.

In the above examples we have stated the work in full, but practically all the operations should be performed mentally and only the result be written down.

240. In converting a decimal of a pound into shillings and pence, it is generally sufficient to obtain a result correct to the nearest farthing. Three decimal places are sufficient for this purpose, and we use the method of Art. 238.

241. The pupil must be careful not to employ the approximate methods of Arts. 234 and 237, if the sum of money is to be multiplied, even though the multiplier be small. For instance, £2. 13*s.* $1\frac{3}{4}d. \times 25 =$ £66. 8*s.* $7\frac{3}{4}d.$; but by the method of Art. 237

$$£2.\ 13s.\ 1\tfrac{3}{4}d. \times 25 = £2\cdot657 \times 25 = £66\cdot425 = £66.\ 8s.\ 6d.$$

If the product were required to the nearest penny the result obtained by this method would therefore be 2*d.* wrong, and moreover the error would be greater if the multiplier were greater.

We have also seen that in order to find the sum, correct to 3 places, of several decimals, we must have the decimals themselves carried to 5 places; if then we add together a large number of sums of money decimalized to only 3 places, we cannot expect to obtain a result correct to the nearest farthing. If, however, the number of sums of money be not very large, the sum may be accurate to the nearest penny.

On the other hand, if a decimal correct to 3 places be *divided* by 10, the quotient is correct to 4 places; and if it be divided by 100, the quotient is correct to 5 places, and so on. In this case, therefore, the method of decimalization at sight is of great value, and saves much labour.

EXAMPLES. 97.

Reduce to pounds and the decimal of a pound, correct to 5 places:

1	£7. 4*s*. 1*d*.	2.	£5. 6*s*. 1¼*d*.	3.	£2. 8*s*. 1¾*d*.
4.	£8. 14*s*. 2¼*d*.	5.	£9. 16*s*. 2½*d*.	6.	£1. 18*s*. 2¾*d*.
7.	£9. 6*s*. 3¼*d*.	8.	£4. 10*s*. 4½*d*.	9.	£3. 12*s*. 5¾*d*.
10.	£2. 3*s*. 7½*d*.	11.	£6. 5*s*. 8¼*d*.	12.	£5. 7*s*. 10½*d*.
13.	£4. 17*s*. 9¾*d*.	14.	£7. 15*s*. 11½*d*.	15.	£3. 13*s*. 7¾*d*.

Reduce to pounds and the decimal of a pound exactly:

16.	3*d*.	17.	5¼*d*.	18.	1*s*. 3¾*d*.	19.	7*s*. 4½*d*.
20.	16*s*. 2¼*d*.	21.	£3. 8*s*. 9*d*.	22.	£5. 14*s*. 1½*d*.
23.	£2. 12*s*. 3¾*d*.	24.	£5. 0*s*. 0¾*d*.	25.	£10. 0*s*. 8¼*d*.
26.	£14. 18*s*. 7½*d*.	27.	£1. 1*s*. 1½*d*.	28.	£2. 19*s*. 10½*d*.
29.	£7. 13*s*. 11¼*d*.	30.	£8. 7*s*. 9¾*d*.	31.	£10. 10*s*. 10½*d*.

32. Divide £57. 12*s*. 6*d*. by 137, to the nearest farthing.

33. Divide £1378. 5*s*. 7*d*. by 213, to the nearest farthing.

34. Divide £178. 15*s* 8½*d*. by 57, to the nearest farthing.

Find, to the nearest penny, the value of:

35. $\frac{7\times 3}{100}\times$ £275. 15*s*. 7½*d*.

36. $\frac{3\times 3\frac{1}{2}}{100}\times$ £576. 18*s*. 9*d*.

37. $\frac{4\times 2\frac{3}{4}}{100}\times$ £1761. 15*s*. 6*d*.

38. $\frac{13}{21}$ of £14. 5*s*. 4*d*.

39. $\frac{16}{19}\times\frac{1}{80}\times$ £1000. 10*s*. 5*d*.

40. £575. 15*s*. 3¾*d*. × 995.

XXII. (a). RECURRING DECIMALS.

242. We will now consider cases of division in which the quotient does not terminate. For example,

$\frac{8}{3}$ = 2·6666...,

$\frac{1}{7}$ = ·142857142857142857...,

$\frac{21}{22}$ = ·9545454...,

in none of which cases will the decimal ever end.

Whenever this happens it will however be found that a digit, or a set of digits, is repeated continually. In $\frac{8}{3}$ the digit 6 is repeated, in $\frac{1}{7}$ the series 142857, and in $\frac{21}{22}$ the digits 54.

243. A decimal in which a digit, or a set of digits, is repeated continually is called a **Recurring**, or a **Periodic**, or a **Circulating** decimal.

244. Recurring decimals are written in a shortened form, the digits which are repeated being marked by dots placed over the first and last of them, thus:

$\frac{8}{3}$ is written $2{\cdot}\dot{6}$, $\frac{1}{7}$ is written $\cdot\dot{1}42857\dot{}$, and $\frac{21}{22}$ is written $\cdot\dot{9}\dot{5}\dot{4}$.

Such a decimal as $\cdot\dot{1}42857\dot{}$, in which all the digits recur, is called a **pure circulator**; and such a decimal as $\cdot\dot{9}\dot{5}\dot{4}$, in which some do not recur, is called a **mixed circulator.**

The digit, or set of digits, which is repeated is called the **Period** of the decimal. In the decimal equivalent to $\frac{8}{3}$ the period is 6; in $\frac{1}{7}$ it is 142857; in $\frac{21}{22}$ it is 54.

245. We have seen that any vulgar fraction, whose denominator is the product of any number of 2's and 5's, can be expressed as a terminating decimal, and that, if the denominator contains any other prime factor than 2 and 5, the equivalent decimal will *not* terminate.

It will easily be seen that if the decimal *does not* terminate, it *must* recur. For, to find the decimal equivalent to a vulgar fraction, we divide the numerator by the denominator; the operation, and therefore the decimal, will terminate if we ever arrive at a remainder 0. If the denominator be represented by n, and if the operation does not terminate, the only possible remainders are 1, 2, 3, 4, ... and $n-1$, one of which is the original dividend. One of these remainders must come a *second* time when we have obtained $n-1$ figures of the decimal, if not before; and when a remainder appears a second time the corresponding figure in the quotient will appear a second time, and from this point the figures in the quotient will repeat themselves in the same order.

EXAMPLES. 98.

Express each of the following fractions as a recurring decimal:

1.	$\frac{4}{15}$.	2.	$\frac{1}{15}$.	3.	$\frac{13}{15}$.	4.	$10\frac{7}{18}$.	5.	$5\frac{17}{30}$.
6	$\frac{7}{12}$.	7.	$\frac{97}{900}$.	8.	$3\frac{5}{37}$.	9.	$\frac{14}{33}$.	10.	$\frac{5}{99}$.
11.	$\frac{7}{75}$.	12.	$\frac{13}{22}$.	13.	$\frac{10}{99}$.	14	$\frac{12351}{11111}$.	15.	$\frac{1}{41}$.
16.	$\frac{13}{41}$.	17.	$\frac{2}{13}$.	18.	$\frac{11}{13}$.	19	$\frac{16}{13}$.	20.	$3\frac{4}{21}$.
21.	$\frac{1}{13}$.	22.	$3\frac{6}{35}$.	23.	$\frac{3}{148}$.	24.	$\frac{5}{56}$.	25.	$\frac{13}{77}$.

XXII. (b). TO FIND A VULGAR FRACTION EQUIVALENT TO A RECURRING DECIMAL.

246. The general rule will be considered again later, but if we consider a few examples we shall be able to see what the rule is.

Example i.

$$\dot{5} = \cdot 555555\ldots;$$
$$\therefore\ 10 \times \dot{5} = 5 \cdot 555555\ldots;$$
$$\therefore \text{ by subtraction, } 9 \times \dot{5} = 5;$$
$$\therefore\ \dot{5} = \tfrac{5}{9}.$$

Example ii.

$$\dot{7}\dot{9} = \cdot 79797979\ldots;$$
$$\therefore\ 100 \times \dot{7}\dot{9} = 79 \cdot 79797979\ldots;$$
$$\therefore\ 99 \times \dot{7}\dot{9} = 79;$$
$$\therefore\ \dot{7}\dot{9} = \tfrac{79}{99}.$$

Example iii.

$$\cdot 0\dot{2}\dot{1} = \cdot 021021021\ldots;$$
$$\therefore\ 1000 \times \cdot 0\dot{2}\dot{1} = 21 \cdot 021021\ldots;$$
$$\therefore\ 999 \times \cdot 0\dot{2}\dot{1} = 21;$$
$$\therefore\ \cdot 0\dot{2}\dot{1} = \tfrac{21}{999} = \tfrac{7}{333}.$$

Hence we see that *a pure recurring decimal is equivalent to a vulgar fraction which has the period of the decimal for its numerator, and for its denominator the number which has for its digits as many nines as there are digits in the period.*

NOTE.—A method similar to that of Art. 221 may be used to express a vulgar fraction as a recurring decimal. For example,

$$\tfrac{5}{37} = \tfrac{15}{111} = \tfrac{135}{999} = \cdot 1\dot{3}\dot{5}.$$

This method cannot often be used, because it is difficult to find the proper factors by which to multiply.

247. A *mixed* recurring decimal becomes the sum of a whole number and a *pure* circulator, when it is multiplied by such a power of 10 as will bring the decimal point to the left of the first recurring figure. We can then find the equivalent vulgar fraction by the process of the preceding article.

Example i.

Let $x = \cdot 1\dot{2}$;
then $10x = 1 \cdot \dot{2} = 1\tfrac{2}{9} = \tfrac{11}{9}$;
$\therefore\ x = \tfrac{11}{90}$;
$\therefore\ \cdot 1\dot{2} = \tfrac{11}{90}$.

Example ii.

Let $x = \cdot 1\dot{2}\dot{3}$;
then $10x = 1 \cdot \dot{2}\dot{3} = 1\tfrac{23}{99} = \tfrac{122}{99}$;
$\therefore\ x = \tfrac{122}{990}$;
$\therefore\ \cdot 1\dot{2}\dot{3} = \tfrac{122}{990}$.

Example iii.

Let $x = \cdot 421\dot{3}\dot{7}$;
then $1000x = 421 \cdot \dot{3}\dot{7} = 421\tfrac{37}{99}$
$= \tfrac{42137}{99} - 421$ *
$= \tfrac{41716}{99}$;
$\therefore\ x = \tfrac{41716}{99000}$.

* We multiply by 9, 99, 999, etc., by the method explained in Art. 37.

248. It follows from the rule that $\dot{9} = \frac{9}{9} = 1$, a result which should not be passed over without notice. We know that $\cdot\dot{9}$ denotes $\cdot 99999\ldots$. Now

$$\cdot 9 = \tfrac{9}{10}, \quad \text{and differs from unity by } \tfrac{1}{10},$$
$$\cdot 99 = \tfrac{99}{100}, \quad \ldots\ldots \tfrac{1}{100},$$
$$\cdot 999 = \tfrac{999}{1000}, \quad \ldots\ldots \tfrac{1}{1000},$$
$$\cdot 9999 = \tfrac{9999}{10000}, \quad \ldots\ldots \tfrac{1}{10000};$$

hence, for each additional nine the difference between the decimal and unity becomes one-tenth of what it was before. As we increase the number of nines this difference rapidly diminishes. And if we take an indefinitely large number of nines, the difference becomes so small as to be inappreciable; that is, the decimal becomes equivalent to unity itself.

Similarly, $\cdot 0\dot{9} = \cdot 1$, $\cdot 06\dot{9} = \cdot 07$, $\cdot 013\dot{9} = \cdot 014$; and whenever $\dot{9}$ occurs at the end of a decimal, it should be omitted, and the preceding digit increased by unity.

249. The student should verify, and be able to identify at once, any of the following equivalent forms:

$\frac{1}{4} = \cdot 25$,	$\frac{1}{3} = \cdot\dot{3}$,	$\frac{1}{9} = \cdot\dot{1}$,	$\frac{1}{11} = \cdot 0\dot{9}$,
$\frac{1}{2} = \cdot 5$,	$\frac{2}{3} = \cdot\dot{6}$,	$\frac{2}{9} = \cdot\dot{2}$,	$\frac{2}{11} = \cdot 1\dot{8}$,
$\frac{3}{4} = \cdot 75$.	$\frac{1}{6} = \cdot 1\dot{6}$,	$\frac{3}{9} = \cdot\dot{3}$,	$\frac{3}{11} = \cdot 2\dot{7}$,
	$\frac{5}{6} = \cdot 8\dot{3}$.	$\frac{4}{9} = \cdot\dot{4}$,	$\frac{4}{11} = \cdot 3\dot{6}$,
		$\frac{5}{9} = \cdot\dot{5}$,	$\frac{5}{11} = \cdot 4\dot{5}$,
$\frac{1}{8} = \cdot 125$,	$\frac{1}{7} = \cdot 142857\dot{}$,	$\frac{6}{9} = \cdot\dot{6}$,	$\frac{6}{11} = \cdot 5\dot{4}$,
$\frac{3}{8} = \cdot 375$,	$\frac{2}{7} = \cdot 285714\dot{}$,	$\frac{7}{9} = \cdot\dot{7}$,	$\frac{7}{11} = \cdot 6\dot{3}$,
$\frac{5}{8} = \cdot 625$,	$\frac{3}{7} = \cdot 428571\dot{}$,	$\frac{8}{9} = \cdot\dot{8}$,	$\frac{8}{11} = \cdot 7\dot{2}$,
$\frac{7}{8} = \cdot 875$.	$\frac{4}{7} = \cdot 571428\dot{}$,	$1 = \cdot\dot{9}$.	$\frac{9}{11} = \cdot 8\dot{1}$,
	$\frac{5}{7} = \cdot 714285\dot{}$,		$\frac{10}{11} = \cdot 9\dot{0}$.
	$\frac{6}{7} = \cdot 857142\dot{}$.		

The decimals equivalent to fractions with denominator 7 should be carefully noticed. They are all pure circulators, and the same digits 142857 occur in all. If we arrange these digits in a circle, it will be seen that by beginning in turn with 1, 2, 4, 5, 7, 8, and in each case reading off the remaining digits in the order in which they stand round the circle, we obtain the decimals equivalent respectively to $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$.

It should be observed too, (i) that we need know only the first three digits of $\cdot\dot{1}42857\dot{1}$; the last three can be obtained by subtracting the first three, in turn, from 9; for 8, 5 and 7 are respectively $9-1$, $9-4$, and $9-2$; (ii) that the sum of the first three digits is 7, since $1+4+2=7$; (iii) that if the digits 142857 be arranged in order of magnitude—1, 2, 4, 5, 7, 8—it will be clear that these digits must be respectively the first digits of the decimals equivalent to $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$ and $\frac{6}{7}$; the remaining digits of each decimal can then be written down in the manner explained above.

Similar properties hold in the case of $\cdot\dot{0}76923\dot{1}$, which is the decimal equivalent to $\frac{1}{13}$. For example, 9, 2 and 3 are respectively $9-0$, $9-7$ and $9-6$; and $0+7+6=13$. Such cases will be considered more fully later.

Example i. *What vulgar fraction is equivalent to* $\cdot025\dot{8}57142\dot{1}$?

Let $x=\cdot025\dot{8}57142\dot{1}$;

then $1000x=25\cdot\dot{8}57142\dot{1}=25\frac{6}{7}=\frac{181}{7}$;

$\therefore\ x=\frac{181}{7000}$.

Example ii. *Find the vulgar fraction equivalent to* $3\cdot21\dot{0}\dot{4}$.

Let $x=\cdot21\dot{0}\dot{4}$;

then $100x=21\cdot\dot{0}\dot{4}=21\frac{4}{99}=\frac{2083}{99}$;

$\therefore\ x=\frac{2083}{9900}$;

$\therefore\ 3\cdot21\dot{0}\dot{4}=3\frac{2083}{9900}$.

EXAMPLES. 99.

Determine the vulgar fractions equivalent to the following recurring decimals, and reduce them to their lowest terms:

1.	$\cdot\dot{3}$.	2.	$\cdot\dot{6}$.	3.	$\cdot3\dot{6}$.	4.	$\cdot\dot{6}\dot{3}$.	5.	$\cdot\dot{3}\dot{3}$.
6.	$\cdot4\dot{2}$.	7.	$\cdot\dot{6}4\dot{8}$.	8.	$\cdot\dot{1}0\dot{8}$.	9.	$\cdot\dot{6}2\dot{1}$.	10.	$\cdot\dot{2}1\dot{6}$.
11.	$\cdot\dot{7}67\dot{1}$.			12.	$\cdot\dot{1}88\dot{1}$.	13.	$5\cdot\dot{2}85714\dot{2}$.		
14.	$10\cdot571428\dot{5}$.			15.	$91\cdot\dot{1}1115\dot{5}$.	16.	$\cdot\dot{0}2439\dot{3}$.		
17.	$\cdot7\dot{3}$.			18.	$\cdot0\dot{5}\dot{6}$.	19.	$\cdot1\dot{5}\dot{6}$.		
20.	$\cdot00\dot{3}\dot{3}$.			21.	$15\cdot8\dot{6}$.	22.	$5\cdot011\dot{3}\dot{6}$.		
23.	$3\cdot0\dot{7}67\dot{1}$.			24.	$2\cdot37\dot{4}8\dot{1}$.	25.	$9\cdot98\dot{7}5\dot{4}$.		
26.	$2\cdot3905\dot{4}8\dot{4}$.			27.	$15\cdot194\dot{3}9\dot{8}$.	28.	$3\cdot1902\dot{3}2\dot{7}$.		
29.	$1\cdot2013\dot{4}3\dot{8}$.			30.	$6\cdot10\dot{0}4273\dot{5}$.	31.	$5\cdot16\dot{1}88\dot{1}$.		

XXII. (c). ADDITION, ETC., OF RECURRING DECIMALS.

250. In dealing with concrete quantities it is seldom necessary, as we have explained in Art. 224, to express the result of an operation as a recurring decimal. It is generally sufficient if we obtain the result correct to a few decimal places. In such cases therefore we merely write down each decimal to 2 places more than we require in the result. We will apply this principle in dealing with the addition, subtraction, multiplication and division of recurring decimals.

Example i. *Find, correct to* 3 *decimal places, the sum of* $3{\cdot}251$, $2{\cdot}5\dot{4}6\dot{2}$ *and* $\cdot3\dot{4}7\dot{2}$.

We carry each decimal to 5 places, and we find the sum to be 6·145, correct to 3 places.

3·251	
2·546	24
·347	27
6·145	

Example ii. *Find, correct to* 5 *decimal places, the difference between* $9{\cdot}2\dot{1}\dot{4}$ *and* $2{\cdot}4\dot{6}2\dot{1}\dot{6}$.

We carry each decimal to 7 places, and we find the result to be 6·75205, correct to 5 places.

9·21421	42
2·46216	62
6·75205	

Example iii. *Find, correct to* 3 *decimal places, the value of* $1{\cdot}8\dot{4}\dot{7} \times 6{\cdot}3185$

We retain 5 figures in the multiplicand; the additional figures 47 are required only to obtain the carrying number. The product, correct to 3 places, is 11·673.

Rough approximation.

The product must lie between 1×6 and 2×7, *i.e.* between 6 and 14.

1·84747	47
6	·3185
11·08485	
·55424	
·01847	
·01478	
·00092	
11·673	

EXAMPLES. 100.

Find, correct to 5 decimal places, the sum of

1. $\cdot\dot{3} + \cdot\dot{6} + \cdot1\dot{2}$.
2. $\cdot\dot{6}\dot{4}\dot{8} + \cdot2\dot{6}2\dot{1} + \cdot\dot{1}0\dot{8}$.
3. $\cdot00\dot{5} + \cdot\dot{1}\dot{8} + \cdot\dot{4}\dot{2}$.
4. $\cdot\dot{7}64\dot{1} + 1{\cdot}\dot{1}88\dot{1} + 6{\cdot}\dot{7}67\dot{1}$.
5. $2{\cdot}\dot{1}88\dot{1} + \cdot0\dot{8} + 3{\cdot}\dot{1}\dot{6}$.
6. $7{\cdot}50\dot{0}\dot{1} + \cdot800\dot{2}\dot{0} + \cdot7\dot{8}\dot{9} + 85{\cdot}1\dot{2}$.
7. $2{\cdot}1\dot{0}\dot{1} + \cdot243\dot{1}\dot{8}\dot{3} + \cdot12\dot{3}\dot{6} + 45{\cdot}2\dot{9}$.
8. $\cdot0\dot{3}\dot{6} + 5{\cdot}015 + \cdot0\dot{5}\dot{9}$.
9. $\cdot6\dot{4}\dot{8} + \cdot0\dot{2} + 1{\cdot}006$.
10. $193{\cdot}\dot{0}\dot{9}\dot{0} + \cdot4\dot{0}7\dot{1} + 35{\cdot}13 + 76{\cdot}\dot{5}$.
11. $\cdot6\dot{4}\dot{8} + \cdot0\dot{1}\dot{6} + 3{\cdot}5\dot{6}$.
12. $6{\cdot}5\dot{6}\dot{3} + 1{\cdot}\dot{5}\dot{6} + \cdot0\dot{1}\dot{8}$.

13. $2{\cdot}00358 + {\cdot}083889941 + 35{\cdot}1612 + 1{\cdot}006$.
14. $32{\cdot}01011 + 76{\cdot}09141 + 5{\cdot}1375 + 98{\cdot}863$.
15. $3{\cdot}7671 + {\cdot}0621 + {\cdot}016 + {\cdot}24$.

EXAMPLES. 101.

Find, correct to 5 decimal places, the difference of

1. $9{\cdot}141$ and $43{\cdot}15$.
2. $17{\cdot}25$ and $20{\cdot}31$.
3. $5{\cdot}5$ and $6{\cdot}41$.
4. $143{\cdot}029$ and $174{\cdot}1$.
5. $13{\cdot}251$ and $18{\cdot}876$.
6. $25{\cdot}2086$ and $32{\cdot}181$.
7. $5{\cdot}7164$ and $76{\cdot}23564$.
8. $15{\cdot}283648341$ and $39{\cdot}51$.
9. $2{\cdot}763210941$ and 3.
10. $23{\cdot}691$ and $36{\cdot}6571428$.
11. $1{\cdot}6341$ and $3{\cdot}1853$.
12. $1{\cdot}2012561$ and $3{\cdot}2085625$.
13. $\cdot3514218$ and $\cdot87654892$.
14. $\cdot35061$ and $10{\cdot}101$.
15. $1{\cdot}520561$ and $3{\cdot}23451$.
16. $3{\cdot}3$ and $3{\cdot}142857$.
17. $15{\cdot}0951$ and $7{\cdot}37$.
18. $21{\cdot}5$ and $19{\cdot}019$.
19. $4{\cdot}732101$ and $3{\cdot}1416$.
20. $\frac{2}{7}$ and $\frac{355}{113}$.

EXAMPLES. 102.

Find, correct to 3 decimal places, the following products:

1. $7{\cdot}6$ by 3, 5, 7.
2. $\cdot15$ by 2, 6, 8.
3. $1\,082$ by 365.
4. $7{\cdot}3674$ by 560.
5. $\cdot00704$ by 834.
6. $\cdot00837$ by 762.
7. $\cdot523809$ by $7{\cdot}62$.
8. $\cdot764321$ by $5{\cdot}625$.
9. $3{\cdot}097$ by $\cdot00061$.
10. $6{\cdot}34287$ by $5{\cdot}01723$.
11. $\cdot321472$ by $1{\cdot}82$.
12. $45{\cdot}62018$ by $28{\cdot}31$.
13. $7{\cdot}7652$ and $3{\cdot}567481$.
14. $184{\cdot}0923$ and $5{\cdot}756$.
15. $189{\cdot}3514$ and $846{\cdot}521$.
16. $49{\cdot}49$ and $612{\cdot}57525$.

251. To divide a recurring decimal by a whole number or by a terminating decimal, we proceed as in ordinary division; but instead of bringing down a zero each time, we bring down the digits of the period in rotation.

Example i. *Divide* $15{\cdot}76203$ *by* 5, *to* 8 *places.*

$$5\,)\,15{\cdot}762036203\ldots$$
$$3{\cdot}152407240\ldots$$

The quotient is $3{\cdot}15240724$.

Example ii. *Divide* $28{\cdot}342012$ *by* 141, *to* 5 *places.*

$$\text{Quot.} = {\cdot}201007\ldots = {\cdot}20101.$$
$$141\,)\,28{\cdot}342012\ldots$$
$$142$$
$$1012$$
$$252$$

EXAMPLES. 103.

Divide, and obtain the quotient to 3 decimal places:

1. $\cdot\dot{3}$ by 3, 5, 7.
2. $\cdot 3325\dot{}$ by 5, 25, 125.
3. $37\cdot\dot{0}8\dot{7}$ by 5, 9, 45.
4. $38\cdot 0\dot{6}9\dot{4}$ by ·003, ·025.
5. $\cdot 1010\dot{1}$ by 18, ·00036.
6. $\cdot 2\dot{0}0\dot{1}$ by 1001, 100·1, 1·001.
7. $20\cdot 13\dot{9}7\dot{2}$ by 42·1.
8. $645\cdot 7536\dot{7}\dot{3}\dot{6}$ by 14·155.
9. $318\cdot 2365206\dot{3}\dot{9}$ by 1·003446.

252. When the divisor, or when both multiplier and multiplicand, are recurring decimals, **we reduce both to equivalent vulgar fractions** if we require an *exact* result.

Example i. *Multiply* $\cdot\dot{3}$ *by* $1\cdot\dot{0}\dot{6}$.

$$\cdot\dot{3}\times 1\cdot\dot{0}\dot{6}=\tfrac{3}{9}\times 1\tfrac{6}{9\,9}$$
$$=\tfrac{1}{3}\times\tfrac{3\,5}{3\,3}$$
$$=\tfrac{3\,5}{9\,9}$$
$$=\cdot 3\dot{5}.$$

Example ii. *Divide* $\cdot\dot{3}$ *by* $1\cdot\dot{0}\dot{6}$.

$$\cdot\dot{3}\div 1\cdot\dot{0}\dot{6}=\tfrac{3}{9}\div 1\tfrac{6}{9\,9}$$
$$=\tfrac{1}{3}\times\tfrac{3\,3}{3\,5}$$
$$=\tfrac{1\,1}{3\,5}=\tfrac{2\cdot 2}{7}$$
$$=\cdot 3\dot{1}42857\dot{}.$$

EXAMPLES 104.

Find the value of

1. $\cdot\dot{6}\div\cdot\dot{7}$.
2. $\cdot 05\dot{4}\div\cdot 0\dot{2}$.
3. $\cdot 0\dot{1}\dot{8}\times\cdot 0\dot{3}$.
4. $\cdot 08\dot{9}\times\cdot 02\dot{8}$.
5. $\cdot\dot{5}\times\cdot\dot{4}$.
6. $\cdot 00\dot{0}\dot{9}\div\cdot 0\dot{9}0\dot{9}$.
7. $\cdot 0006\dot{}\times 1\cdot 8\dot{7}$.
8. $6\cdot 835\dot{1}\times\cdot\dot{7}$.
9. $\cdot 3\dot{9}36117\dot{}\div\cdot 1\dot{8}$.
10. $\cdot 013726\dot{}\div\cdot 0332\dot{7}$.

XXIII. (a). TO FIND WHAT FRACTION ONE QUANTITY (A) IS OF ANOTHER (B) OF THE SAME KIND.

253. If we express the quantities in terms of a common unit, *the required fraction has for its numerator the number of these units contained in* A, *and for its denominator the number contained in* B.

Example 1. *What fraction of* £5 *is* 4*s.* 6*d.*?

We take *sixpence* for the common unit; then since

4*s.* 6*d.* = 9 sixpences,

and £5 = 200 ,, ;

$\therefore$ 1 sixpence $=\frac{1}{200}$ of £5;

and 9 sixpences $=\frac{9}{200}$ of £5;

$\therefore$ 4*s.* 6*d.* $=\frac{9}{200}$ of £5;

$\therefore$ **fraction required** $=\frac{9}{200}$.

Example ii. *What fraction of* 8*s.* 4*d. is* 3*s.* 6*d.?*

$$\text{The fraction} = \frac{3s.\ 6d.}{8s.\ 4d.} = \frac{42}{100} = \frac{21}{50}.$$

Example iii. *Reduce* 2 *qrs.* 8 *lb. to a fraction of* 2 *cwt.*

$$\text{The fraction} = \frac{2 \text{ qrs. } 8 \text{ lb.}}{2 \text{ cwt.}} = \frac{64}{224} = \frac{2}{7}.$$

Example iv. *Reduce* 5 *m.* 1 *dm. to the fraction of* 17 *m.* 85 *cm.*

$$\text{The fraction} = \frac{5 \text{ m. } 1 \text{ dm.}}{17 \text{ m. } 85 \text{ cm.}} = \frac{510 \text{ cm.}}{1785 \text{ cm.}} = \frac{510}{1785} = \frac{2}{7}.$$

EXAMPLES. 105. *Oral.*

What fraction of

1. 9*d.* is 4*d.*?
2. 18*d.* is 6*d.*?
3. 20*d.* is 15*d.*?
4. 4*d.* is 9*d.*?
5. 6*d.* is 18*d.*?
6. 2*s.* is 10*d.*?
7. 1*s.* is 5*d.*, $5\frac{1}{2}d.$, $\frac{1}{4}d.$?
8. £1 is 13*s.* 6*d.*?
9. 5*s.*, 7*s.* 6*d.*, 10*s.*, 12*s.* 6*d.*, 15*s.*, 17*s.* 6*d.* is 2*s.* 6*d.*?
10. £1 is 2*s.* 6*d.*, 5*s.*, 7*s.* 6*d.*, 10*s.*, 12*s.* 6*d.*, 15*s*, 17*s.* 6*d.*?
11. 15*s.* is 2*s.* 6*d.*, 5*s.*, 17*s.* 6*d.*?
12. 17*s.* 6*d.* is 2*s.* 6*d.*, 5*s.*, 12*s.* 6*d.*?
13. 6*s.* 8*d.*, 10*s.*, 13*s.* 4*d.*, 16*s.* 8*d.* is 3*s.* 4*d.*?
14. £1 is 3*s.* 4*d.*, 6*s.* 8*d.*, 13*s.* 4*d.*, 16*s.* 8*d.*?
15. £1. 7*s.* 6*d.*, £2. 12*s.* 6*d.*, £1. 17*s.* 6*d.* is 2*s.* 6*d.*?
16. £1. 3*s.* 4*d.*, £1. 6*s.* 8*d.*, £2. 3*s.* 4*d.* is 3*s.* 4*d.*?
17. £3. 3*s.* 4*d.* is £2. 3*s.* 4*d.*?
18. £4. 17*s.* 6*d.* is £1. 5*s.*?
19. 12*s.* 6*d.* is 5*s.* 6*d.*, 7*s.*, 10*s.* 6*d.*?
20. 15*s.* is 1*s.* 3*d.*, 3*s.* 9*d*?

EXAMPLES. 106.

What fraction of

1. 2*s.* 6*d.* is $7\frac{3}{4}d.$?
2. 7*s.* 10*d.* is $8\frac{1}{4}d.$?
3. 10*s.* is $\frac{1}{4}d.$?
4. 3*s.* 5*d.* is 1*s.* $4\frac{1}{2}d.$?
5. 6*s.* 2*d.* is $1\frac{3}{8}d.$?
6. 6*s.* $5\frac{1}{2}d.$ is $\frac{3}{4}d.$?
7. 3*s.* $4\frac{1}{2}d.$ is 2*s.* $8\frac{1}{4}d.$?
8. 8*s.* $10\frac{3}{4}d.$ is 3*s.* $7\frac{1}{4}d.$?
9. 6*s.* $2\frac{1}{2}d.$ is 1*s.* $2\frac{1}{4}d.$?
10. 6*s.* $7\frac{3}{4}d.$ is 3*s.* $2\frac{1}{4}d.$?
11. 7*s.* $1\frac{3}{4}d.$ is $6\frac{3}{4}d.$?
12. 10*s.* $5\frac{1}{2}d.$ is 4*s.* $6\frac{3}{4}d.$?
13. 19*s.* $5\frac{3}{4}d.$ is 11*s.* $8\frac{1}{4}d.$?
14. £1. 5*s.* 1*d.* is 2*s.* $7\frac{1}{2}d.$?
15. £1. 5*s.* $7\frac{1}{2}d.$ is 15*s.* $4\frac{1}{2}d.$?
16. £1. 4*s.* $6\frac{1}{4}d.$ is 6*s.* $7\frac{3}{4}d.$?

What fraction of

17. £48. 19*s*. $10\frac{1}{2}d$. is £20. 14*s*. $6\frac{3}{4}d$.?

18. 3 lb. 14 oz. is 16 lb. 1 oz.? 19. 1 qr. 3 lb. is 26 lb ?

20. 1 cwt. 1 qr. 3 lb. is 1 qr. 5 lb.?

21. 2 cwt. 3 qrs. 11 lb. is 1 qr. 5 lb.?

22. 5 cwt. 3 qrs. 6 lb. is 3 qrs. 20 lb.?

23. 1 ton 1 qr. 10 lb. is 5 cwt. 3 qrs. 2 lb.?

24. 150 grs. is 1 oz. troy? 25. 150 grs. is 1 oz. avoir.?

26. 63 oz. 10 dwt. is 15 oz ? 27. 39 m. 93 cm. is 20 m. 35 cm.?

28. 27*s*. is $\frac{9}{11}$ of 1*s*. $10\frac{1}{2}d$.? 29. £3 is $\frac{2}{5}$ of $3\frac{1}{3}$ of £1. 11*s*. 6*d*.?

XXIII. (b). TO FIND WHAT DECIMAL ONE QUANTITY (A) IS OF ANOTHER (B) OF THE SAME KIND.

254. (a) When B is a compound quantity we proceed as in Examples (i) and (ii).

Example i. *Reduce* 5*s*. *to the decimal of* 13*s*. 4*d*.

The vulgar fraction $= \frac{5}{13\frac{1}{3}} = \frac{1\,5\,0}{4} = \frac{3}{8}$;

$\therefore$ the decimal = ·375.

Example ii. *Reduce* 2 *qrs*. 8 *lb*. *to the decimal of* 1 *cwt*. 3 *qrs*. 24 *lb*.

The vulgar fraction $= \frac{6\,4}{2\,2\,0} = \frac{1}{5}\frac{6}{5}$;

$\therefore$ the decimal $= \frac{3 \cdot 2}{1\,1} = \cdot 2\dot{9}\dot{0}$.

(b) When B is a simple quantity we proceed as in Examples (iii) and (iv).

Example iii. *Express* £1. 17*s*. $3\frac{3}{4}d$. *as a decimal of* £5.

12 | 3·75 pence
20 | 17·3125 shillings
5 | 1·865625 pounds
·373125 of £5

$\therefore$ the decimal = ·373125.

Example iv. *Reduce* 1 *qr*. 25 *lb*. 3·2 *oz*. *to the decimal of* 1 *ton*.

16 { 4 | 3·2 ounces
4 | ·8

28 { 4 | 25·2 pounds
7 | 6·3

4 | 1·9 quarters
20 | ·475 cwt.
·02375 of a ton

$\therefore$ the decimal = ·02375.

In Ex. iii., we first express $3\frac{3}{4}d$. as the decimal 3·75*d*. We then divide by 12 to reduce to shillings, and get ·3125*s*. We add the 17*s*. from the given amount and get 17·3125*s*.; this, reduced to pounds, becomes £·865625. We then add the £1 from the given amount and divide by 5.

In Ex. iv. the process is similar.

EXAMPLES. 107.

In the following examples reduce the first of the two given quantities to the decimal of the second, correct to 5 places.

1. 2*s.* 6*d.*; £5.
2. $\frac{3}{4}$ of 12*s.*; £2.
3. 7*s.* 6*d.*; £2.
4. £5. 13*s.* 4*d.*; £1.
5. £3. 4*s.* 7$\frac{1}{2}$*d.*; £1.
6. £1. 11*s.* 1$\frac{1}{2}$*d.*; £2.
7. 10*s.* 9$\frac{1}{4}$*d.*; £3.
8. £5. 8*s.* 8$\frac{1}{4}$*d.*; £4.
9. 1*s.* 3$\frac{1}{2}$*d.*; £25.
10. 16*s.* 9$\frac{1}{2}$*d.*; £1.
11. £36. 17*s.* 8$\frac{3}{4}$*d.*; £5.
12. 7*s.* 9$\frac{3}{4}$*d.*; £8.
13. 3*s.* 3$\frac{3}{4}$*d.*; 4*s.* 5*d.*
14. 6*s* 6$\frac{3}{4}$*d*; $\frac{1}{2}$ guinea.
15. 5*s.*; 13*s.* 4*d.*
16. £1. 19*s.* 4$\frac{1}{2}$*d.*; £3. 3*s.*
17. £1. 4*s.* 10$\frac{3}{4}$*d.*; £3. 19*s.* 8*d*
18. 7*s.* 2*d.*; £3. 15*s.* 3*d.*
19. 1*s.* 9*d.*; 3 guineas.
20. 9*s.* 4$\frac{1}{2}$*d.*; £1. 7*s.*
21. 13*s.* 4*d.*; £2. 6*s.* 8*d*
22. 16*s.* 1$\frac{1}{2}$*d.*; £5. 7*s.* 6*d.*
23. 3*s.* 3$\frac{3}{4}$*d.*; £2. 13*s.* 4*d*
24. £3. 10*s.* 7*d.*; 11 guineas.
25. 3 qrs. 22 lb. 6 4 oz.; 1 cwt.
26. $2\frac{3}{176}$ of 104 lb.; 1 ton.
27. 1 lb. 5 oz.; 1 cwt.
28. 2 cwt. 3 qrs. 3 lb. 8 oz.; 1 ton.
29. 4 cwt. 3 qrs. 8 lb. 12 oz.; 1 ton.
30. 6 lb.; 1 cwt.
31. 3 cwt. 3 qrs. 5 lb. 4 oz.; 1 ton.
32. 345 g.; 16 Kg.

XXIII. (c). TO FIND THE VALUE OF A GIVEN DECIMAL OF A QUANTITY.

255. We know (Art. 142) how to find the value of a given vulgar fraction of a quantity; we may therefore express the decimal as a vulgar fraction, and proceed as before. Otherwise we may operate directly with the decimal itself.

Example 1. *Find the value of* ·1375 *of half a sovereign.*

(a) ·1375 of 10*s.* = 1·375 of 1*s.* = 1$\frac{3}{8}$ of 1*s.* = 1*s.* 4$\frac{1}{2}$*d.*

(b) Otherwise we may proceed thus.

We first multiply the decimal by 10, and so get 1·375 shillings.

We then multiply ·375*s.* by 12 to reduce it to pence.

The result is, as before, 1*s.* 4$\frac{1}{2}$*d.*

·1375 half sovs.
10
1·375 shillings
12
4·500 pence.

Example ii. *Find the value of* 2·1372 *of* 2 *tons* 5 *cwt.* 1 *lb.*

We multiply 2·1372 by 2 and get 4·2744 tons. We then multiply ·2744 of a ton by 20 to reduce to cwt., and to the result we add 5 times 2·1372 cwt. We thus get 16·1740 cwt. We multiply ·1740 cwt. by 4 and get ·6960 qr. We multiply ·6960 of a qr. by 28 to reduce to lb., and to the result we add 2·1372 lb.

The required value is 4 tons 16 cwt. 21·6252 lb.

2·1372	
2	
4·2744	tons
20	
5·4880	
10·6860	
16·1740	cwt
4	
0·6960	qr.
28	
19·4880	
2·1372	
21·6252	lb

NOTE.—We might otherwise reduce 2 tons 5 cwt. 1 lb. to lb.; multiply by 2·1372, and reduce the result back again to tons, etc.

Example iii. *Find the value of* 1·234375 *of* £2. 6*s.* 8*d.*

£2. 6*s.* 8*d.* = £2⅓.

1·234375	
2⅓	
2·468750	
·411458	
Value =	£2·880
=	£2. 17*s.* 7½*d.*

EXAMPLES. 108. *Oral.*

What is the value of

1.	·5 of 1*s.*?	2.	·5 of £1?	3.	·5 of 10*s.*?
4.	·5 of 6*s.* 8*d.*?	5.	·5 of 17*s.* 9*d.*?	6.	·25 of 1*s.*?
7.	·25 of 2*s.* 6*d.*?	8.	·25 of £1?	9.	·25 of £3. 4*s.* 8*d.*?
10.	·75 of 1*s.*?	11.	·75 of £1?	12.	·75 of 16*s.*?
13.	·75 of £1. 4*s.*?	14.	·125 of 8*d.*?	15.	·125 of 1*s.*?
16.	·125 of £1?	17.	·125 of £1. 4*s.*?	18.	·125 of 13*s.* 4*d.*?
19.	·375 of 6*s.* 8*d.*?		20. 3·625 of £1?		
21.	4·375 of 1 lb. avoir.?		22. ·625 of 16*s.* 8*d.*?		

EXAMPLES. 109.

Find the value of

1. 9·68125 of £5.	2. ·0625 of £24. 7*s.* 4*d.*
3. ·875 of £3. 5*s.* 6*d.*	4. 3·03 of 10*s.* 5*d.*
5. ·01625 of £204. 13*s.* 4*d.*	6. 1·146875 of £16. 6*s.* 8*d.*
7. 171·15625 of 6*s.* 8*d.*	8. 1·0625 of 14 guineas.

9. 2·8125 of a cwt. 10. ·0583 of a cwt.
11. ·15 of a ton. 12. ·0141 of a cwt.
13. 18·225 of a ton. 14. ·0625 of 1 ton 2 cwt. 3 qrs. 12 lb.
15. 3·3275 of 3 qrs. 6 lb. 16. ·856 of 2 cwt. 26 lb.
17. 2·2183098 of 1 cwt. 1 qr. 2 lb. 18. 5·62625 of 1 mile.
19. ·042625 of 3 mi. 3 fur. 4 ch. 20. 3·0125 of 2 yds. 2 ft. 8 in.

256. The following examples are somewhat more complicated:

Example i. *Find the value of*

·00325 *of* £1. 10*s.* + 1·075 *of* 5*s.* + 13·25 *of* 2*s.*

The value = ·00325 of 30*s.* + 1·075 of 5*s.* + 13·25 of 2*s.*
= ·0975*s.* + 5·375*s.* + 26·5*s.*
= 31·9725*s.*
= £1. 11*s.* 11·67*d.*

Example ii. *Find the value of*

7·375 *tons* + ·625 *of a cwt.* + 31¼ *qrs.* + ·75 lb.

(a). The expression

= $7\frac{3}{8}$ tons + $\frac{5}{8}$ cwt. + 31 qrs. + $7\frac{3}{4}$ lb.
= 7 tons + $\frac{6\,5}{8}$ cwt. + 31 qrs. + $7\frac{3}{4}$ lb.
= 7 tons + $15\frac{1}{8}$ cwt. + 3 qrs. + $7\frac{3}{4}$ lb.
= 7 tons 15 cwt. 3 qrs. $21\frac{3}{4}$ lb.

(b). Otherwise:

	cwt.	qrs.	lb.
7·375 tons = $7\frac{3}{8}$ tons =	147	2	0
·625 cwt. = $\frac{5}{8}$ cwt. =		2	14
31¼ qrs. =	7	3	7
·75 lb. = $\frac{3}{4}$ lb. =			$\frac{3}{4}$
∴ sum =	155	3	$21\frac{3}{4}$

(c). A very simple way of working examples of this kind is the following:

Multiply 7·375 tons by 20 to reduce to cwt., and add the ·625 cwt. Multiply the result by 4 to reduce to qrs., and add 31¼ qrs. Multiply this result by 28 to reduce to lb., and add ·75 lb. Finally, reduce the last result back again to tons, cwt., qrs., and lb.

Example iii. *Find the value of*

$\cdot 0\dot{1}4857\dot{1}4\dot{2}$ *of* £2. 14*s.* $8\frac{1}{4}d.$ + $1\cdot 1\dot{0}\dot{7}14285\dot{7}$ *of* 3 *guineas.*

The expression = $14\frac{6}{7}$ / 1000 of £2. 14*s.* $8\frac{1}{4}d.$ + $1\frac{10\frac{5}{7}}{100}$ of 63*s.*
= $\frac{1\,0\,4}{7\,0\,0\,0}$ of £2. 14*s.* $8\frac{1}{4}d.$ + $1\frac{7\,5}{7\,0\,0}$ of 63*s.*
= $\frac{1\frac{3}{8}}{7\,5}$ of £2. 14*s.* $8\frac{1}{4}d.$ + $1\frac{3}{2\,8}$ of 63*s.*
= $9\frac{3}{4}d.$ + £3. 9*s.* 9*d.*
= £3. 10*s.* $6\frac{3}{4}d.$

We find the value of $\frac{1\frac{3}{8}}{7\,5}$ of £2. 14*s.* $8\frac{1}{4}d.$ by multiplying £2. 14*s.* $8\frac{1}{4}d.$ by 13, and dividing the product by 875; and we treat the other term in a similar way.

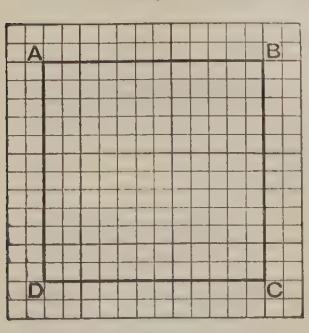
EXAMPLES 110.

Find the value of

1. ·5 of 1*s.* + ·7 of 5*s* + ·125 of £1
2. ·0625 of 1*s* + ·0375 of £1. 3 $3\frac{3}{4}$ of 7*s.* 6*d.* − ·875 of 21*s.*
4. ·55 of 7*s.* 6*d.* + 5·32 of 17*s.* 3*d.*
5. ·125 of a guinea + $\frac{5}{8}$ of £1 + $\frac{7}{8}$ of 1*s.*
6. $\frac{3}{8}$ of 15*s* + $\frac{3}{8}$ of 4*s.* 6*d.* − $\frac{3}{4}$ of 4*s* 11*d.*
7. $\frac{1}{3}$ of £1 + $\frac{2}{7}$ of a guinea − $\frac{4}{9}$ of 15*s.*
8. $\frac{2}{5}$ of £2 + $\frac{4}{9}$ of £39. 0*s.* $6\frac{3}{4}$*d.* + $2\frac{1}{2}$ of 10*s* 6*d*
9. $\frac{3}{8}$ of 2*s.* 6*d* + 8 of a guinea + $\frac{5}{3}$ of £1 − $\frac{1}{10}$ of 1*d*
10. $\frac{5}{6}$ of £2 9*s* 4*d* − ·6 of £4 + $2\frac{1}{2}$ of $\frac{3}{2}$ of 21*s.*
11. $\frac{4}{9}$ of a guinea + ·375 of $\frac{1}{2}$ of £1 − $\frac{1}{1\frac{1}{2}}$ of $\frac{3}{4}$ of 5*s.* − $\frac{1}{8}$ of $\frac{1\frac{3}{4}}{1\frac{1}{8}}$ of 1*s*
12. ·375 of £1 10*s.* + $\frac{2}{3}$ of $1\frac{1}{2}$ of $\frac{4}{5}$ of £1 5*s* − ·5625 of £1.
13. $\frac{15\frac{3}{5}}{7\frac{4}{5}}$ of £1 + ·2 of £140. 10*s.* 6*d.* + ·4 of a guinea.
14. $1\frac{3}{2\frac{3}{8}}$ of £1. 10*s.* 11*d.* + $\frac{7\cdot546}{11\frac{11}{13}}$ of 700*d.*
15. ·6 of £1 6*s.* $6\frac{3}{4}$*d.* + $\frac{11}{3\frac{3}{8}}$ of 21*s.* + ·078125 of £1.
16. 8·71875 of 8*d* + 1·146875 of 6*s.* 8*d.* − ·0625 of 21*s.*
17. 2·5 of 1 ton 6 cwt. + 3·125 of 2 qrs. 16 lb. + 3·75 of 448 oz.

XXIV. MEASURES OF AREA—BRITISH AND METRIC.

257. The principal measure of **area** is a square which has each of its sides equal to the principal measure of length, one yard. It is therefore called a **square yard**.



In the margin is drawn on squared paper a square ABCD, such that each side contains 12 units of length. The pupil will see that the whole square is made up of 12 rows, each row containing 12 of the small squares, and each side of a small square being equal to a unit of length. Each of the small squares is a unit of area, and it is clear that in ABCD there are 12 × 12, or 144, small squares, and therefore 144 units of area.

If the pupil will draw squares whose sides contain 3 and 22 units of length respectively, he can satisfy himself in the same way

that these squares contain 3×3 and 22×22, *i.e.* 9 and 484, units of area respectively; and, whatever lengths he may take, he will find that the number of units of area in a square is always equal to the square of the number of units of length in each side of the square

258. It is easy with the aid of the above principle to deduce a table of Square measures from a table of Linear measures.

Thus 1 *square foot* = 144 *square inches*;

and 1 *square yard* = 9 *square feet.*

These and certain other measures of area are arranged in the following table:

SQUARE MEASURE.

144 **sq. inches** make 1 **sq. foot.**
9 **sq. feet** ..... 1 **sq. yard.**
484 **sq. yards** ..... 1 **sq. chain.**
10 **sq. chains** } ..... 1 **acre.**
or 4840 **sq yards** }
640 **acres** ..... 1 **sq. mile.**

We have also

10000 *sq links* = 1 *sq. chain.*
$30\frac{1}{4}$ *sq. yards* = 1 *sq. pole*, or *perch.*
40 *perches* = 1 *rood.*
4 *roods* = 1 *acre.*

Reduction is effected in the same manner as in former cases.

Example i. *Reduce* 5 *ac.* 3 *sq. ch.* 298 *sq. yds.* 7 *sq. in. to sq. inches.*

ac. sq. ch. sq. yds. sq. in.
5 . 3 . 298 . 7
10
53 sq. chains.
484
212
424
510 (including the 298).
25950 sq. yds.
9
233550 sq. ft. × 144.
934200
934207 (including the 7).
33631207 sq. inches.

Example ii. *Reduce* 33631207 *sq. in. to acres.*

144 { 12 | 33631207 sq. inches.
12 | 2802600 ... 7
9 | 233550 sq. ft. ... 0 } 7 sq. inches.
484 | 25950 sq. yds. 7 sq. in.
10 | 53 sq. ch. 298 sq. yds. 7 sq. in.
5 ac. 3 sq. ch. 298 sq. yds 7 sq. in.

484) 25950 (53
1750
298

NOTE.—We have given this example in order to show the working of the whole table; but, in practical work, the whole of the table would never be required in one operation; large areas would be stated in acres, sq chains, and sq. yds.; smaller areas in sq. yards, sq. feet, and sq inches.

259. In the Metric system the table for measuring areas is obtained from the Metric Linear Measure in the same way as the British Square Measure is obtained from the British Linear Measure. By drawing on squared paper a square, each of whose sides contains 10 units of length, the pupil should satisfy himself that the square contains 10×10 small squares, *i.e.* 100 units of area.

METRIC MEASURES OF AREA.

100 sq. **millimetres** (sq. mm.) = 1 sq. **centimetre** (sq. cm.).
100 sq. **centimetres** = 1 sq. **decimetre** (sq. dm.).
100 sq. **decimetres** = 1 sq. **metre** (sq. m.) = 1 **centiare**.
100 sq. *metres* = 1 sq. *decametre* (sq. Dm.) = 1 **are**.
100 sq. *decametres* = 1 sq. *hectometre* (sq Hm.).
100 sq. *hectometres* = 1 sq. **Kilometre** (sq. Km.).

The names *square decametre* and *square hectometre* are seldom used. In measuring land, a square decametre is called an **are** The terms derived from this are the **centiare** and the **hectare.**

METRIC LAND MEASURE.

100 **centiares** (ca.) = 1 **are** (a.).
100 **ares** = 1 **hectare** (Ha.).
100 **hectares** = 1 sq. **Kilometre** (sq. Km.).

NOTE.—A **centiare** is a **square metre,** and a **hectare** is a **square hectometre.**

260. In the reduction of metric square measures we must remember always to have *two* figures to express the number of each denomination, and to supply the place of a missing denomination by *two* 0's. Thus we would write 5 sq. dm. 7 sq. cm. in the form 5 sq. dm. 07 sq. cm. for purposes of reduction, and we then see that it is equivalent to 507 sq. cm.

Example i. *Reduce* 5 *sq. m.* 3 *sq. dm.* 51 *sq. cm. to sq. cms.*

5 sq. m. 3 sq. dm. 51 sq. cm $= (5 \times 10^4 + 3 \times 10^2 + 51)$ sq. cm.
$= (50000 + 300 + 51)$ sq. cm.
$= 50351$ sq. cm.

Thus we see that reduction could be effected at once by expressing the measure in the form 5 sq. m. 03 sq. dm. 51 sq. cm., and then writing down the figures in the order in which they occur.

Example ii. *Reduce* 5 *sq. dm.* 21 *sq. mm. to sq. mm.*

5 sq. dm. 21 sq. mm. = 5 sq. dm. 00 sq. cm. 21 sq. mm. = 50021 sq. mm.

Example iii. *Reduce* 17684900 *ares to sq. Km.*

The divisors in this case are each 100, hence

17684900 ares $= (1768 \times 10^4 + 49 \times 10^2)$ ares = 1768 sq. Km. 49 hectares.

EXAMPLES. 111. *Oral.*

1. How many square inches in $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$, $\frac{1}{4}$ of a sq. foot?
2. How many sq. feet in $\frac{1}{3}$ of a sq. yard?
3. How many sq. feet in 2 sq. yds., 4 sq. yds., $5\frac{1}{2}$ sq. yds.?
4. How many sq. yards in 45, 81, 126 sq. ft.?
5. How many perches in 1 ac., $\frac{1}{4}$ ac., $\frac{1}{2}$ ac., $\frac{3}{4}$ ac.?
6. How many sq. yards in 1 acre, and in $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ of an acre?
7. How many perches in 2 roods?
8. How many acres in $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$ of a square mile?
9. How many sq. yards in 2 square chains?
10. How many sq. chains in 3, 5, 8, 10 acres?

EXAMPLES. 112.

Reduce

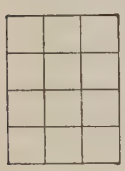
1. 3 sq. yds. 6 sq. ft. 75 sq. in. to sq. inches.
2. 8 sq. ch. 275 sq. yds. 7 sq. ft. to sq. ft.
3. 9 ac. 5 sq. ch. 125 sq. yds. to sq. yds.
4. 5 sq. mi. 56 ac. 9 sq. ch. to sq. chains.

Reduce

5. 123 ac. 7 sq. ch. 56 sq. yds. to sq. yds.
6. 5 ac. 4 sq. ch. 355 sq. yds. 8 sq. ft. to sq. ft.
7. 235 sq. yds. 5 sq. ft. 105 sq. in. to sq. inches.
8. 7321 sq. in. to sq. yds.
9. 10486 sq. in. to sq. yds.
10. 3156704 sq in. to chains.
11. 8456789 sq. yds. to acres.
12. 7845687 sq. links to acres.
13. 34568 sq. chains to sq. miles
14. 12345678 sq. links to acres.
15. 5 sq. m 12 sq. cm. to sq cm.
16. 5 sq. Km. 7 hectares 5 ares to ares.
17. 3 sq. m. 5 sq. dm. 8 sq. cm. to sq. cm.
18. 5 sq. m. 15 sq. cm. 9 sq. mm. to sq. mm.
19. 176849 sq. mm. to sq. dm., etc.
20. 568459 sq. cm. to sq. metres, etc.
21. 786845 ares to sq. Km., etc.
22. 1234587 centiares to hectares, etc.

Applications of Measures of Area.

261. Suppose that a rectangular surface, such as the floor of a room, is 4 yds. long and 3 yds. wide; we can divide the sides into 4 equal parts and 3 equal parts respectively, each part being 1 yard. If now through the points of division we draw lines parallel to the sides in the manner shewn in the margin, it is clear that we divide the surface into 4 rows of smaller areas, and that there are 3 of the smaller areas in each row. Moreover, each of the smaller areas is a square yard, and therefore the area of the surface is 3×4 square yards.



Hence, to find the number of *square yards* in a rectangular area, we multiply the number of *yards* in the length by the number of *yards* in the breadth.

If the length and breadth be given in *inches*, the product will give the number of *square inches* in the area: if in *feet*, the product will give the number of *square feet*.

We must take care that the length and breadth are expressed in measures of the same name; both in yards, or both in feet, or both in terms of some common measure: the product will then give the area in terms of the *corresponding* square measure.

EXAMPLES. 113.

On squared paper draw rectangles whose dimensions are as follows, and count the number of units of area in each:

1.	2 by 4.	2.	3 by 5.	3.	4 by 7.	4.	5 by 6.
5.	8 by 9.	6.	7 by 13.	7.	10 by 12	8.	10 by 17.
9.	13 by 11.	10.	5 by 20.	11.	18 by 8.	12.	21 by 27.

262 If we use the words *length, breadth* and *area* to denote the *number of units* in each respectively, the relation between them may be stated thus:

$$\text{length} \times \text{breadth} = \text{area},$$

$$\text{length} = \frac{\text{area}}{\text{breadth}}, \quad \text{and} \quad \text{breadth} = \frac{\text{area}}{\text{length}};$$

hence, if any two be given, we can always find the third.

The length and the area, or the breadth and the area, must be expressed respectively in feet and square feet, or in yards and square yards, or in some other measure of length and the square measure that corresponds to it

If A represent the number of units in the area of a rectangle, and l and b represent the numbers of units of length in adjacent sides, the connection between them may be stated thus:

$$\mathsf{A} = l \times b, \quad l = \frac{\mathsf{A}}{b}, \quad b = \frac{\mathsf{A}}{l}.$$

NOTE.—The pupil must be careful not to write 13 ft. × 15 ft. instead of 13 ft. by 15 ft. The former cannot represent an arithmetical operation, for, since multiplication is merely a short method of addition, a multiplier must always be an abstract number. The latter denotes the dimensions of a rectangular room whose sides are 13 ft. and 15 feet.

Example i. *Find the area of the floor of a room,* 12 *ft.* 8 *in. long and* 10 *ft.* 9 *in. broad.*

Length of room $= 12\frac{2}{3}$ ft., and breadth $= 10\frac{3}{4}$ ft.;

$\therefore$ area $= 12\frac{2}{3} \times 10\frac{3}{4}$ sq. ft.

$= \frac{38}{3} \times \frac{43}{4}$ sq. ft.

$= \frac{817}{6}$ sq. ft.

$= 136$ sq. ft. 24 sq. in.

Example ii. *Find the breadth of a court-yard,* 25 *sq. yds.* 8 *sq. ft.* 48 *sq. in in area, and* 17 *ft.* 6 *in. in length.*

Area $= (25 \times 9 + 8 + \frac{48}{144})$ sq. ft. $= 233\frac{1}{3}$ sq. ft.,

and length $= 17\frac{1}{2}$ ft.;

$\therefore$ breadth $= \frac{233\frac{1}{3}}{17\frac{1}{2}}$ ft. $= \frac{1400}{105}$ ft. $= 13\frac{1}{3}$ ft. $= 13$ ft. 4 in.

Example iii. *Find in ares the area of a field,* 175 *m. by* 144 *m.*

The length = 17·5 Dm., and the breadth = 14·4 Dm.;

∴ the area = 17·5 × 14·4 sq. Dm.

= 252 ares

= 2 Ha. 52 a.

```
 17·5
 14·4
-----
 175
 70·0
  7·00
-----
 252
```

NOTE.—When the result is required in *ares* it is best to express the length and breadth in *decametres.* The product will then be the number of sq. decametres., *i.e. ares.*

Example iv. *The length of a field is* 775 *m., and its area is* 14 *Ha.* 67 *a.* 85 *ca.; find its breadth.*

Area = 1467·85 ares = 1467·85 sq. Dm.;

Length = 775 m. = 77·5 Dm.;

∴ Breadth = (1467·85 ÷ 77·5) Dm. = 18·94 Dm. = 189·4 metres.

```
      18·94
    -------
775 ) 14678·5
      6928
      -----
       7285
       3100
```

EXAMPLES. 114.

(*In these examples all areas may be assumed to be rectangular.*)

1. Find the area of a room whose length and breadth are respectively

(i) 15 ft. and 10 ft. (ii) 14 ft. and 12 ft.
(iii) 14 ft. 6 in. „ 13 ft. 2 in. (iv) 16 ft. 8 in. „ 12 ft. 6 in.
(v) 18 ft. 10 in. „ 15 ft. 7 in. (vi) 8 ft. 7½ in. „ 11 ft. 4½ in.

2. Find the length of a room when

(i) Area = 225 sq. ft. and breadth = 15 ft.:
(ii) Area = 252 sq. ft. and breadth = 12 ft.;
(iii) Area = 372 sq. ft. 36 sq. in. and breadth = 13 ft. 6 in.;
(iv) Area = 475 sq. ft. 72 sq. in. and breadth = 15 ft. 3 in.

3 Find in sq. metres, etc., the area of a room whose length and breadth are respectively

(i) 17 m. and 10 m. 5 dm. (ii) 14 m. 25 cm. and 8 m. 40 cm.
(iii) 16·84 m. and 12·75 m. (iv) 15·73 m. and 13·37 m.

4. Find the length of a room whose area and breadth are respectively

(i) 174·8 sq. m. and 11·5 m. (ii) 190 sq. m. and 12 m. 50 cm.
(iii) 123 sq. m. 75 sq. dm. 50 sq. cm. and 9 m. 3 dm. 4 cm.

5. Find in acres, roods and perches the area of a piece of land whose length and breadth are respectively

(i) 5 fur. 15 p. and 2 fur. 32 p.
(ii) 7 fur. 18 p. and 4 fur. 22 p.
(iii) 3 fur. 25 p. and 3 fur. 25 p.

Express the above areas also in acres and sq. chains.

6. Find in acres, sq chains and sq. links the area of a piece of land whose length and breadth are respectively

(i) 3 fur. 7 ch. and 8 ch. 25 links.
(ii) 2 fur. 6 ch. 45 links and 9 ch. 64 links.
(iii) 1 mi. 7 fur. 5 ch. and 6 fur. 4 ch. 50 links.
(iv) 5 fur. 3 ch. 75 links and 2 fur. 9 ch. 24 links.

7. Find in Hectares, ares and centiares the area of a piece of land whose length and breadth are respectively

(i) 75·5 m. and 63·4 m. (ii) 5 Hm. 27 m. and 13·35 Dm.
(iii) 1 Km. 378 m. and 9 Hm. 64 m. (iv) 3·756 Km. and 786·5 m.

8. If a school-room be 42 ft. long and 20 ft. wide, how many boys will it accommodate, allowing to each boy at least 8 sq. ft.?

9. In a row of 25 houses each house has 17 windows, each window 4 panes, and each pane measures 18 in. by 9 in. What will be the cost of glazing all these windows at 6*d.* per sq. foot?

10. A field can be divided into 10000 squares, each square measuring 100 inches along a side; find the area of the field.

11. If 2240 stones, each 9 inches square, will pave a court, find the area of the court.

12. Find the cost of laying a plot of ground, 40 yds. long and 100 ft. wide, with turfs each 1 yard long and 1 foot wide; the turfs when laid costing 6*s.* 9*d.* per hundred.

13. How many marble slabs, 1 ft. long and 9 in. broad, would pave a hall, 50 yds. long and 50 ft. broad? What would the cost be if the price of the slabs is £5 per dozen?

14. How many panes of glass, each 2 ft. long and 1 ft. 5 in. wide, will glaze a window 8 ft. high and 5 ft. 8 in. wide?

15. What will be the cost of varnishing the floor of a room, 14 ft. 6 in. broad and 15 ft. 4 in. long, at 6*d.* per sq. yard?

16. What will be the cost of covering a room, 25 ft. long and 13 ft. 6 in. broad, with carpet at 10*s.* a sq. yard?

17. Find the cost of paving a court, 59 ft. 4 in. long and 48 ft. 9 in. wide, at 4*s.* 6*d.* per sq. yard.

18. What will it cost to pave an area, 266 ft. 3 in. long and 48 ft. 9 in. broad, at $11\frac{1}{4}$*d.* per sq. yard?

19. The area of a piece of land is 732 Ha. 31 a. 65 ca., and its length is 7 Km. 46·5 Dm. Find the breadth.

20. The area of a piece of land is 585 Ha. 91 a. 35 ca., and its length is 5·865 Km. Find the breadth.

21. The area of a piece of land is 781 acres, and the breadth is 78 ch. 40 links. Find the length to the nearest link.

22. The area of an estate is 8 sq. mi. 540 ac., and its length is 3 mi. 3 fur. 5 ch. Find the breadth to the nearest chain.

23. The area of a rectangle is 845·76 sq. metres, and the length of one side is 32 m. 38 cm. Find the length of the other side to the nearest centimetre.

24. The length of one side of a rectangle is 1·756 Km. and the other is 3 054 Km. Find the area to the nearest hectare.

25. The lengths of the sides of a rectangle are 5 fur. 7 ch. 30 links and 3 fur. 4 ch. 75 links. Find the area to the nearest acre.

26. If a plan be drawn to a scale of $\frac{1}{1000}$, how many acres does a field contain, which on the plan measures 4 in. by $3\frac{1}{8}$ in.?

27. If a plan be drawn to a scale of $\frac{1}{10000}$, how many hectares, etc., does a field contain, which on the plan measures 7·5 cm. by 4·8 cm.?

28. If a map be drawn to a scale of $\frac{1}{4\,608\,000}$, how many square miles are contained in an area which measures on the map 5·5 in. by 4·125 in.?

29. If a map be drawn to a scale of $\frac{1}{20\,000\,000}$, how many square kilometres are contained in an area which measures on the map 7·25 cm. by 4·35 cm.?

30. An estate containing 394 ac. 2 sq. ch. is divided into lots, each lot containing 27·375 sq. ch. How many lots are there?

31. Find the cost of a field, ·625 Km. long and ·38 Km. broad, at 730 f. per hectare.

32. Find the cost of a field, 25 ch. by 28 ch., at £95. 5*s.* per acre.

33. Find the cost of paving a court, 27 m. 35 cm long and 19 m. 64 cm. wide, at 5 f. 25 c. per sq. metre.

34. A piece of land, 2354 metres square, is sold at 275 f. 25 c. per hectare. How much does the whole sell for?

35. Find in hectares, etc., the area of a field whose length is 2 fur. 5 ch. 75 links and breadth 1 fur. 2 ch. 45 links, if 1 acre = 40·4671 ares. (Answer to nearest centiare)

36. If 1 hectare = 2·47711 acres, find in acres the equivalent of 45 Ha 37 a. 25 ca. to the nearest sq. chain.

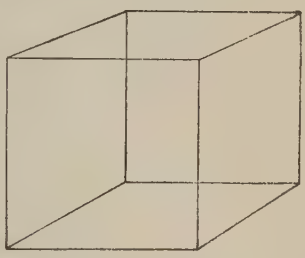
37. If 1 metre = 39·3708 inches, find to 4 decimal places the equivalent of 1 sq. metre in sq. ft. and also in sq. yards.

38. If 1 foot = 3 0479 dm., find to 4 decimal places the number of sq. feet in a square metre.

XXV. MEASURES OF CAPACITY—BRITISH AND METRIC.

263. The principal measure of **volume** is a solid similar to that described in the margin. Its length, breadth and height are each *one yard*, and the volume is called a **cubic yard.**

The drawing in the margin represents an exact **cubic inch**, its length, breadth and height being each *one inch.*



If we arrange 144 *cubic inches* in 12 rows, each row containing 12 cubes, we form a solid, one inch high, with a top surface whose area is a square foot.

If again we form 12 such solids, and place them one above another, we build up a cube whose length, breadth and height are each one foot. It will therefore be a **cubic foot**; and, since we have used 12×144 cubic inches in making it, it is clear that

1728 **cubic inches** make 1 **cubic foot.**

Similarly it may be shewn that

27 **cubic feet** make 1 **cubic yard.**

264. The principal measure of liquids is called a **gallon.**

LIQUID MEASURE.

4 **gills** make 1 **pint.**
2 **pints** ..... 1 **quart.**
4 **quarts** ..... 1 **gallon.**
36 **gallons** ..... 1 **barrel.**

A quarter barrel (9 gallons) is also called a **Firkin,** and a half barrel (18 gallons) a **Kilderkin.**

A **Hogshead** (hhd.) of ale contains 54 gallons, or $1\frac{1}{2}$ barrels; a **Butt** of ale 3 barrels, and a **Pipe** 6 barrels. These terms are used also in measuring wine, but they vary with the kind of wine.

"**A pint of pure water weighs a pound and a quarter**"; therefore a gallon of water weighs 10 lb.

A gallon contains 277·274 cubic inches: hence **a cubic foot of pure water weighs very nearly 1000 ounces.**

NOTE.—Apothecaries subdivide the pint as follows: 60 **minims** (♏) or **drops** make **1 fluid drachm** (ʒ); 8 **drachms** make 1 **fluid ounce** (ʒ); 20 **ounces** make **1 pint.** A fluid ounce, being $\frac{1}{20}$ of a pint, contains an ounce *by weight* of pure water.

265. The following measures are used for certain *dry* goods such as grain and peas, and for some kinds of fruit and vegetables. But the measuring of such things by *volume* is being gradually superseded by their measure by *weight.*

DRY OR CORN MEASURE.

4 **gills**	make	1 **pint.**
2 **pints**	.....	1 **quart.**
4 **quarts**	.....	1 **gallon.**
2 **gallons**	.....	1 **peck.**
4 **pecks**	.....	1 **bushel.**
8 **bushels**	.....	1 **quarter.**
4 **quarters**	.....	1 **ton** (cargo measurement).

Formerly there were in use the following measures of capacity, each of which is exactly twice as large as that which precedes it: *noggin, gill, pint, quart, pottle, gallon, peck, tuffet, bushel, strike, coomb, and quarter.*

266. We give a few special measures for reference. A **stone of butcher's meat** weighs 8 lb.; a **cental of corn** weighs 100 lb.; a **sack of coals** weighs 2 cwt.; a **peck of flour** weighs 14 lb.; a **barrel of flour** weighs 196 lb.; a **sack of flour,** 280 lb.; a **barrel of gunpowder,** 100 lb.; a **pack of wool,** 240 lb.; a **pocket of hops,** 168 lb.; and a **quartern loaf,** 4 lb. A **ton of shipping** is a measure of capacity and contains 40 cubic ft.

A **rod of brickwork** is 272 sq. feet of brickwork, 14 inches thick. All brickwork, whatever its thickness may be, is expressed in terms of this unit. It is then called *reduced work.*

267. The metric measures of volume are derived from the corresponding measures of length in the same way as the British measures of volume (Art. 211).

METRIC MEASURES OF VOLUME.

10^3, or 1000, **cubic millimetres** (c.mm.)	= 1 **cubic centimetre** (c.cm.).
10^3, or 1000, **cubic centimetres**	= 1 **cubic decimetre** (c.dm.).
10^3, or 1000, **cubic decimetres**	= 1 **cubic metre** (c.m.).

In measuring logs of wood a cubic metre is called a **stere**. The only multiple of this measure ever used is the **decastere** (10 steres), and the only submultiple is the **decistere** ($\frac{1}{10}$ stere). These are however seldom met with.

268. In the metric measures of capacity the same prefixes are employed as are used in the tables of weight and length.

The principal unit is a **litre**, which is equal to the volume of **1 cubic decimetre**, or to 1000 cubic centimetres; and since the **gram** was chosen as the weight of 1 c.cm. of water at 4° C., it follows that one *litre* of water weighs 1000 g. or 1 *kilogram*.

Thus the **litre** is practically defined as *the volume of* **one kilogram of water at 4° C.**

METRIC MEASURES OF CAPACITY.

10 **centilitres** (cl.) = 1 **decilitre** (dl.).
10 **decilitres** = 1 **litre** (l.).
10 *litres* = 1 *decalitre* (Dl.).
10 *decalitres* = 1 **hectolitre** (Hl.).

It should be remembered that 1 **litre** = $1\frac{3}{4}$ **pints** (nearly).

EXAMPLES. **115.** *Oral.*

1. In $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{2}$ of a cubic yd. how many cubic ft.?
2. In 2, 3, 4, 5 cubic yards how many cubic ft.?
3. In 5, 6, 9 pints how many gills?
4. In $5\frac{1}{2}$, $7\frac{1}{2}$, $12\frac{1}{2}$ pints how many gills?
5. In 2, 3, 4, 5 quarts how many gills?
6. How many quarts in 8, 24, 64, 96 pints?
7. How many gallons in 40, 88, 96 quarts?
8. How many quarts in 4, 5, 6, 7, $6\frac{1}{2}$, $7\frac{1}{4}$, $9\frac{3}{4}$ gallons?
9. How many gallons in 72, 80, 104 pints?
10. How many gallons in 2, 3, 8 barrels of beer?
11. How many pecks in 32, 96, 44, 60 quarts?
12. How many bushels in 32, 44, 60, 96 pecks?
13. What is the weight in pounds of 5, 20, 50, 216, 400 gallons of water?
14. Express in cubic centimetres,
    3 c.m. 152 c.dm., 72 c.dm. 571 c.cm., 87 c.dm. 25 c.cm.,
    1 c.m. 50 c.dm. 50 c.cm., 3 c.m. 100 c.dm. 7 c.cm.

15. Express in cubic metres and cubic decimetres,
37215 c.cm., 1003461 c.cm., 78684172 c.cm., 1800018008 c.cm., 9090909 c.cm.

16. Express in litres and decimals of a litre,
5 Hl. 17 l.; 8 Hl 5 l; 7 Hl. 25 l. 3 dl.; 8 l. 9 dl. 7 cl. 3 Hl. 14 l. 25 cl.; 25 Hl. 5 cl

17. Express in hectolitres, litres, etc.
1725 cl., 137 l., 857·5 l, 127·15 l., 84597 dl., 12345 cl., 1·5764 Hl., ·0025 Hl.

EXAMPLES. 116.

Reduce

1. 2 quarts 1 pint to pints.
2. 3 quarts 2 gills to gills.
3. 1 hogshead of ale to quarts.
4. 5 galls. 1 pint to gills.
5. 15 galls. 3 qts. 2 gills to gills.
6. 2 galls. 1 qt. 3 gills to gills
7. 8 bush. 2 pks. to pecks.
8. 2 qrs. 3 bush. 2 pks. to gals.
9. 3 qrs. 7 bush. 1 pk. to galls.
10. 4 qrs. 6 bush. 1 pk. to galls.
11. 3 qrs. 7 bush. 2 pks. to pints.
12. 2 qrs. 5 bush. 3 pks. 1 gall. to pints.
13. 4 qrs. 3 bush. 2 pks. 1 gall. to pints.
14. 5 quarters to gills.
15. 8 galls. 2 fl. oz. to fl. drms
16. 3842 gills to quarts.
17. 5629 gills to gallons.
18. 7428 gills to gallons.
19. 62741 gills to gallons.
20. What is the weight of 14 galls. 3 pints of water?
21. In 10 sacks of flour how many cwt.?
22. In 125 packs of wool how many tons?
23. How many litres are there in 7545 bottles, if each bottle contains 34 cl.?
24. A number of casks contain 402 Hl. 5 l. of wine. If each cask contains 2 Hl. 15 l., how many casks are there?
25. A man has to carry 670 Hl. 35 l. of wheat from one place to another; if he carries 1 Hl. 35 l. each time, how many journeys must he make? How much will he carry the last time?
26. A pile of wood containing 14 decasteres 6 steres $2\frac{1}{2}$ decisteres is divided into lots, each containing $7\frac{1}{2}$ decisteres. How many lots are there?

27. How much wood is burnt in five fireplaces, which burn respectively, 18 decisteres, $2\frac{1}{4}$ steres, 1·65 steres, 27·5 decisteres, and 3 steres 8 decisteres? How much would each have burnt, if they had all burnt equal quantities?

28. A man, who had 31 Hl. of wine, put 17 dl. into each of 1728 bottles. The remainder of the wine he put into bottles, each of which held 7 dl. How many bottles were filled in all?

29. 4840 stones, each containing 325 c.dm., are used in a building. How many cubic metres are used in all?

30. Express, in litres and centilitres, $\frac{3}{32}$ of 12 Hectolitres.

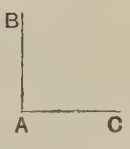
XXVI. MEASURES OF ANGLES, NUMBER AND TIME.

269. Euclid defines a *plane rectilineal angle* as the "inclination of two straight lines to one another, which meet together, but are not in the same straight line."

If two adjacent angles are equal to one another, each is a **right angle.**

Measures of Angles.

The angle BAC is a **right angle.** A right angle is subdivided thus:



60 **seconds** (60″) make 1 **minute** (1′).
60 **minutes** ..... 1 **degree** (1°).
90 **degrees** ..... 1 **right angle.**

A **quadrant** is a quarter of the circumference of a circle. It subtends a right angle at the centre of the circle.

Measures of Number.

12 **units** make 1 **dozen.**
20 **units** ..... 1 **score.**
12 **dozen** ..... 1 **gross.**
12 **gross** ..... 1 **great gross.**
Also 24 **sheets of paper** ..... 1 **quire.**
20 **quires** ..... 1 **ream.**
10 **reams** ..... 1 **bale.**

Measures of Time.

270. The chief measure of time is a **day**, which is the average time in which the earth turns round on its axis.

The time in which the earth travels round the sun is a **solar year**, and is equal to 365·2422414 days.

The **common year** consists of 365 days. The difference between a common year and a solar year is therefore ·2422414 of a day, and we take account of it by adding a whole day to every *fourth* year. Consequently in every 4th year there are 366 days.

The years which have the extra day are called *bissextile* or **Leap Years.** The day is inserted at the end of February, and is called the 29th of February.

The difference between 4 common years and 4 solar years is ·9689656 of a day. If therefore we add a whole day to every 4th year, we add too much by ·0310344 of a day. To take account of this we omit the extra day three times every 400 years, and in this way we make the number of common years in any long period very nearly equal to the number of solar years. The object is to ensure that each season may fall at the same time of the year in all years. In course of time, without these corrections, we should have winter in July and summer in January.

With a slight variation the present divisions of the year are those arranged in B.C. 46 by Julius Cæsar as *pontifex maximus*; to him is due also the insertion of the extra day every fourth year. The omission of the extra day three times in 400 years is called the **Gregorian Correction.** It was ordered in 1582 by Pope Gregory XIII., in order that certain positions in the course of the earth's journey round the sun, upon which the seasons depend, might ever after be passed on the same days of the year as in 325, when the Council of Nice was held. The years in which the day is omitted are the last year of each century, except those which are divisible by 400. For example, 1600, 2000, 2400, are leap years, but 1700, 1800, 1900, 2100 are not.

The Gregorian correction was adopted at once in 1582 in Roman Catholic countries, but not in England until 1752. In that year the day after Sept. 2nd was called Sept. 14th. At the same time it was also arranged that the legal year should begin on the 1st of January, instead of on the 25th of March. The year 1751 therefore, which began on the 25th of March, ended on the 31st of December. The next day

became the 1st of January, 1752. The Gregorian mode of reckoning is called the **New style**; the former, the **Old style.**

The New style has not yet been adopted in Russia, so that since 1752 they have had two more leap years (1800 and 1900) than we have, and they are now 13 days behind us: what we call Sept. 25th, they call Sept. 12th. "Old Christmas Day" is now the 7th of January, and "Old Michaelmas Day" the 12th of October.

We can ascertain whether a year is a leap year or not, by dividing the number formed of the last two digits by 4, except when they are both zeros. In the exceptional case, when the last two digits are both zeros, we divide the first two by 4. If in either case there is a remainder, the year is a common year; if not, it is a leap year.

271. The divisions and multiples of the day are given in the following table:

MEASURES OF TIME

60	**seconds** (60^s)	make 1	**minute** (1^m).
60	**minutes**	 1	**hour.**
24	**hours**	 1	**day.**
7	**days**	 1	**week.**
28, 29, 30 or 31	**days**	 1	**calendar month.**
12	**calendar months**	 1	**year.**
365	**days**	 1	**common year.**
366	**days**	 1	**leap year.**
100	**years**	 1	**century.**

Of the Calendar months **February** has 28 days in a common year, and 29 days in a leap year; **April, June, September** and **November** have each 30 days; the others have each 31 days.

In England and Ireland the **Quarter Days** are four days of the year, on which rent, salaries, etc., for the preceding quarter become due. They are March 25th, called **Lady Day**; June 24th, **Midsummer Day**; September 29th, **Michaelmas Day**; and December 25th, **Christmas Day.**

In Scotland the **Quarter** or **Term** days are **Candlemas**, February 2nd; **Whitsunday**, May 15th; **Lammas**, August 1st; and **Martinmas**, November 11th. The **Removal Terms** are May 28th and November 28th.

A **solar year**, or the time in which the earth travels round the sun, is equal to 365 days 5 hrs. 48 min. $47\frac{1}{2}$ secs. nearly.

A **lunar month**, or the time in which the moon travels round the earth, is equal to 28 days nearly.

272. It will be found that

$$365 \text{ days} = 52 \text{ weeks} + 1 \text{ day},$$

and

$$366 \text{ days} = 52 \text{ weeks} + 2 \text{ days},$$

and that in 28 consecutive years (not including a *century year*) there are 21 common years and 7 leap years. In 28 consecutive years there are therefore 1456 weeks and $(21 + 7 \times 2)$ days, *i.e.* 1456 weeks and 35 days, *i.e.* 1461 complete weeks exactly. Consequently, after a period of 28 years, the days of the month will again fall on the same days of the week as in this year. Any date in the years 1932, 1960, etc., for example, will fall on the same day of the week as in 1904, and the calendars for these years will be identical.

This statement does not hold good if the period of 28 years includes a *century year* such as 1900, which is *not* a leap year; but it would still hold if the century year were 2000 or 2400.

In 400 consecutive years there is *always* an exact number of weeks, whatever be the year with which the period begins.

These facts are useful in calculating the day of the week on which a certain date fell in the past, or will fall in the future.

Example. *The battle of Waterloo was fought on June 18th, 1815. What was the day of the week?*

Adding 28 to 1815, we see that June 18th fell on the same day of the week in 1815, 1843, 1871, and 1899. Hence the problem is reduced to finding the day of the week on which June 18, 1899, fell.

Now June 18th, 1899, is 168 days, *i.e.* 24 weeks after January 1, 1899. Hence June 18th fell on the same day as January 1 in 1899. But January 1, 1904, fell on a Friday; therefore January 1, 1899, fell 5 days earlier, *i.e.* on a Sunday. Sunday was therefore the day on which the battle of Waterloo was fought.

273. A rod of a certain length suspended freely by one end and set in motion, will vibrate in exactly one second. Such a rod is called a **seconds' pendulum**. The length varies for different places, but is always the same at the same place. At London, at the level of the sea, its length is 39·1393 inches.

EXAMPLES. 117. *Oral.*

1. How many degrees in 2, 3, 4, 8 right angles?
2. How many degrees in $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{1}{4}$ of a right angle?

3. How many minutes in 2, 3, 4 degrees?
4. How many minutes in $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ of a degree?
5. How many seconds in 5, 6, 7 minutes?
6. How many seconds in $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ of a minute?
7. How many units in 5, 6, 8, 10 dozen?
8. How many units in $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{6}$ of a dozen?
9. How many dozen in 5, 8, 12 gross?
10. How many units in 10, 12 score?
11. How many sheets in 1 ream?
12. How many sheets in 1 bale?
13. How many minutes in $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ of an hour?
14. How many days in 6, 10, 13 weeks?
15. How many days in 2 common years?
16. How many days in 4 consecutive years?
17. How many days in 1 century?

EXAMPLES. 118.

Reduce

1. 45° to seconds
2. 60° to seconds
3. 29° 15′ to minutes.
4. 24° 21′ to seconds.
5. 38° 43′ 25″ to seconds.
6. 57° 16′ 30″ to seconds.
7. 5 hrs. 3^m 40^s to seconds.
8. 13 wks. to seconds.
9. 47632 secs. to degrees.
10. 563472 secs. to degrees.
11. 894631 secs. to days.
12. 1084203 secs. to days.
13. 3440707 secs. to weeks.
14. 59018732 secs. to weeks.
15. In 56 reams of paper, how many sheets?
16. In 250 right angles, how many seconds?
17. How many days are there from Nov. 16, 1902, to Sept. 5, 1904 (inclusive)?
18. How many days were there in the 19th century?
19. How many days will there be from 1st Jan. 1886 to 31st Dec. 2000 (inclusive)?
20. Find the number of hours from Dec. 1st, 1765, to Nov. 30th, 1865 (inclusive).

On what days of the week did the following events take place?

21. The battle of La Hogue, June 1, 1794.
22. The battle of Trafalgar, October 21, 1805.

On what days of the week did the following events take place?

23. The battle of Vittoria, June 21, 1813.

24. The battle of Gravelotte, August 18, 1870.

25. The relief of Lucknow, September 25, 1857.

26. The battle of Austerlitz, December 2, 1805.

MISCELLANEOUS EXAMPLES. 119.

1. The combined weight of 3 sacks is 2 cwt. If one of them is 10 lb. less than each of the other two, which are of equal weight, find the weight of each sack.

2. A truck is loaded with 105 sacks Each sack, when empty, weighs 6 lb. and contains 3 qr. 6 lb. of grain. Find the total weight of the load.

3. If 5 sovereigns weigh 40 g., how many sovereigns will weigh 15 Kg. 16 Dg.?

4. If a sovereign weighs 123 grs., how many sovereigns will weigh 142 oz. 4 dwt. 9 grs.?

5. 115 tons of earth are to be carried away by carts which, when full, hold 1 ton 2 cwt. 1 qr. 7 lb. How many journeys must be made? What weight will the *last* cart carry?

6. How many days were there in England in the years 1751, 1752 and 1753 respectively?

7. The beginning and the end of a straight course, and the end of each chain, are marked by posts. If the course is $1\frac{1}{2}$ miles long, how many posts are there? If the course were circular, how many posts would there be?

8. How much too long is a two-foot rule which shows that a distance of 12 ft. 6 in. is apparently 12 ft.?

9. Find the L.C.M. of the number of farthings in a sovereign, of pounds in a ton, and of inches in a mile.

10. The breadth of a railway track is 22 yds. How many acres does one mile of it cover?

11. How often is 15 inches contained in 210 yards?

12. How often is 7 ft. 6 in. contained in 195 m. 7 fur.?

13. If 17 sovereigns form a column one inch high, how many would it take to represent the height of Snowdon (3451 ft.)?

14. The diameter of a sovereign being $\frac{7}{8}$ of an inch, how many placed in contact along a straight line would extend from London to Liverpool, a distance of 196 miles?

15. How many chains are there in $12\frac{1}{2}$ miles?

16. How often will a wheel, 2 yds. 2 ft. 3 in. in circumference, turn in a distance of $12\frac{1}{2}$ miles?

17. How many times will a coach wheel, 7 ft. 6 inches in circumference, turn in a distance of 391 miles 6 furlongs?

18. The circumference of a carriage wheel is 7 ft. 4 in.; how often will it turn round in travelling a league?

19. A bicycle wheel has a circumference of $3\frac{1}{4}$ yards; how many times will it turn in travelling 26 miles?

20. A coach wheel is 17 ft $7\frac{1}{5}$ in. in circumference; how often will it turn in 8 miles 264 feet?

21. What is the circumference of a wheel which turns 880 times in a quarter of a mile?

22. How many cable's lengths are there in 5 miles?

23. A vessel holds $2\frac{1}{3}$ pints; how many times can it be filled from a cask of 56 gallons? will there be any remainder?

24. If 3 pennies weigh one ounce, find the weight of £10 worth of pennies.

25. How many seconds are there in the month of November?

26. How many minutes are there in the month of February in the years 2000 and 1700 respectively?

27. One man is 25 years 33 weeks and 6 days old, and another is 31 years 49 weeks and 2 days old; a third is just as much older than the first as he is younger than the second. What is the age of the third?

28. In how many ordinary years would there be an exact number of lunar months?

29. If a pendulum vibrate 5 times in two seconds, how many times will it vibrate in 48 hours?

30. If a man sleep 7 hours and 25 minutes each night, how much time does he spend in sleep during 1904?

31. A speed of how many feet per second is equivalent to a speed of a mile per hour?

32. A boy running a mile in 5 minutes advances 3 ft. 8 in. each step; how many steps per minute does he take?

33. A crew rows 35 strokes in a minute, the boat travelling 7 yards in each; how far will it travel in 25 minutes?

34. Find the number of hours from noon on Dec. 11th, 1809, to noon on Dec. 11th, 1909. What would be the amount of one farthing for each hour?

35. On the 15th of May, 1877, a person had lived 30,000 days; what was his birth-day?

36. A train travels 45 miles an hour; how many revolutions does the driving-wheel of the engine make in a second, its circumference being 22 feet?

37. A watch, set right at noon on March 1st, loses $31\frac{1}{2}$ seconds daily; what will be the time by the watch at noon on March 31st?

38. A magazine contains 120 pages, each 9 in. long and $7\frac{1}{2}$ in. wide, and weighs 5 oz. What is the weight of a square yard of the paper on which it is printed?

39. A waggon contains a certain number of bales, each weighing 1 qr. 3 lb.; 5 times as many, each weighing 1 qr. 14 lb.; and 7 times as many, each weighing 1 qr. 10 lb. If the total load be 2 tons 14 cwt. 1 qr. 8 lb., how many of each kind are there?

40. A courtyard, 13 yds. 2 ft. 1 in. long by 4 yds. 2 ft. 2 in. broad, is to be paved with square tiles. If the tiles are as large as possible, how many must be used?

41. A hole, 8 yds. 2 ft. 7 in. long, 5 yds. 1 ft. 11 in. broad and 2 yds. 1 ft. 3 in. deep, is to be filled up with cubical blocks of stone. If the blocks are as large as possible, find the number required and the number of cubic inches in each block.

42. The circumference of a circle is 3·14159 times the diameter. How many times will a wheel, whose diameter is 4 ft. $7\frac{1}{2}$ in., revolve in a journey of $2\frac{3}{4}$ miles? (Answer to nearest integer.)

43. Assuming that a kilogram is 2·204621 lb., express as a decimal of an ounce, to 2 places, the difference between 1 ton and 1016 Kg.

44. If a gallon contains 277·274 cu in., how many cubic feet are there in 100 bushels? (Answer to nearest integer.)

45. A measure containing 1 l 57 cl. is filled as often as possible from a cask containing 3 Hl. 52 l. How often can it be filled, and how much remains over?

46. A takes 116 strides per minute and the length of his stride is 30 inches. B takes 112 strides per minute and the length of his stride is 80 cm. If they start together, find, to the nearest foot, their distance apart at the end of an hour. (1 cm. = ·3937 in.)

47. A and B measure the side of a square plot with a tape supposed to be one chain long; but the tape is 3 inches short. The apparent area is 14 ac. 4 sq. chains. What is the true area?

48. The rails of a railway are 5 yards long at 0° C. and ·0006435 of their length longer at 55° C. To allow for this increase, find, to two decimal places of an inch, the least distance between two consecutive rails at 0° C.

49. The area of a square iron plate is 75 sq. cm. If the plate expand equally so that the length of each edge is increased by ·000585 of itself, find the new area to the nearest square millimetre.

50. If 1 foot = ·3047945 metres, find, to three decimal places, the number of Kilometres in a knot; given that 1 knot = 6081 ft.

51. Express $\cdot 2\dot{6}\dot{3}$ of £1. 7*s*. 6*d*. as a decimal of £20.

52. A boy after giving away ·25 of his pocket money to one companion, and ·875 of the remainder to another, has 9*d*. left. How much had he at first?

53. Find the value of $\frac{3}{8}$ of 30*s*. + $\frac{8}{15}$ of $1\frac{1}{2}$ of £1. 5*s*. − ·5625 of £1.

54. Simplify $\cdot 6\dot{5}$ of $4\cdot 1\dot{1}$ of $\frac{11}{39}$ of $2\cdot 43\dot{2}$.

55. Two persons take steps of 2·45 ft. and 2·85 ft. respectively; if they start 'in step,' how far will they go before they are 'in step' again for the first time?

56. What fraction of *their sum* is the *difference* of $\frac{7}{8}$ of 10*s*. and $\frac{8}{7}$ of 3*s*. 6*d*.?

57. Find the value of a ton of gold at £3. 17*s*. 6*d*. per ounce troy.

58. Prove that the square of every odd number when divided by 8 leaves a remainder 1.

59. Find the missing figures in the following sums:

(i)
$$\begin{array}{r} **4* \\ 5* \\ \hline ***** \\ 23**7 \\ \hline 410*** \end{array}$$

(ii)
$$*\,\big)\,*****$$
$$*\,\big)\,81*1 - 2$$
$$*** - 3$$
$$\Big\} \; 23 = \text{remainder.}$$

60. If A gives B 24 oxen worth £16 10*s*. each, and receives in exchange 240 sheep worth £1. 12*s*. each, does he gain or lose, and how much?

61. A grocer buys 572 lb. of tea at 2*s*. $2\frac{1}{4}$*d*. per lb., and sells it at 2*s*. 6*d*. per lb.; what does he gain altogether?

62. A dealer buys 15 horses at £28. 13*s*. each; he sells 8 at £35 each, and the rest at £42. 10*s*. each; find his total gain.

63. Ten tons of goods, bought at 27*s.* 6*d.* per cwt., are sold at £30 per ton; what is the whole gain?

64. A grocer buys a chest of tea containing 180 lb. at 3*s.* 7¼*d.* per lb. If 10 lb. be spoiled, what does he gain by selling the remainder at 4*s.* 4*d.* per lb.?

65. A grocer buys a cwt. of sugar for £1. 13*s.* 4*d.*, and sells it for £2. 2*s.* 8*d.*; what does he gain per pound?

66. A grocer gave £25 for 8 cwt. 1 qr. 9 lb. of sugar, and lost £5. 11*s.* 3*d.* by retailing it; at what rate per lb. was it sold?

67. A grocer buys 6½ cwt. of tea at 17 guineas per cwt. He sells 3 cwt. of it at 3*s.* 3*d.* a lb., and the rest at 3*s.* 9½*d.* a pound; how much does he gain?

68. A dealer bought wheat at 41*s.* 3*d.* per qr.; he subsequently sold it at 49*s.* 7*d.* per qr., and made a profit of £277. 10*s.* altogether; how many quarters did he buy and sell?

69. A draper bought 600 yds. of silk at 3*s.* 4½*d.* per yd.; he sold 360 yds. at 4*s.* 6*d.* per yd., 81 yds. at 3*s.* 9*d.* per yd., and was robbed of the rest; what was his whole gain or loss?

70. A grocer buys 96 lb. of tea at 3*s.* 6*d.* per lb., 24 lb. of which, being damaged, he sells at 3*s.* per lb.; at what price must he sell the rest, so as to gain two guineas altogether?

71. A man buys 500 qrs. of wheat at 28*s.* a quarter, and sells one-half of it at 3*s.* a bushel; at what rate must he sell the remainder so as to gain £25 altogether?

ARITHMETIC.

PART II.

XXVII. RECTANGULAR AREAS.

274. Carpets are now very often made rectangular in shape, and cover only the middle portion of the floor. Sometimes they are made in one piece, and sometimes they are made from a roll of definite width. In the latter case a length equal to the *length* of the area to be covered is cut off the roll; and similar lengths are cut off, laid alongside and attached, until the required width is attained. If it should happen that the width of the space to be covered is an exact multiple of the breadth of the carpet, there would be no waste; but otherwise, a portion of the last strip has to be cut off from end to end.

We will find the cost of a rectangular carpet, $10\frac{1}{2}$ ft. by $15\frac{1}{2}$ ft., made in this way from a roll 27 inches wide at 4*s.* 6*d.* per yard.

$$\text{Width of area} = \frac{10\frac{1}{2}}{2\frac{1}{4}} \times \text{breadth of carpet}$$
$$= \tfrac{42}{9} \times \text{breadth of carpet}$$
$$= 4\tfrac{2}{3} \times \text{breadth of carpet.}$$

Practically, therefore, the quantity of carpet required will be 5 strips, each of the same length as the area to be covered, *i.e.* $15\frac{1}{2}$ feet; and $\frac{1}{3}$ of the breadth of the fifth strip is cut off and wasted. Hence

$$\text{length of carpet cut from roll} = 5 \times 15\tfrac{1}{2} \text{ feet} = 77\tfrac{1}{2} \text{ feet};$$
$$\therefore \text{ the cost of the carpet} = 77\tfrac{1}{2} \times 1s.\ 6d. = £5.\ 16s.\ 3d.$$

It is clearly impossible to consider here any waste of material due to the requirements of the pattern.

NOTE.—The strips must be put together and laid down *lengthways*, not *across* the areas.

275. If, again, we wish to find how many square tiles would be required to pave a certain courtyard, we must remember that unless both the length and the width of the courtyard are exact multiples of the length of a tile, the work cannot be done with whole tiles only. Either one or two of the borders will consist of parts of tiles, and, in reckoning the number of tiles required for the courtyard, we must generally reckon each of these parts as a whole tile.

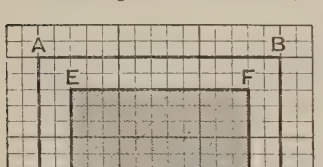
Example. *How many tiles, 6 inches square, will be required to pave an area 15 ft. 10 in. by 12 ft.?*

24 tiles placed side by side will make up the width, namely, 12 feet; but we shall need 32 tiles to make up the length. Hence the number of tiles required $= 24 \times 32 = \underline{768}$.

276. In the following examples we have to deal with the difference of two areas. In all such questions the pupil will find it of great assistance to draw a plan on squared paper. When the areas are squares, the formula $x^2 - y^2 = (x + y)(x - y)$ may be used with advantage.

Example i. *The length of a room is 15 ft. and the breadth 12 ft. A carpet is laid down so as to leave a margin 2 ft. wide all round it. Find the area of the carpet and the area of the margin.*

In the figure drawn below, ABCD represents the floor of the room and EFGH the carpet. A side of a small square represents 1 foot.



It will be clear that

EF = (15 − 2 − 2) ft. = 11 feet,

and FG = (12 − 2 − 2) ft. = 8 feet;

∴ area of EFGH = 11 × 8 sq. ft

= $\underline{88}$ sq. feet.

Area of room = 12 × 15 sq. ft.

= 180 sq. feet;

∴ area of margin = (180 − 88) sq. ft.

= $\underline{92}$ sq. feet.

Example ii. *A room, 20 feet square, is carpeted so as to leave a margin 2 feet wide all round. Find the area of the margin.*

The length of each side of the carpet is clearly 16 feet;

∴ area of margin $= (20^2 - 16^2)$ sq. ft. $= (20 + 16)(20 - 16)$ sq. ft. $= \underline{144}$ sq. feet.

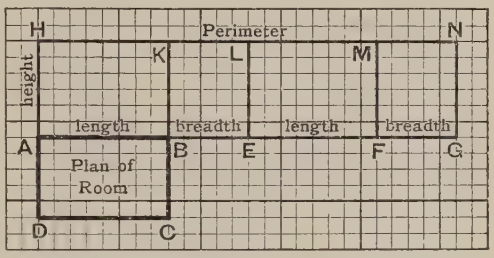
277. To find the area of a wall we multiply the length of the wall by the height. In the rooms which we consider here, we assume that the floor is rectangular and that there are 4 walls, those walls which are opposite to one another being equal.

We will suppose the length of a room to be 16 feet, the breadth, 10 feet, and the height, 12 feet.

In the figure in the margin we will take a side of a small square to represent 2 feet. ABCD then represents the floor of the room and AH the height; ABKH will represent the area of the wall whose lower edge is AB.

BE, EF and FG are equal respectively to BC, CD and DA. Consequently, BELK, EFML and FGNM represent the areas of the other three walls, and HAGN represents the area of all four walls.

Thus, the area of the four walls
= area of AGNH
= AG × AH
= 2(AB + BC) × AH
= 2 × 26 × 12 sq. ft.
= 624 sq. feet.



It will be evident that if l, b and h represent the number of units of length in the length, breadth and heights respectively, then

$$\text{the area of the four walls} = 2(l+b) \times h,$$
$$\text{the area of floor, or ceiling} = l \times b,$$
$$\text{and the distance round the room} = 2(l+b).$$

The distance round the room is called the **perimeter.**

Example i. *Find the area of the walls of a room* 12 *ft.* 6 *in. long,* 10 *ft.* 6 *in. wide and* 10 *ft. high.*

Perimeter $= 2(12\frac{1}{2} + 10\frac{1}{2})$ ft. $= 46$ ft.;
height $= 10$ ft.;
$\therefore$ area of walls $= \underline{460 \text{ sq. ft.}}$

Example ii. *If in the room in Ex.* i. *there is one door* 7 *ft. by* 4 *ft. and one window* 6 *ft. by* $3\frac{1}{2}$ *ft., what would it cost to paint the walls at* 3*d. per square foot?*

Area of walls $= 460$ sq. ft.,
area of door $= 7 \times 4$ sq. ft. $= 28$ sq. ft.,
area of window $= 6 \times 3\frac{1}{2}$ sq. ft. $= 21$ sq. ft.;
$\therefore$ area to be painted $= (460 - 28 - 21)$ sq. ft. $= 411$ sq. ft.;
$\therefore$ cost of painting $= 411 \times 3d. = 102s.\ 9d. = \underline{£5.\ 2s.\ 9d.}$

278. When we have to deal with the papering of a room, the circumstances to be considered are similar to those which we

explained when we dealt with the carpeting of a floor; but we have also to bear in mind the fact that wallpaper is not sold by the linear foot or yard, but in *pieces*, each piece being 12 yards long and of a certain width. If the area to be covered would require, for example, a length of 87 yards of paper, it would be necessary to reckon the quantity of paper required as 8 pieces, not $7\frac{1}{4}$ pieces.

Example. *Find the cost of papering a room* 12 *ft.* 6 *in. long,* 10 *ft.* 6 *in. wide and* 10 *ft. high, with paper* 2 *ft.* 6 *in. wide, at* 5*s. per piece of* 12 *yards.*

The perimeter of the room $= 2(12 \text{ ft. } 6 \text{ in.} + 10 \text{ ft. } 6 \text{ in.}) = 46 \text{ ft.}$;

$\therefore$ no. of strips of paper 10 feet long $= \frac{46}{2\frac{1}{2}} = \frac{92}{5} = 19$ (to next integer);

$\therefore$ length of paper required $= 190$ feet;

$\therefore$ no. of pieces required $= \frac{190}{36} = 6$ (to next integer);

$\therefore$ cost of paper $= 5s. \times 6 = 30s. = £1.\ 10s.$

NOTE.—We do not consider here any possible waste of material due to the requirements of the pattern.

EXAMPLES. 120.

1. Find how many sq. yards of carpet will be wanted to cover a floor 34 ft. 9 in. long and 26 ft. 6 in. wide? What will be the cost at 4*s.* 6*d.* per sq. yard?

2. What length of carpet, 2 ft. 6 in. wide, will be used in covering a floor whose area is 22 sq. yds. 7 sq. ft. 114 sq. in., if there is no waste?

3. How many yards of drugget, $5\frac{1}{4}$ ft. wide, would be needed to cover a floor 22 ft. 6 in. long and 16 ft. 4 in. wide?

4. What length of carpet, $\frac{3}{4}$ yd. wide, will be required in order to cover the floor of a room 12 ft. 4 in. long and 12 ft. 6 in. broad?

5. How many yards of carpet, 2 ft. wide, will be required to cover a room 8 yds. long and $7\frac{1}{4}$ yds. wide?

6. How many planks, each $13\frac{1}{2}$ ft. long and $10\frac{1}{2}$ in. wide, will be required for the construction of a platform 54 yds. long and 21 yds. broad? What will be the cost at $5\frac{1}{2}d.$ per sq. ft.?

7. Find the cost of carpeting a room 18 ft. 9 in. long and 17 ft. 6 in. broad with carpet, 2 ft. wide, at 4*s.* 9*d.* a yard.

8. Find the area of a room 17 ft. 4 in. long and 14 ft. 3 in. broad, and the cost of carpet to cover it, 1 ft. 9 in. broad, at 5*s.* $2\frac{1}{2}d.$ per yard.

9. An upholsterer covers a floor, 21 ft. 8 in. by 16 ft. 6 in., with carpet 27 in. wide. Find the cost at 3*s.* $4\frac{1}{2}d.$ per yard.

10. How much carpet will be wanted to cover the floor of a square room 17 ft. 6 in. long, the width of the carpet being 2 ft. 4 in.? Find the cost of the carpet at 3*s.* 9*d.* per yard.

11. Find the cost of carpeting the floor of a room, 16 ft. 8 in. long and 13 ft. 5 in. broad, leaving the hearthstone, which is 4 ft. 2 in. long and 1 ft. 8 in. broad, uncovered; the price of the carpet being 8*s.* per square yard.

12. Find the quantity of carpet required for the central portion of a room, this portion being 14 ft. 3 in. wide and 19 ft. long, and the carpet being $\frac{3}{4}$ yd. wide. Find also the cost at 4*s.* 6*d.* per yard. If between the carpet and each wall there is a distance of $2\frac{1}{2}$ ft., how much of the floor will be uncovered?

13. How many tiles, each 7 in. square, will be required to pave a courtyard 112 ft. by 84 ft.?

14. How many tiles, each 9 in. by 5 in., will be required to pave a courtyard 175 ft. by 93 ft.? How must they be laid?

15. On the floor of a square room a margin 4 ft. 6 in. wide is left uncarpeted. The cost of polishing this at 2*s.* a square yard is £2. 10*s.* Find the size of the room.

16. A field is measured to be 13 ch. 12 yds. by 9 ch. 15 yds. It is afterwards found that the first measurement is 2 yds. short and the second is 3 yds. too long; what is the error in the area?

17. In measuring the side of a square field, the side is taken 2 chains too long and the calculated area is consequently 36 ac. 4 sq. chains too large. What is the true area of the field?

18. A rectangular grass plot, 32 yds. 1 ft. long by 20 yds. 2 ft. 6 in. broad, is surrounded by a path, 10 ft. wide along the two longer sides, and 6 ft. 3 in. wide along the two shorter sides. Find the area of the path.

19. A man, in charge of a horse roller, 40 in. wide, walks by the side of the roller, while it rolls a square 10-acre field. How far does he walk? How far would he walk if the field were 1100 yds. long by 44 yds. wide, and were rolled in the direction of its length?

EXAMPLES. 121.

[*A piece of paper is* 12 *yards long.*]

1. If the length of a room is 24 ft., its breadth 18 ft. and its height 15 ft., what is the area of its 4 walls?

2. Find the area of the four walls of a room 24 ft. 9 in. long, 17 ft. 3 in. wide and 11 ft. 6 in. high; and find the cost of painting the walls at 5*d.* per sq. foot.

3. How many sq. yards are there in the walls of a room 30 ft. $6\frac{1}{4}$ in. long, 18 ft. $9\frac{3}{4}$ in. wide and 10 ft. 6 in. high? What will be the cost of painting them at 1*s.* $3\frac{1}{2}$*d.* per sq. yard?

4. How many stamps, 1 in. long and $\frac{4}{5}$ in. wide, will be required to paper a room 18 ft. 10 in. long, 16 ft. 9 in. wide and 9 ft. 6 in. high?

5. How many pieces of wall paper, 27 in. broad, will be required for a room 18 ft. long, 12 ft. broad and 11 ft. high?

6. How many pieces of wall paper, $22\frac{1}{2}$ in. wide, would be required for a room 18 ft. 9 in. long, 13 ft. 3 in. broad and 14 ft. 6 in. high?

7. How many sq. yds. are there in the walls of a room 8 yds. 2 ft. long, 3 yds. 1 ft. high and 5 yds. 1 ft. wide? How many pieces of paper, $\frac{3}{4}$ yd. wide, will be required to cover the walls? What will be the cost of the paper at 7*s.* 6*d.* per piece?

8. Find the area of the walls of a room 15 ft. long, 13 ft. broad and 10 ft. high. What length of paper 14 in. wide will be needed to cover the walls? What will it cost at 4*s.* 6*d.* per piece?

9. A room is 28 ft. 6 in. long, 18 ft. 9 in. wide and 12 ft high; how many pieces of paper 1 ft. 9 in. wide will be needed to cover the walls? Find the cost of the paper at 2*s.* 6*d.* per piece.

10. What is the area of the walls of a room 21 ft. 5 in. long, 18 ft. 7 in. wide and 10 ft. high? How many pieces of wall paper 2 ft. 1 in. wide would be required? What would it cost at 2*s.* 6*d.* per piece?

11. Find the cost of papering a room 5 yds. 1 ft. $2\frac{1}{2}$ in. by 5 yds. $3\frac{1}{2}$ in. and 4 yds. high, with paper 2 ft. wide, at $2\frac{1}{2}$*d.* a yd.

12. Find the cost of papering a room 37 ft. 3 in. long, 18 ft. 4 in. wide and 14 ft. high, with paper 2 ft. wide, at 6*d.* per yard.

13. Find the cost of papering a room 25 ft. long, 18 ft. 6 in. wide and 10 ft. high, with paper 2 ft. wide, at 3*s.* per piece.

14. How many pieces of paper, $\frac{3}{4}$ yd. broad, will be sufficient to paper a room 22 ft. 5 in. long, 12 ft. 1 in. broad and 11 ft. 3 in high? What will it cost at 4*s.* 6*d.* per piece?

15. What is the cost of papering a room 6 yds. 1 ft. 1 in. long, 6 yds. 4 in. broad and 12 ft. high, with paper 21 in. wide, at 3*s.* per piece?

16. A cistern 12 ft. 9 in. long, 8 ft. 3 in. broad and 6 ft. 6 in. deep is to be lined with lead. How many sq. feet of lead will be required? What will be the weight of the lead, if each sq. foot weigh 8 lb.? What will it cost at £1. 8*s.* per cwt.?

17. A room is a perfect cube, each wall being 15 ft. high and 15 ft. broad; there is a door 6 ft. high and 4 ft. wide, and a window 7 ft. high and 3 ft. wide. Find the cost of painting the walls at $2\frac{1}{2}d.$ per square yard.

18. If the cost of papering a room $8\frac{1}{4}$ yds. long and $6\frac{2}{3}$ yds. wide with paper, 2 ft. wide, at $4d.$ per yd., be £3, find the height of the room.

19. The walls of a room 21 ft. long, 15 ft. 9 in. wide and 11 ft. 8 in. high, are painted at an expense of £9. 12*s.* 6*d.* Find the cost per sq. ft., and the cost also of painting the ceiling.

20. Find the cost of carpeting and papering a room 32 ft. 6 in. long, 27 ft. 3 in. broad and 16 ft. high, the cost of carpet $\frac{3}{4}$ yd. wide being 4*s.* $4\frac{1}{2}d.$ per yd., and the cost of paper 2 ft. wide being $10\frac{1}{2}d.$ per yd.

21. A closed vessel is 8 ft. 3 in. long, 7 ft. 5 in. wide and 4 ft. 3 in. high, measured externally. What is the area of the external surface? If the material of the box be one inch thick, what is the area of the internal surface?

22. A cistern, without a lid, whose floor and walls are $1\frac{1}{2}$ in. thick, is 5 ft. 3 in. long, 3 ft. 7 in. wide and 2 ft. $5\frac{1}{2}$ in. high in its external dimensions. Find the area of its inside surface, and the cost of painting the inside at the rate of $4d.$ per sq. foot.

23. A box with a lid is made of planking $1\frac{1}{2}$ in. thick; if the external dimensions be 3 ft. 6 in., 2 ft. 6 in. and 1 ft. 9 in., find how many square feet of planking are used in the construction.

XXVIII. SQUARE ROOTS.

Prime Factors.

279. The **square root** of a given number is a number such that when it is multiplied by itself, the product is the given number.

We know, for example, that $3^2 = 9$; consequently, 9 is the *square* of 3, and 3 is the **square root** of 9. Generally, if $a^2 = x$, the *square root* of x is a. The square root of a number is denoted by the symbol $\sqrt{\ }$ placed in front of, and over, the number; thus,

$$\sqrt{9} = 3, \ \sqrt{16} = 4, \text{ etc.}$$

The student will find it convenient to make himself gradually acquainted with the following list, which he may easily verify by multiplication.

The square root of

1 is 1	36 is 6	121 is 11	256 is 16	441 is 21
4 ... 2	49 ... 7	144 ... 12	289 ... 17	484 ... 22
9 ... 3	64 ... 8	169 ... 13	324 ... 18	529 ... 23
16 ... 4	81 ... 9	196 ... 14	361 ... 19	576 ... 24
25 ... 5	100 ... 10	225 ... 15	400 ... 20	625 . . 25

280. When a number can be easily separated into prime factors its square root may often be determined by inspection.

Thus $7056 = 2^4 \times 3^2 \times 7^2,$

$$\therefore \sqrt{7056} = 2^2 \times 3 \times 7 = 84.$$

In a number which has an exact square root, *every prime factor which occurs at all must occur raised to an* **even** *power.*

Example. *What is the least number by which* 8085 *must be multiplied so that the product may be a perfect square?*

We have $8085 = 3 \times 5 \times 7^2 \times 11.$ (Art. 51.)

The least multiplier which will make this into a perfect square

$$= 3 \times 5 \times 11 = \underline{165}.$$

281. We know that $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$, and generally that the square of any vulgar fraction is a fraction whose numerator is the square of the given numerator, and denominator the square of the given denominator.

Conversely, the square root of $\frac{4}{9}$ is $\frac{2}{3}$; and generally, whenever the numerator and denominator are square numbers, the square root of the fraction is a fraction whose numerator is the square root of the original numerator, and denominator the square root of the original denominator. The fraction whose square root is required must first be reduced to its lowest terms.

Example. *Find the square root of* $\frac{49}{256}$.

$$\sqrt{\frac{49}{256}} = \frac{\sqrt{49}}{\sqrt{256}} = \frac{7}{16}.$$

EXAMPLES. 122.

Find, by factors, the square root of

1.	36.	2.	900.	3.	1764.	4.	7056.
5.	11025.	6.	17424.	7.	53361.	8.	63504.
9.	99225.	10.	122500.	11.	148225.	12.	245025.
13.	480249.	14.	571536.	15.	680625.		
16.	2480625.	17.	12446784.	18.	18593344.		
19.	22325625.	20.	300155625.	21.	5294745225.		

22. 54218191104.
23. 720673655625.
24. 1204971634944.
25. 4943396390625.
26. $\frac{121}{169}$.
27. $\frac{256}{841}$.
28. $\frac{7056}{9216}$.
29. $\frac{450}{2048}$.
30. $\frac{11025}{96004}$.
31. $1\frac{7}{9}$.
32. $39\frac{1}{16}$.
33. $210\frac{1}{4}$.
34. $37\frac{36}{49}$.
35. $2756\frac{1}{4}$.
36. $65\frac{64}{81}$.
37. $269\frac{44}{49}$.

Find the least multipliers which will make the following numbers into perfect squares:

38. 567. 39. 1573. 40. 3549. 41. 92950. 42. 26299.

General Method.

282. Since $\sqrt{100}=10$, and $\sqrt{10000}=100$, and $\sqrt{1000000}=1000$, etc., it follows that the square roots of all numbers between 100 and 10000 lie between 10 and 100, and consist of *two* digits: that the square roots of all numbers between 10000 and 1000000 lie between 100 and 1000, and consist of *three* digits, and so on. Hence we see that *two* additional digits in a number lead to *one* additional digit in the square root.

If therefore we mark off the digits of a number in periods of *two*, beginning at the right hand, the number of these periods will be the same as the number of digits in the square root. The left-hand period will sometimes however consist of only *one* digit.

283. Let us now consider the number 1444, obtained by multiplying 38 by 38. Its square root is 38.

In extracting the square root we determine first the number of *tens*, and then the number of *units*. If we write 38 in the form $30+8$, and multiply it by $30+8$ as in the margin, we see that

$$1444=30^2+(2\times30+8)\times8.$$

$$\begin{array}{rr} & 30+8 \\ & 30+8 \\ \hline & 30\times8+8^2 \\ 30^2+ & 30\times8 \\ \hline & 30^2+2\times30\times8+8^2 \end{array}$$

We can find the number of *tens* in the square root by ascertaining what multiple of 10 has its square *next less than* 1444: this is clearly **30**, for $40^2=1600$, and is too great.

Having found the *tens*, we next find the *units*. Subtracting 30^2 or 900 from 1444, the remainder is 544; hence $544=(2\times30+8)\times8$.

If we could divide 544 by $2\times30+8$, we should get the number of units **8** at once. But since the divisor itself involves this unascertained number **8**, the plan adopted is to use the part 2×30 as a **trial** divisor, to find by means of it a **trial** quotient, and then to see whether

$$(2\times30+\text{trial quotient})\times\text{trial quotient}=544.$$

The first trial quotient may not prove correct; if it is too great, we try the number next less, and so on.

In this particular case, if we divide 544 by 2×30, *i.e.* by 60, we get a trial quotient **9**. But $(60+9)\times9=621$, and is too great; we therefore try **8**,

and we find that $(60+8)\times 8=544$. The digits of the square root 38 are thus found in succession.

The operations described above may be stated concisely thus:

	14,44 | 30+8,		or thus		3 8
	900				14,44
60+8	544			68	544
	544				544

The latter is the practical form, and the process is briefly as follows:

(i) We mark off the digits in twos, beginning at the right hand.

(ii) The greatest square in 14 is 9, and its square root is 3. We place 3 in the root place. We multiply 3 by 3 and subtract the product from 14. The remainder is 5.

(iii) To this remainder we annex the period 44, and our dividend becomes 544. Twice the root digit 3 is 6, so we put 6 in the divisor.

Instead of 60 we take 6 as a *trial divisor*, and we take 54 as a *trial dividend*. Proceeding as explained above, we find that 8 is the true quotient. We annex 8 to the 3 in the root place, and also to the divisor. We multiply 68 by 8 and, subtracting the product from 544, we get no remainder. The square root is thus 38.

284. Treating a number of 3 digits, 238 for example, in a similar way, we find that

$$238^2=200^2+(2\times 200+30)\times 30+(2\times 200+2\times 30+8)\times 8.$$

Also $238^2=56644$. Hence to find the square root of 56644 the method is similar to that in Art. 283.

The work necessary in this calculation may be written thus:

	5,66,44 | 200+30+8;		or thus		2 3 8
	40000				5,66,44
400+30	16644				4
	12900			43	166
460+8	3744				129
	3744			468	3744
					3744

The latter is the practical form, and may be described thus:

(i) The integer next less than the root of the first period 5 is 2. The square of 2 is 4. Subtracting 4 from 5 the remainder is 1. We bring down the second period, and we get 166 for the next dividend.

(ii) The *trial* divisor is 2×2 or 4, and the *trial* dividend is 16. The true quotient is 3. We place 3 as the second digit in the root, and we also annex 3 to the trial divisor.

(iii) In the next step the *trial* divisor is 46, obtained by doubling the 23 in the root; and the *trial* dividend is 374.

285. The methods of extracting the square root of a number are based upon the formula

$$(a+b)^2=a^2+2\times a\times b+b^2=a^2+(2\times a+b)\times b,$$

This seems inconsistent with the next

and the more extended formula

$$(e+d+c+b+a)^2 = e^2 + (2e+d)\times d + \{2(e+d)+c\}\times c + \{2(e+d+c)+b\}\times b + \{2(e+d+c+b)+a\}\times a;$$

which leads directly to the method described above, supposing that a, b, c, d, e stand for the units, tens, hundreds, etc., respectively.

The following example, with the aid of Arts. 283 and 284, will explain the application of the principle. We will consider the square root of 3886899025.

$e + d + c + b + a$

$60000 + 2000 + 300 + 40 + 5$

	3886899025
$e^2 =$	3600000000
	286899025
Trial divisor $= 2e = 120000$	
Trial quotient $= d =$ 2000	
$(2e+d)\times d = 122000\times 2000 =$	244000000
	42899025
Trial divisor $= 2(e+d) = 124000$	
Trial quotient $= c =$ 300	
$(2e+2d+c)\times c = 124300\times 300 =$	37290000
	5609025
Trial divisor $= 2(e+d+c) = 124600$	
Trial quotient $= b =$ 40	
$(2e+2d+2c+b)\times b = 124640\times 40 =$	4985600
	623425
Trial divisor $= 2(e+d+c+b) = 124680$	
Trial quotient $= a =$ 5	
$(2e+2d+2c+2b+a)\times a = 124685\times 5 =$	623425

Usual form.

	6 2 3 4 5
6	38,86,89,90,25
	36
122	286
	244
1243	4289
	3729
12464	56090
	49856
124685	623425
	623425

Example 1. *Extract the square root of* 10588516.

Sq. root = 3 2 5 4

3	10,58,85,16
	9
62	158
	124
645	3485
	3225
6504	26016
	26016

There are 4 periods, and therefore 4 digits in the square root.

We find the digits 3, 2, 5, 4 successively, the *trial* divisors being 6, 64, 650, and the trial dividends 15, 348, 2601.

To the successive remainders, as before, we annex the periods in turn.

The required square root = 3254.

Example ii. *Extract the square root of* 674541903025.

Sq. root = 8 2 1 3 0 5

8	67,45,41,90,30,25
	64
162	345
	324
1641	2141
	1641
16423	50090
	49269
1642605	8213025
	8213025

When the digits 8213 have been obtained as in Example i., the *trial* divisor is 16426, and the *trial* dividend is 8213; the former is greater than the latter, so we put the digit 0 in the root and *also at the end of the divisor*, which becomes 164260. We then bring down the next period 25 and proceed as before.

The required square root is 821305.

Example iii. *Extract the square root of* 4160580062500.

Sq. root = 2 0 3 9 7 5 0

2	4,16,05,80,06,25,00
403	1605
4069	39680
40787	305906
407945	2039725

We take account of the final period 00, by putting the digit 0 at the end of the root; and we do this again for each period 00 at the end of a number. We may write down the remainders only as in the Italian method of long division.

The required square root is 2039750.

EXAMPLES. 123.

Find the square root of

1.	361.	2.	529.	3.	1681.	4.	3481.
5.	6889.	6.	8649.	7.	14161.	8.	22201.
9.	22801.	10.	64009.	11.	146689.	12.	226576.
13.	249001.	14.	253009.	15.	375769.		
16.	546121.	17.	1261129.	18.	1121481.		
19.	14837904.	20.	39740416.	21.	22099401.		
22.	64432729.	23.	72471169.	24.	182493081		
25.	930677049.	26.	373301041.	27.	893830609.		
28.	3915380329.	29.	3914879761.	30.	1383542416.		
31.	103797374976.	32.	10074538384.	33.	17167550625.		

Square Root of a Decimal.

286. It follows from the nature of decimals that the method just explained is applicable to pure and to mixed decimals in

exactly the same way as to integers. One or two points however must be noticed: (i) We mark off the periods not from the *end* of the decimal, but *from the decimal point* both ways, right and left; thus: 2,17,53,21,·54,28,4. (ii) We put a decimal point in the square root immediately before the digit obtained from using the first period in the decimal part. (iii) If the given decimal has an *odd* number of decimal places, we annex a zero in order to make an exact number of pairs; and we may then annex as many additional pairs of zeros as we please. (iv) If the square root of an integer does not come out exactly, we may approximate to it by annexing a decimal point and zeros, and then continuing the process.

Example i. *Find the square root of* 1127·6164.

Sq. root = 3 3 · 5 8

3	11,27,·61,64
	9
63	227
	189
665	3861
	3325
6708	53664
	53664

Before we bring down the period 61, which comes next after the decimal point, we put a decimal point in the root.

The required square root is 33·58.

Example ii. *Find the square root of* ·000625.

Sq. root = ·0 2 5

2	·00,06,25
	4
45	225
	225

The first period ·00 gives ·0 in the root, and the second 06 gives the digit 2.

The square root is ·025.

Example iii. *Find the square root of* 49·434961.

Sq. root = 7 ·0 3 1

7	49,·43,49,61
	49
1403	4349
	4209
14061	14061
	14061

The square root is 7·031.

Example iv. *Find the square root of* 321·73025.

Sq. root = 1 7 ·9 3 6 8

1	3,21,·73,02,50,00,00
27	221
349	3273
3583	13202
35866	245350
358728	3015400
	145576

We annex a 0 to make the number of decimal places even.

When we have brought down the last period we find that there is still a remainder.

We may then annex pairs of zeros, and continue the process as far as we please.

The square root is 17·9368....

We may notice here that if a decimal, in its simplest form, has an *odd* number of places, its square root cannot be obtained exactly.

EXAMPLES. 124.

Find the square root of

1.	1·96.	2.	3·24.	3.	86·49.
4.	1·4161.	5.	28·8369.	6.	331·24.
7.	12·0409.	8.	1015·6969.	9.	170·3025.
10.	121·2201.	11.	8·678916.	12.	100·861849.
13.	144·600625.	14.	8264·446281.	15.	676·208016.
16.	26·04469156.	17.	2·94328336.	18.	4·06506244.
19.	71·9224421041.	20.	206·5316768641.	21.	25·3130740641.
22.	·0043046721.	23.	·00000961.	24.	·00005476.
25.	·00010201.	26.	·00047961.	27.	·0003272481.
28.	·0003418801.	29.	·0000001849.	30.	·0000005184.

Applications of Square Root.

287. Example i. *Find the number of yards in the side of a square park containing* 109 *ac.* 8 *sq. ch.* 9 *sq. yds.*

We first reduce the area to sq. yds.

109 ac. 8 sq. ch. 9 sq. yds.
10
1098 sq. ch.
484
4392
8784
4401 (adding the 9)
531441 sq. yds.

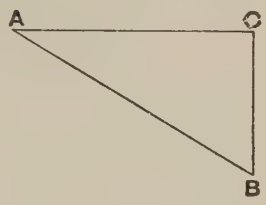
Sq. root = 7 2 9

7	53,14,41
	49
142	414
	284
1449	13041
	13041

Area of park = 531441 sq. yds ;

∴ length of side = $\sqrt{531441}$ yds. = 729 yards.

Example ii. *Find the diagonal of a rectangular field* 80 *yds. long and* 60 *yds. wide.*



If in the triangle ACB the angle ACB be a right angle, we know that the square described upon AB is equal to the sum of the squares described upon AC and BC. Hence

the diagonal $= \sqrt{60^2 + 80^2}$ yds.
$= \sqrt{3600 + 6400}$ yds.
$= \sqrt{10000}$ yds.
$= 100$ yds.

Example iii. *The area of a room is* 56 *sq. yds.* 8 *sq. ft., and the room is twice as long as it is broad. Find the length of the room.*

Let x = the number of feet in the breadth,
then $2x$ = ... length;
$\therefore$ $2x^2$ = the number of sq. feet in the area;
$\therefore$ $2x^2 = 512$;
$\therefore$ $x^2 = 256$;
$\therefore$ $x = 16$.

Hence the breadth is 16 ft. and the length is 32 ft.

EXAMPLES. 125.

1. A certain number of men spent £361, each spending as many pounds as there were men; how many men were there?

2. A square pavement contains 20736 square stones, all of the same size; how many rows of stones are there, and how many stones are there in each row?

3. The area of a square is 2 Ha. 25 a.; find the length of a side in metres.

4. The area of a square is 40 ac.; find the length of a side.

5. A square contains $122\frac{1}{2}$ ac.; find the length of a side.

6. The area of a square field is 9 Ha. 36 a. 36 ca.; find the length of a side in metres.

7. Find the side of a square which is equal in area to a rectangle whose sides are 12·32 m. and 6·93 m.

8. Find the length of the diagonal of a rectangular field whose sides are 924 ft. and 385 ft.

9. A square field whose area is 46656 sq. m. is surrounded by a moat whose area is 26 a. 28 ca. Find the breadth of the moat.

10. How many yards of fencing are required to enclose a square park containing 832 ac. $3176\frac{1}{4}$ sq. yards?

11. Find the length of the side of a square which is equal in area to a rectangle, whose sides are 513 yds. 1 ft. 11 in. and 1628 yds. 11 in.

12. How long will it take a man to walk round a square field whose area is $5\frac{5}{8}$ acres, at the rate of a mile in $10\frac{2}{3}$ minutes?

13. The area of a rectangular field is 30479·805 sq. ft. Its length is twice its breadth. Find its length.

14. The area of a rectangular field is 975744 sq. ft.; one side is $3\frac{1}{2}$ times as long as the other. Find the length of each.

15. A room is half as long again as it is broad, and its area is $412\frac{49}{98}$ sq. yds. Find its length and breadth.

16. It is required to carpet 167·71545025 sq. ft. Find the breadth of the carpet, (i) when square; (ii) when the length is $6\frac{1}{4}$ times the breadth.

17. The diagonal of a square court is 90 ft. Find the area.

18. A map is drawn to a scale of $\frac{1}{1000000}$; find, in centimetres, the side of a square which will represent 166464 sq. kilometres on the map.

19. The foot of a ladder is 7 feet away from a vertical wall, and the height of the top of the ladder, which rests against the wall, is 24 feet. Find the length of the ladder.

20. The height of a wall is 2·64 m. and the breadth of a ditch at its foot is 14·4 m. Find the length of the plank which will just reach from the top of the wall to the other side of the ditch.

21. The perimeter of one square field is 571·2 metres and the perimeter of another is 88·4 metres. Find the perimeter of a square field whose area is equal to the sum of these two.

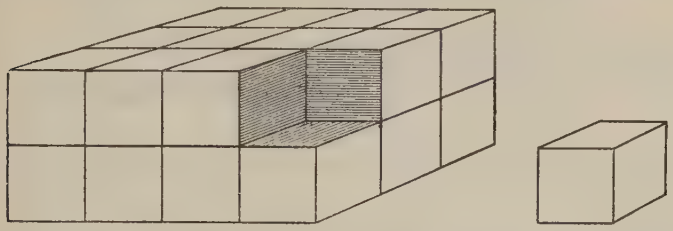
22. A ladder is 24·86 m. long. If its foot be placed 3·3 m. from the foot of a wall, how far up the wall will it reach?

23. A room, three times as long as it is broad, is carpeted at 4*s.* 6*d.* a sq. yd., and the walls are coloured at 9*d.* a sq. yd., the respective costs being £8. 5*s.* $4\frac{1}{2}$*d.* and 4 guineas. Find the dimensions of the room.

XXIX. VOLUME OR CUBICAL CONTENT.

288. If a rectangular solid be 4 feet long, 3 feet wide and 2 feet high, we can divide the edges respectively into 4, 3 and 2 equal parts, each of which parts will be 1 foot: and if we draw planes through the points of division as in the diagram, the solid will be separated into a number of smaller blocks each of which

is a *cubic foot*: and since there are 2 layers, in each of which there are 3×4 blocks, we see that there are $2 \times 3 \times 4$ blocks altogether, and that the solid therefore contains $2 \times 3 \times 4$ cubic feet.



Thus we can find the **volume** or **cubical content** of any rectangular solid by expressing the length, breadth and height in terms of a common unit of measurement, and multiplying the three numbers together. The product will give the volume of the solid in terms of the *corresponding* unit in cubic measure.

If we use the words volume, length, breadth and height, or the letters V, l, b and h, to denote the *number of units* in each respectively, the relation between them may be stated thus:

$$\text{Volume} = \text{length} \times \text{breadth} \times \text{height}, \text{ or } V = l \times b \times h;$$

$$\therefore \text{ length} = \frac{\text{volume}}{\text{breadth} \times \text{height}}, \text{ or } l = \frac{V}{b \times h};$$

$$\text{breadth} = \frac{\text{volume}}{\text{length} \times \text{height}}, \text{ or } b = \frac{V}{l \times h};$$

and

$$\text{height} = \frac{\text{volume}}{\text{length} \times \text{breadth}}, \text{ or } h = \frac{V}{l \times b};$$

hence, if any three are known, the fourth can be determined.

If A denote the number of units of area in the base, expressed in the **corresponding square** measure, then

$$A = l \times b; \quad \therefore \ V = h \times A, \ A = \frac{V}{h} \text{ and } h = \frac{V}{A}.$$

Example i. *Find the cubic content of a box* 1 *m.* 75 *cm. long,* 94 *cm. wide and* 84 *cm. deep.*

Length = 1·75 m.,
breadth = ·94 m.,
depth = ·84 m.;
∴ content = 1·75 × ·94 × ·84 c.m.
= 1·3818 c.m.

```
1·75
 0·94
1·575
 ·0700
1·645
 0·84
1·3160
 6580
1·3818
```

Example ii. *Find the cubic capacity of a rectangular block of wood* 16 *ft.* 6 *in. long,* 9 *in. wide and* 4 *in. thick.*

Length $= 16\frac{1}{2}$ ft., width $= \frac{3}{4}$ ft., thickness $= \frac{1}{3}$ ft. ;

$\therefore$ cubic capacity $= \frac{33}{2} \times \frac{3}{4} \times \frac{1}{3}$ cu. ft. $= \frac{33}{8}$ cu. ft. $= \underline{4\frac{1}{8} \text{ cu. ft.}}$

Example iii. *Find the breadth of a rectangular solid containing* $3\frac{1}{2}$ *cu. feet ; the length being* 5 *ft.* 4 *in. and the height* 9 *in.*

Content $= 3\frac{1}{2}$ cu. ft., breadth $= 5\frac{1}{3}$ ft., height $= \frac{3}{4}$ ft. ;

$\therefore$ length $= \dfrac{3\frac{1}{2}}{5\frac{1}{3} \times \frac{3}{4}}$ ft. $= \frac{7}{2} \times \frac{3}{16} \times \frac{4}{3}$ ft. $= \frac{7}{8}$ ft. $= \underline{10\frac{1}{2} \text{ inches.}}$

Example iv. *The area of one face of a cube is* 92·16 *sq. cm. ; find its volume.*

Let $x =$ no. of centimetres in one edge of cube.

Then $x^2 = 92{\cdot}16$;

$\therefore\ x = 9{\cdot}6$;

$\therefore$ no. of c.cm. in volume $= x \times x \times x = x^3 = (9{\cdot}6)^3$

$= 92{\cdot}16 \times 9{\cdot}6$;

$\therefore$ volume $= 884{\cdot}736$ c.cm.

$= \underline{884 \text{ c.cm. } 736 \text{ c.mm.}}$

92·16
9·6
829·44
55·296
884·736

Rough Approximation.—Answer lies between 92×9 and 93×10 c.cm., *i.e.* between 828 and 930 c.cm.

Example v. *The total rainfall at Lonaon for* 1903 *was* 37 *inches. Find, to the nearest ton, the weight of water which fell on an acre : given that a cubic foot of water weighs* 1000 *oz.*

Area considered $= 4840 \times 9$ sq. ft. ;

$\therefore$ volume of water $= 4840 \times 9 \times \frac{37}{12}$ cu. ft.

$= 1210 \times 3 \times 37$ cu. ft.

$= 134310$ cu. ft. ;

$\therefore$ weight of water $= 134310 \times 1000$ oz.

$= \frac{134310 \times 1000}{16 \times 224}$ tons

$= \underline{3747 \text{ tons.}}$

3747·4...
3584) 13431000
26790
17020
2684
175
32

Example vi. *Find the weight of water contained in a tank* 9·25 *m. long,* 5·4 *m. broad and* 275 *cm. deep.*

Rough Approximation ;

$90 \times 50 \times 20 = 90000$, and $100 \times 60 \times 30 = 180000$;

$\therefore$ the result must lie between 90 t. and 180 t.

Length $= 92{\cdot}5$ dm.,

breadth $= 54$ dm.,

depth $= 27{\cdot}5$ dm. ;

$\therefore$ volume $= 92{\cdot}5 \times 54 \times 27{\cdot}5$ c.dm.

$= 137362{\cdot}5$ litres ;

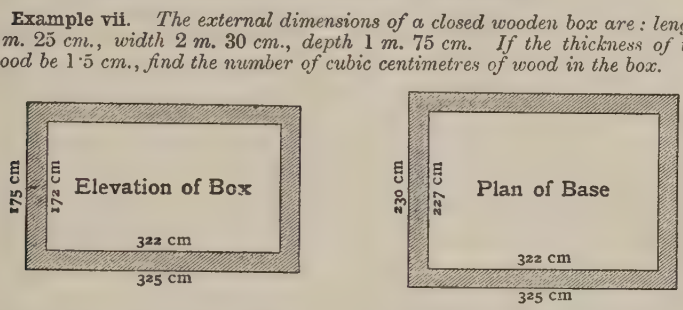
$\therefore$ weight $= 137362{\cdot}5$ Kg.

$= \underline{137 \text{ t. } 362{\cdot}5 \text{ Kg.}}$

92·5
5 4
4625
370·0
4995
27·5
9990
24965
2497·5
137362·5

NOTE.—If the volume is to be found in **litres**, *i.e.* in cubic decimetres, the linear measurements must be expressed in **decimetres**.

Example vii. *The external dimensions of a closed wooden box are: length* 3 *m.* 25 *cm., width* 2 *m.* 30 *cm., depth* 1 *m.* 75 *cm. If the thickness of the wood be* 1·5 *cm., find the number of cubic centimetres of wood in the box.*



The figures represent the plan and elevation of the box. It will thence be seen that the internal dimensions of the box are:

Length = 325 cm. − 2 × 1·5 cm. = 322 cm.,

breadth = 230 cm. − 2 × 1·5 cm. = 227 cm.,

and depth = 175 cm. − 2 × 1·5 cm. = 172 cm.

Thus external volume = 325 × 230 × 175 c.cm. = 13081250 c.cm.,

and internal volume = 322 × 227 × 172 c.cm. = 12572168 c.cm;

∴ volume of wood = 13081250 c.cm. − 12572168 c.cm.

= 509082 c.cm.

EXAMPLES. 126.

1. Find the cubic content of boxes of the following dimensions:

(i)	Length,	5 feet,	width,	3 feet,	depth,	2 feet;
(ii)	,,	7 ft.,	,,	4 ft.,	,,	3 ft.;
(iii)	,,	75 cm.,	,,	24 cm.,	,,	35 cm.;
(iv)	,,	$6\frac{1}{2}$ ft.,	,,	$2\frac{3}{4}$ ft.,	,,	$3\frac{1}{4}$ ft.;
(v)	,,	2·345 m.,	,,	1·64 m.,	,,	·85 m.;
(vi)	,,	3 ft. 4 in.,	,,	2 ft. 5 in.,	,,	1 ft. 7 in.;
(vii)	,,	3·055 m.,	,,	2·15 m.,	,,	1·62 m.;
(viii)	,,	10 ft. 9 in.,	,,	6 ft. 4 in.,	,,	4 ft. 2 in.

2. A block of stone is 4 ft. long, $2\frac{1}{2}$ ft. broad and $1\frac{1}{4}$ ft. thick; how many cu. feet does it contain?

3. Find the height of a room which is 12 ft. long, 10 ft. wide and contains 1440 cu. ft. of air.

4. Find the content of a wooden beam 9·15 m. long, ·64 m. wide and 54 mm. thick.

5. How many cubic metres are there in a regular block 8·5 m. long, 64 dm. wide and 4 m. 24 cm. deep?

6. A tank is 30 ft. 9 in. long, 16 ft. 7 in. wide and 6 ft. 4 in. deep; find how much water it will hold in cu. feet and inches.

7. Find the number of cu. feet in a room 14 ft. 7 in. long, 12 ft. 5 in. wide and 11 ft. 3 in. high.

8. How many cu. feet of brickwork are there in a wall 108 yds. long, 3 ft. 8 in. high and 14 in. thick?

9. How many bricks, each 9 in. by $4\frac{1}{2}$ in. by 3 in., are there in a stack 36 ft. long, 9 ft. wide and 12 ft. high?

10. How many bricks, each 9 in. long, $4\frac{1}{2}$ in. wide and 3 in. deep, will be needed for a wall 25 yds. long, 15 ft. high and 1 ft. $10\frac{1}{2}$ in. thick?

11. A room 42 ft. by 20 ft. accommodates 105 children. What must be the height of the room if each child has $83\frac{1}{3}$ cu. ft. of air?

12. A rectangular cistern 2 yds. 2 ft. long and 2 yds. wide contains a depth of 4 ft. of water. How many cu. feet of water are there? If a cu. foot of water weighs 1000 oz., find the weight of the water in the cistern.

13. Find the weight of water in a tank, 10·25 m. long, 4·6 m. wide and 215 cm. deep.

14. What is the value of a block 5 ft. 3 in. long, 2 ft. 4 in wide and 1 ft. 2 in. thick, at 4 guineas per cu. foot?

15. Find the value of a balk of timber, 39 ft. 6 in. long and 3 ft. 7 in. thick each way, at 2*s.* 6*d.* per cu. foot.

16. A cu. foot of water weighs $62\frac{1}{2}$ lb., and a room 18 ft. 9 in. by 13 ft. 4 in. is flooded to a depth of 2 in. What is the weight of water in the room?

17. Find the weight of a block of marble 9 ft. 6 in. long, 2 ft. 3 in. broad and 2 ft. thick, supposing the weight of a cu. foot of marble to be 2716 oz.

18. If 17 cu. ft. 594 cu. in. of oak are required to floor a room 13 ft. 6 in. long and 12 ft. 4 in. wide, what is the thickness of the boards?

19. A rectangular cistern, $13\frac{3}{5}$ ft. long and 6 ft. broad, contains $294\frac{1}{4}$ cu. ft. of water; what is the depth of the water?

20. If a pint of water weighs $1\frac{1}{4}$ lb. and a cu. foot weighs 1000 oz., find how many gallons a cistern 3 ft. 4 in. long, 2 ft. 8 in. broad and 2 ft. 3 in. deep will hold.

21. A closed vessel is 8 ft. 3 in. long, 7 ft. 5 in. wide and 4 ft. 3 in. high, measured externally. If the material be 1 in. thick, how many cu. feet will the vessel contain?

22. The rainfall on a certain day was 1·35 inches; find, to the nearest ton, the weight of water which fell on a square mile.

23. The rainfall on a certain day was 3·45 cm.; find the weight of water which fell on a hectare.

24. A tank contains 993 Hl. 30 l. of water. The area of the base is 376·25 sq. m. Find the depth of the water.

25. The area of the face of a cube is 128 sq. m. 18 sq. dm. 81 sq. cm.: find the volume to the nearest cubic metre.

26. The water in a tank is 135 cm. deep. After 68 Hl. 46 l. have been added, the depth of the water is found to be 1·5 m. How many hectolitres did the tank contain at first?

27. If iron weighs 7·8 times as much as the same volume of water, find, correct to the nearest tenth of an inch, the length of a bar of iron, 2 ft. 4 in. by 1 ft. 8 in., which weighs 18 tons.

28. Find, to the nearest centimetre, the depth of a tank, 7·15 metres long and 3·24 metres wide, which holds 160 hectolitres.

29. Find the number of cubic centimetres of wood in a closed box whose external dimensions are 2 m. 56 cm. by 1 m. 32 cm. by 105 cm., the thickness of the wood being 1·25 cm.

30. A cubic metre of iron weighs 8000 Kg. It is rolled into a bar, of square cross section, 16 metres long. The length of the bar is then divided into equal portions, so that each part cut off is a perfect cube. How many of these cubes are there? How many kilograms does each weigh?

31. How many rods of reduced brickwork are there in a wall a mile long, 8 feet high and 9 in. thick? (Answer to nearest integer.)

XXX. CUBE ROOT BY FACTORS, AND BY INSPECTION.

289. The **cube root** of a given number is that number which when raised to the *third* power will produce the given number.

Thus $4^3 = 64$; hence 64 is the *cube* of 4, and 4 is the *cube root* of 64. Generally, if $a^3 = x$, the *cube root* of x is a.

The symbol $\sqrt[3]{\ }$ is used to denote the cube root of a number; thus $\sqrt[3]{64} = 4$ and $\sqrt[3]{125} = 5$.

The following table will be found useful.

The cube root of			
1 is 1	64 is 4	343 is 7	1000 is 10
8 ... 2	125 ... 5	512 ... 8	1331 ... 11
27 ... 3	216 ... 6	729 ... 9	1728 ... 12

290. If a number can be easily separated into prime factors, its cube root may be determined by inspection. Thus

$$\sqrt[3]{5832} = \sqrt[3]{2^3 \times 3^6} = 2 \times 3^2 = 18.$$

In every number whose cube root can be exactly determined *every prime factor which occurs at all must occur raised to a power which is a multiple of* 3.

291. If the numerator and the denominator of a fraction are cubes, its cube root is a fraction, which has for its numerator the cube root of the original numerator, and for its denominator the cube root of the original denominator. For example,

$$\sqrt[3]{\tfrac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \tfrac{2}{3}.$$

The cube root of a mixed number may sometimes be found in this manner by first expressing the number as an improper fraction. For example,

$$\sqrt[3]{2\tfrac{10}{27}} = \sqrt[3]{\tfrac{64}{27}} = \tfrac{4}{3} = 1\tfrac{1}{3}.$$

292. When a number of not more than 6 digits is known to be a perfect cube, its cube root can be determined by inspection. For $1000 = 10^3$ and 1000000 is 100^3; hence the cube root of any number of 4, 5 or 6 digits must lie between 10 and 100, and must therefore be expressed by 2 digits only.

From the table of cube roots it will be seen that if a perfect cube ends in 1, 2, 3, 4, 5, 6, 7, 8, 9, its cube root ends in 1, 8, 7, 4, 5, 6, 3, 2, 9, respectively; so that the right-hand digit of the cube root can never be mistaken.

The left-hand digit also can always be determined by striking off the last three digits of the number and ascertaining the greatest digit which, when cubed, will be less than the number formed by the 1, 2 or 3 figures which remain. For example, since $27000 = 30^3$ and $64000 = 40^3$, it follows that any number which lies between 27000 and 64000 has a cube root which lies between 30 and 40, and therefore has 3 for its left-hand digit.

Example 1. *Find the cube root of* 4913.

It will be seen from the table that the right-hand digit of the cube root is 7. The cube root of 4 lies between 1 and 2. The left-hand digit required is therefore 1. Hence *if* 4913 *has a cube root*, that cube root is 17.

By multiplication we find that $17 \times 17 \times 17 = 4913$. Hence 17 is the cube root required.

Example ii. *Find the cube root of* 704969.

We see from the table that the right-hand digit of the cube root is **9**. And since 704 lies between 512 and 729, the left-hand digit of the cube root must be **8**. Thus the cube root of 704969, *if there is one*, must be **89**.

By multiplication we find that $89^3 = 704969$. Hence 89 is the cube root required.

EXAMPLES. 127.

Find, by factors or by inspection, the cube root of

1.	216.	**2.**	1728.	**3.**	2744.
4.	3375.	**5.**	9261.	**6.**	10648.
7.	13824.	**8.**	35937.	**9.**	91125.
10.	166375.	**11.**	216000.	**12.**	456533.
13.	1157625.	**14.**	3652264.	**15.**	4492125.
16.	5359375.	**17.**	11390625.	**18.**	3456649728.
19.	114154707051.			**20.**	106712258605568.
21.	$3\frac{3}{8}$.	**22.**	$\frac{189}{875}$.	**23.**	$42\frac{7}{8}$.
24.	$49\frac{8}{27}$.	**25.**	$6\frac{539}{729}$.	**26.**	$30\frac{265}{512}$.
27.	$4\frac{11191}{27744}$.	**28.**	$2\frac{13371}{27744}$.	**29.**	$\frac{136}{12393}$.
30.	$\frac{68921}{593119}$.	**31.**	$405\frac{28}{125}$.	**32.**	$2326\frac{13}{64}$.
33.	$\frac{185193}{493039}$.	**34.**	$3460\frac{41}{512}$.	**35.**	$\frac{28372625}{87528384}$.
36.	$714516\frac{123}{125}$.	**37.**	$423987\frac{457}{512}$.	**38.**	$1277289\frac{1}{8}$.

Find the least multipliers which will make the following numbers perfect cubes:

39. 968. **40.** 11979. **41.** 49392. **42.** 314928.

43. A cubical tank holds 1331 cu. yards: how deep is it?

44. A cube contains $2326\frac{13}{64}$ cu. in.; what is the length of an edge and the area of a face?

45. A cube contains 103823 cu. in.; what is the sum of the areas of its faces?

46. A cubical cistern holds 3430 Hl.: how many square metres are there in the base?

47. The weight of water in a cubical tank is 5639 t. 752 Kg. Find the dimensions of the tank.

48. Three cubes, the areas of whose faces are 36 sq. cm, 64 sq. cm. and 100 sq. cm., are melted down and recast into a single cube. Find the length of the edge of the new cube.

49. A closed cubical box whose external volume is 41 c.m. 63 c.dm. 625 c.cm. holds 393 Hl. 4 l. when full of water Find the thickness of the material of which the box is made.

XXXI. PRACTICE.

(a). Simple Practice.

293. An **aliquot** part of a quantity is such that if we take it a certain integral number of times we make up the quantity. Its relation to the whole is expressed by a fraction which has unity for its numerator and an integer for its denominator.

For example, 2*s.* 6*d.* is an *aliquot* part, namely $\frac{1}{8}$, of £1; and 28 lb. is an *aliquot* part, namely $\frac{1}{4}$, of 1 cwt.

294. **Practice** is a method of calculating *by addition of aliquot parts* the value of a simple or compound quantity when the value of a unit of one denomination is given. The method is *simple* or *compound* according as the quantity is simple or compound, and will easily be understood from the following examples.

295. We will first consider a few examples of simple practice.

Example i. *Find the value of* 25 *things at* £2. 10*s. each.*

We find the value of 25 things at £2. 10*s.* each by adding their value at 10*s.* each to their value at £2 each; 10*s.* being an aliquot part of £2. The work appears thus:

	£	*s.*	*d.*	
	25	0	0	= value at £1 each.
			2	
	50	0	0	= ..... £2 .....
10*s.* = $\frac{1}{2}$ of £1, ∴	12	10	0	= ..... 10*s.* .....
∴	62	10	0	= value at £2. 10*s.* each.

This result may be verified by multiplying £2. 10*s.* by 25.

It is generally best to work in decimals of a pound, retaining 5 places in order to obtain the result correct to 3 places. We express the pounds and shillings as pounds and the decimal of a pound; we then multiply the number of things by this decimal, and we take parts for the *pence* only.

Example ii. *Find the value of* 243 *things at* £4. 14*s.* 8*d. each.*

We see that £4. 14*s.* = £4·7.

	£.	
	243	= value at £1 each.
	4·7	
	972	= ..... £4 .....
	170·1	= ..... 14*s.* .....
8*d.* = $\frac{1}{30}$ of £1, ∴	8·1	= ..... 8*d.* .....
Value required =	£1150·2	= £1150. 4*s.*

Rough approximation. The true value lies between £4 × 243 and £5 × 243, *i.e.* between £972 and £1215.

NOTE.—A beginner may find the process easier to understand if he adds the lines 972 and 170·1, obtained from the multiplication, before he proceeds to take parts.

Example iii. *Find the value of* 5641 *things at* £75. 17*s.* 4½*d.* each.

We see that £75. 17*s.* = £75·85.

	£.	
	5641	= value at £1 each
	75·85	
	394870	= ..... £70 .....
	28205	= ..... £5 .....
	4512·8	= ..... 17*s.* .....
	282·05*	
4*d.* = $\frac{1}{60}$ of £1, ∴	94·01666	= ..... 4*d.* .....
½*d.* = $\frac{1}{8}$ of 4*d.*, ∴	11·75208	= ..... ½*d.* .....
Required value = £427975·619		= £427975. 12*s.* 4½*d.*

*See note on Example ii.

Rough approximation. The value lies between £70 × 5600 and £80 × 5700, *i.e.* between £392000 and £456000.

If the number of things is not integral, and the fractional part of it can easily be expressed as an exact decimal, we may express the fraction in decimal form and proceed as above. If, for example, the number of things in Ex. iii. were 5641$\frac{3}{8}$, we might have stated the value at £1 each as £5641·375.

296. When the number of things is partly fractional and the value of the fraction can be expressed as an exact amount in shillings and pence, we may proceed as below. In other cases, it is generally best to calculate the value of the fraction separately.

Example i. *Find the value of* 512$\frac{3}{8}$ *things at* 16*s.* 4¾*d. each.*

Rough approximation. The value lies between 16*s.* × 500 and 17*s.* × 600, *i.e.* between £400 and £510.

	£	*s.*	*d.*	
	512	13	4	= value at £1 each
10*s.* = ½ of £1, ∴	256	6	8	= ..... 10*s.* .....
5*s.* = ½ of 10*s.*, ∴	128	3	4	= ..... 5*s.* .....
1*s.* = ⅕ of 5*s.*, ∴	25	12	8	= ..... 1*s.* .....
4*d.* = ¼ of 1*s.*, ∴	8	10	10·66	= ..... 4*d.* .....
½*d.* = ⅛ of 4*d.*, ∴	1	1	4·33	= ..... ½*d.* .....
¼*d.* = ½ of ½*d.*, ∴		10	8·16	= ..... ¼*d.* .....
∴	£420	5	7	= value at 16*s.* 4¾ each.

In the addition, the pupil must be careful here not to include the first line, which is the value at £1 each. The result may be verified by multiplying 16*s.* 4¾*d.* by 512$\frac{3}{8}$. The fraction $\frac{3}{8}$ is made up of the aliquot parts ¼ and ⅛.

Example ii. *Find the value of* $847\frac{7}{8}$ *things at* £2. 18*s.* 10*d. each.*

Rough approximation. The value lies between £2 × 800 and £3 × 900, *i.e.* between £1600 and £2700.

In this case we may conveniently find the value at £3 each, and subtract from it the value at 1*s.* 2*d.* each.

£	*s.*	*d.*		
847 .	17 .	6	= value at £1	each
		3		
2543 .	12 .	6	= £3	.....
1*s.* = $\frac{1}{20}$ of £1, ∴ 42 .	7 .	10·50	= ..... 1*s.*	.....
2*d.* = $\frac{1}{8}$ of 1*s.*, ∴ 7 .	1 .	3·75	= ..... 2*d.*	.....

∴ by subtraction, £2494 . 3 . 3·75 = value at £2. 18*s.* 10*d.* each.

EXAMPLES. 128.

Find, by Practice, the value of the following things to the nearest farthing; the given price being the price of *one*.

1.	18	at £1. 10*s.*	2.	24	at £1. 5*s.*
3.	22	at £1. 2*s.* 6*d.*	4.	16	at £1. 3*s.* 4*d.*
5.	27	at £1. 6*s.* 8*d.*	6.	35	at £2. 7*s.* 6*d.*
7.	42	at £2. 12*s.* 6*d.*	8.	89	at £3. 13*s.* 4*d.*
9.	38	at £3. 17*s.* 6*d.*	10.	53	at £2. 16*s.* 8*d.*
11.	73	at 5*s.* 6*d.*	12.	35	at 2*s.* $3\frac{1}{2}d.$
13.	112	at 18*s.* 9*d.*	14.	6420	at 4*s.* $3\frac{1}{2}d.$
15.	952	at 3*s.* $8\frac{1}{2}d.$	16.	63	at 16*s.* $8\frac{1}{2}d.$
17.	357	at 6*s.* $9\frac{3}{4}d.$	18.	511	at 7*s.* $10\frac{3}{4}d.$
19.	540698	at 13*s.* $4\frac{1}{2}d.$	20.	25	at £1. 2*s.* 7*d.*
21.	47	at £2. 14*s.* 10*d.*	22.	564	at £3. 14*s.* 4*d.*
23.	1978	at £3. 9*s.* 2*d.*	24.	1842	at £2. 6*s.* $9\frac{1}{4}d.$
25.	357	at £3. 7*s.* $2\frac{1}{2}d.$	26.	327	at £4. 17*s.* $10\frac{1}{2}d.$
27.	$235\frac{1}{2}$	at £4. 8*s.* 4*d.*	28.	$362\frac{1}{2}$	at £1. 3*s.* $8\frac{1}{2}d.$
29.	$278\frac{1}{2}$	at £1. 16*s.* 9*d.*	30.	$351\frac{3}{4}$	at £5. 18*s.* 9*d.*
31.	$137\frac{1}{2}$	at £9. 13*s.* 6*d.*	32.	$2037\frac{1}{2}$	at £1. 19*s.* $4\frac{1}{2}d.$
33.	$2395\frac{7}{8}$	at £2. 17*s.* 10*d.*	34.	$835\frac{9}{10}$	at £2. 13*s.* 4*d.*

35. $7364\frac{2}{7}$ things at £14. 17*s.* 4*d.* each.

36. $1157\frac{11}{13}$ things at £5. 18*s.* $2\frac{1}{2}d.$ each.

Find, by Practice, the value of

37. 5 tons 16 cwt. 2 qrs. 7 lb. × 5371.

38. 8 mi. 7 fur. 2 ch. 11 yds. × 4367.

39. 5 hrs. 20 min. 15 sec. × 365.

40. 3 sq. yds. 6 sq. ft. 75 sq. in. × 1005.

(b). Compound Practice.

297. In **Compound** Practice we find the value of a compound quantity, when the value of a unit of *one* denomination is given.

When the measure of the quantity can be expressed easily as a decimal of the unit whose value is given, we may proceed as in Simple Practice.

Example i. *Find the rent of* 15 *ac.* 3 *ro.* 36 *per. at* 9*s.* 4*d. per rood.*

We see that 15 ac. 3 ro. 36 per. = 63·9 roods.

	£.	
	63·9	= rent at £1 per rood.
	0·45	
	25·56	= ..... 9*s.* .....
	3·195*	
4*d.* = $\frac{1}{240}$ of £1, ∴	1·065	= ..... 4*d.* .....
Required rent =	£29·820	
=	£29. 16*s.* 4$\frac{4}{5}$*d.*	

{*See note on Art. 295.}

Rough approximation. The rent lies between 9*s.* × 63 and 10*s.* × 64; *i.e.* between £28. 7*s.* and £32.

Example ii. *If a man's debts amount to* £57201. 16*s. and he can pay only* 5*s.* 7$\frac{3}{4}$*d. for each pound, how much will his creditors get?*

We see that £57201. 16*s.* = £57201·8.

	£.	
	57201·8	= amount of debts in full
5*s.* = $\frac{1}{4}$ of £1, ∴	14300·45	= amt. obtd. at 5*s.* for each £1
6*d.* = $\frac{1}{20}$ of 5*s.*, ∴	1430·045	= ..... 6*d.* .....
1$\frac{1}{2}$*d.* = $\frac{1}{4}$ of 6*d.*, ∴	357·51125	= ..... 1$\frac{1}{2}$*d.* .....
$\frac{1}{4}$*d.* = $\frac{1}{6}$ of 1$\frac{1}{2}$*d.*, ∴	59·58521	= ..... $\frac{1}{4}$*d.* .....
∴ Amt. received by creditors =	£16147·591	= ..... 5/7$\frac{3}{4}$ .....
=	£16147. 11*s.* 10*d.*	to the nearest penny.

Rough approximation. Amount lies between 5*s.* × 57000 and 6*s.* × 58000, *i.e.* between £14250 and £17400.

298. When the money can be expressed exactly as the decimal of £1, *i.e.* when the number of pence and farthings is exactly divisible by 3, it is best to proceed as follows:

Example. *Find the value of* 5 *mi.* 6 *fur.* 8 *ch.* at £37. 17*s.* 6*d. per mile.*

		£.		
		37·875	= value of 1 mi.	
		5		
		189·375	=	5 mi.
4 fur. = ½ of 1 mi.,	∴	18·9375	=	4 fur.
2 fur. = ½ of 4 fur.,	∴	9·46875	=	2 fur.
8 ch. = ¼ of 4 fur.,	∴	3·7875	=	8 ch.
		Value = £221·569	= £221. 11*s.* 4½*d.*	

Rough approximation. The value lies between £37 × 5 and £38 × 6, *i.e.* between £185 and £228.

Note.—In this case, since 5 mi. 6 fur. 8 ch. = 5·85 mi., we might have obtained the result by finding the product £37·876 × 5·85, *correct to* 3 *places.*

299. When neither the money nor the measure of the quantity can be expressed exactly as a decimal, we may leave the money expressed in shillings and pence, and take parts for the quantity as in the following example.

Example. *Find the value of* 9 *cwt.* 3 *qrs.* 24 *lb. at* £3. 5*s.* 8*d. per cwt.*

Rough approximation. The value lies between £3 × 9 and £4 × 10, *i.e.* between £27 and £40.

The value of 9 cwt. is obtained by multiplying £3. 5*s.* 8*d.* by 9. To the product we add the value of 3 qrs. 24 lb., obtained by taking aliquot parts. Thus:

		£.	s.	d.		
		3 .	5 .	8	= value of	1 cwt.
				9		
		29 .	11 .	0	=	9 cwt.
2 qrs. = ½ of 1 cwt.,	∴	1 .	12 .	10	=	2 qrs.
1 qr. = ½ of 2 qrs.,	∴		16 .	5	=	1 qr.
16 lb. = ¼ of 1 cwt.,	∴		9 .	4·57	=	16 lb.
8 lb. = ½ of 16 lb.,	∴		4 .	8·28	=	8 lb.
	∴	£32 .	14 .	3·85	= value of	9 cwt. 3 qrs. 24 lb.

300. When the measure of the quantity contains a large number of that denomination whose value is given, the process followed in previous examples would involve a long compound multiplication. This can be avoided by using the following method.

Example. *Find the value of* 537 *tons* 12 *cwt.* 60 *lb. at* £2. 4*s.* 2*d. per ton.*

By the method of Art. 254, we find that

537 tons 12 cwt. 60 lb. = 537·62678... tons,

and we proceed as follows. The result will generally be correct to the nearest penny if the value per ton is not greater than £10.

	£.	
	537·62678	= value at £1 per ton.
	2·2	
	1075·25356	= ..... £2 .....
	107·52536	= ..... 4*s.* .....
$2d. = \frac{1}{120}$ of £1, ∴	4·48022	= ..... 2*d.* .....
Required value =	£1187·259	= £1187. 5*s.* 2¼*d.*

EXAMPLES. 129.

Find, by Practice, the value to the nearest penny of

1. 111 oz. of silver plate at 7*s.* 3½*d.* per oz.
2. 86 oz. 5 dwt. 4 grs. at 3*s.* 7½*d.* per dwt.
3. 8 oz. 12 dwt. 18 grs. of gold at £3. 18*s.* 4*d.* per oz.
4. 8 oz. 6 dwt. 16 grs. of silver at 5*s.* 6*d.* per oz.
5. 40 oz. 7 dwt. at 6*s.* 10½*d.* per oz.
6. 2 qrs. 9 lb. 13 oz. at 15*s.* 9*d.* per qr.
7. 56 cwt. 2 qrs. 14 lb. at £5. 9*s.* 6*d.* per cwt.
8. 2 cwt. 3 qrs. 7 lb. at £4. 15*s.* 8*d.* per cwt.
9. 13 cwt. 3 qrs. 4 lb. at £2. 18*s.* 4*d.* per cwt.
10. 57 cwt. 3 qrs. 11 lb. at £1. 8*s.* per cwt.
11. 5 cwt. 3 qrs. 19 lb. at £2. 15*s.* per cwt.
12. 24 tons 15 cwt. 2 qrs. 14 lb. at 16*s.* 7¾*d.* per cwt.
13. 9 tons 4 cwt. 3 qrs. 21 lb. at £14. 15*s.* 9*d.* per cwt.
14. 9 tons 19 cwt. 3 qrs. 14 lb. at £28 per ton.
15. 23 tons 6 cwt. 2 qrs. 14 lb. at £3. 13*s.* 4*d.* per ton.
16. 5 tons 3 qrs. 17 lb. at £7. 3*s.* 4*d.* per ton.
17. 137 tons 8 cwt. 60 lb of coal at £1. 2*s.* 8*d.* per ton.
18. 3 cwt. 3 qrs. 14 lb. at 16*s.* 8*d.* per ton.
19. 5 cwt. 3 qrs. 21 lb. at £15 per ton.
20. 2 qrs. 14 lb. at £7. 16*s.* 8*d.* per ton.
21. 3 tons 17 cwt. 2 qrs. 13 lb. 14 oz. at £3. 17*s.* 6*d.* per cwt.
22. 2 tons 7 cwt. 22 lb. 5 oz. at £32 per ton.
23. 50 ac. 1 ro. 10 per. at £3. 7*s.* 4*d.* per acre.
24. 7 ac. 3 ro. 10 per. at £396 per acre.
25. 45 ac. 3 ro. 20 per. at £111. 11*s.* 4*d.* per acre.
26. 237 ac. 1 ro. 30 per. at £43. 15*s.* per acre.
27. 23 ac. 3 ro. 16 per. at 2½ guineas per acre.

Find, by Practice, the value to the nearest penny of

28. 145 ac. 1 ro. 22 per. at £10. 5*s.* 3*d.* per acre.
29. 45 ac. 3 ro. 24 per. at £96. 2*s.* 8½*d.* per acre.
30. 3 ac. 2 ro. 17 per. at £53. 3*s.* 4*d.* per acre.
31. 5 qrs. 3 bush. 1 pk. of oats at £1. 14*s.* 8*d.* per quarter.
32. 315 qrs. 7 bush. 1 pk. of malt at £1. 1*s.* 6*d.* per quarter.
33. 710 qrs. 6 bush. 3 pks. at £1. 0*s.* 8*d.* per quarter.
34. 18 gall. 3 qts. 1½ pts. at 17*s.* 10½*d.* per gallon.
35. 3 pks. 1 gal. 3 qts. 1 pt. at £4. 12*s.* per gallon.
36. 36 mi. 3 fur. 22 yds. of wire at £11. 13*s.* 4*d.* per mile.
37. 7000 grains of gold at £3. 16*s.* 10*d.* per ounce.
38. the dividend on £5146. 15*s.* 0*d.* at 17*s.* 9*d.* in the £.
39. the dividend on £2870. 10*s.* 0*d.* at 14*s.* 3*d.* in the £.
40. the dividend on £835. 2*s.* 6*d.* at 13*s.* 10*d.* in the £.
41. the dividend on £5608. 10*s.* 0*d.* at 14*s.* 9½*d.* in the £.
42. 35 Kg. 4 Dg. 8 g. at 12*s.* 7¾*d.* per kilogram.
43. 15 t. 5 q. 85 Kg. 7 Hg. at £3. 12*s.* 5½*d.* per quintal.
44. 24 t. 8 q. 52 Kg. 8 Dg. at £14. 17*s.* 8½*d.* per tonne.
45. 3 Hg. 75 g. 27 cg. at £5. 18*s.* 11*d.* per kilogram.
46. 85 Ha. 15 a. 75 ca. at £45. 15*s.* per hectare.
47. 3 Dm. 5 m. 27 cm. 5 mm. at £1. 5*s.* 8*d.* per metre.
48. 24 mi. 5 fur. 7 ch. at £15. 12*s.* 3*d.* per mile.

Find, by Practice, the weight of

49. 2 cu. yds. 2 cu. ft. 486 cu. in. if 1 cu. ft. weighs 1000 oz.
50. 3 pks. 1 gall. 3 qts. 1 pt. if 1 gallon weighs 10 lb.
51. 5 cu. yds. 9 cu. ft. 492 cu. in. to the nearest ounce, if 1 cu. yd. weighs 1 ton.

(c). Bills of Parcels.

301. It will be a useful exercise for the student to make out the cost of a number of purchases in the form that tradesmen use.

Tradesmen's bills may be classed under two or three heads. When a buyer has completed his purchases, he may be presented with a **bill** in the form of a written list of the articles bought, with a statement of the cost of them attached; and when goods are sent to a buyer's residence there is sent with them a similar list, which is called an **invoice**, each separate entry being called an item. The amount stated on the invoice may be paid at

once; otherwise at the end of a certain period, usually 1, 3, or 6 months, a list of the amounts, with the dates, is sent to the purchaser, sometimes without any specification of the nature of the goods. A periodical statement of this kind is called an **account**: and when, as in many cases may be necessary, the full particulars of the items are given, it is called a **detailed account**. When an account is sent to the purchaser it is said to be **rendered**.

EXAMPLES. 130.

Make out the following bills, and calculate the amount of each item to the next farthing.

1. 27 tons of coal at 17*s.* 9*d.* per ton, 12 tons at £1. 2*s.* 6*d.* per ton, and 11 tons at 19*s.* 4*d.* per ton.

2. 10 Kg. of sugar at 5*d.*; 6 Kg. of tea at 2*s.* 8*d.*; 8 Kg. of coffee at 2*s.* 9*d.*; 12 Kg. of currants at 11*d.*; 10 Kg. of rice at $4\frac{1}{2}d.$; 9 Kg. of cheese at 1*s.* 11*d.*

3. 7 lb. at 2*s.* 3*d.* per lb.; 16 lb. at 7*d.* per lb.; 11 oz. at 1*s.* 7*d.* per oz.; 9 lb. at $3\frac{1}{2}d.$ per lb.; 7 lb. at $7\frac{3}{4}d.$ per lb.; $3\frac{1}{2}$ lb. at 1*s.* 10*d.* per lb.

4. $3\frac{1}{4}$ Kg. of tea at 4*s.* 2*d.* per Kg.; $1\frac{1}{2}$ Kg. of coffee at 3*s.* 10*d.*; $6\frac{3}{4}$ Kg. of loaf sugar at 6*d.*; $\frac{3}{4}$ Kg. of butter at 2*s.* 10*d.*

5. 6 lb. of candles at $7\frac{1}{2}d.$ a lb.; 21 lb. of sugar at $2\frac{1}{2}d.$ a lb.; $8\frac{1}{2}$ lb. of plums at $4\frac{3}{4}d.$ a lb.; $\frac{3}{4}$ lb. of tea at $1\frac{1}{2}d.$ an ounce; 3 lb. 10 oz. of rice at 2*d.* a lb.

6. 3 lb. of coffee at 17*d.*; 2 lb. of tea at 1*s.* 10*d.*; 21 lb. of moist sugar at $2\frac{1}{4}d.$; 10 lb. of rice at $2\frac{1}{2}d.$; 7 lb. of loaf sugar at $2\frac{1}{2}d.$; 9 lb. of raisins at $6\frac{1}{2}d.$; 7 lb. of currants at $5\frac{3}{4}d.$

7. 27 metres of flannel at 1*s.* $6\frac{1}{2}d.$ per metre; 32 metres of calico at $5\frac{1}{2}d.$ per metre; 6 pairs of stockings at 1*s.* $9\frac{3}{4}d.$ per pair; 6 pairs of gloves at 3*s.* 6*d.* per pair; 12 collars at $8\frac{1}{2}d.$ each.

8. $18\frac{1}{2}$ metres of flannel at 1*s.* 4*d.* a metre; 32 metres of calico at $8\frac{1}{4}d.$ a metre; $13\frac{1}{4}$ metres of linen at 3*s.* 10*d.* a metre; 14 metres of carpet at 4*s.* 8*d.* a metre; 17 pairs of stockings at 1*s.* $11\frac{1}{2}d.$ a pair; 7 pairs of gloves at 2*s.* $3\frac{3}{4}d.$ a pair.

9. $4\frac{1}{2}$ yds. of tartan at 2*s.* 11*d.* per yd.; 7 yds. of serge at 2*s.* $4\frac{3}{4}d.$ per yd.; 6 pairs of children's socks at $9\frac{3}{4}d.$ per pair; 3 pairs of silk hose at 4*s.* 6*d.* per pair; 1 doz. pairs of men's socks at 1*s.* $9\frac{1}{2}d.$ per pair; $5\frac{1}{2}$ yds. of flannel at 1*s.* 11*d.* per yd.

Make out the following bills, and calculate the amount of each item to the next farthing.

10. $18\frac{1}{2}$ yds. of silk at 5*s.* 6*d.* per yard; 64 yds. of sheeting at 1*s.* $4\frac{1}{2}d.$; 4 pairs of blankets at 18*s.* 4*d.* per pair; 3 pairs of blankets at 12*s.* 8*d.* per pair; 26 yds. of alpaca at $10\frac{3}{4}d.$; and 9 damask table-cloths, of which 4 are at 19*s.* 6*d.* each, and 5 at 26*s.* 6*d.* each.

11. 5 yds. of muslin at $2\frac{1}{2}d.$ a yd.; 39 yds. of merino at 4*s.* $3\frac{1}{4}d.$ a yd.; 3 doz. buttons at $4\frac{3}{4}d.$ a doz.; 198 yds. of braid at 1*s.* $3\frac{1}{2}d.$ a doz. yds.

12. 2 dozen pairs of gloves at 2*s.* $11\frac{3}{4}d.$ a pair; 37 metres of chintz at $9\frac{3}{4}d.$ a metre; a gross of buttons at $3\frac{1}{4}d.$ a dozen; 164 metres of fringe at 1*s.* $3\frac{3}{4}d.$ a dozen metres.

13. 12 bars of soap, each 3 Kg., at $6\frac{1}{2}d.$ per Kg.; 164 Kg. of tea at 54*s.* per doz. Kg.; 12 packets of candles, each $6\frac{1}{2}$ Kg., at $10\frac{1}{2}d.$ per Kg.

14. $10\frac{1}{4}$ yds. of ribbon at $2\frac{1}{4}d.$ per yd.; $13\frac{1}{4}$ yds. of calico at 8*d.* a yd.; 106 reels of cotton at 2*s.* 3*d.* a dozen; 10 pieces of tape at $1\frac{1}{4}d.$ a piece; and one umbrella at 10*s.* 6*d.*

15. 15 yds. of damask, for curtains, at 4*s.* 9*d.* a yd.; 28 yds. of gimp at $4\frac{1}{2}d.$; 24 rings at 6*d.* per dozen; 24 hooks at $1\frac{3}{4}d.$ per doz.; and workwoman's time, $2\frac{1}{2}$ days, at 2*s.* 9*d.* a day.

16. $5\frac{1}{2}$ doz. lb. of candles at $8\frac{1}{2}d.$ per lb.; $8\frac{1}{4}$ lb. of wax candles at 2*s.* 3*d.* per lb.; 7 galls. $2\frac{1}{2}$ pts. of oil at $11\frac{1}{2}d.$ per gall.; and 2 lamps at 13*s.* 9*d.* each.

XXXII. RATIO.

302. The relation which one quantity bears to another of the same kind with respect to *magnitude*, is called the **ratio** of the one to the other.

Ratio may also be defined as the multiple, part, or parts which one quantity is of another, and may be determined by dividing the first quantity by the second.

The first quantity is called the **antecedent**, the second is called the **consequent**, and the antecedent and the consequent are called the **terms** of the ratio. A ratio may be expressed in the form of a vulgar fraction, of which the *antecedent* is the numerator, and the *consequent* the denominator. Thus

$$\text{the ratio} = \frac{\text{the antecedent}}{\text{the consequent}}.$$

The ratio, for example, of 3 lb. of tea to 5 lb. of tea may be expressed either in the form 3 : 5, or in the form $\frac{3}{5}$.

303. A ratio is one of **greater inequality**, when the antecedent is *greater* than the consequent; and is one of **less inequality**, when the antecedent is *less* than the consequent. The ratio of 5 to 3, for example, is one of *greater* inequality, and that of 3 to 5 is one of *less* inequality; and, generally, the ratio of a to b is of *greater* inequality when a is *greater* than b, and of *less* inequality when a is *less* than b.

304. Since a ratio may be expressed in the form of a vulgar fraction, it follows that the general rules, which we have established for the treatment of fractions (Art. 146), apply also to ratios. Thus a ratio is in its simplest form when both terms are integers, and when these integers are prime to one another; and we may *multiply* or *divide* both terms of a ratio by any, the same number, without affecting the value of the ratio. But the ratio is *not* unchanged, if we *add* equal numbers to, or *subtract* them from the terms, except when the terms are equal.

We will illustrate the last of these statements with the ratio $\frac{3}{5}$. If we *add* 2 to each term, the ratio becomes $\frac{5}{7}$; and, if we *subtract* 2, it becomes $\frac{1}{3}$. But the fractions $\frac{3}{5}$, $\frac{5}{7}$ and $\frac{1}{3}$ can be represented by $\frac{1}{2}\frac{5}{5}$, $\frac{1}{2}\frac{5}{1}$, and $\frac{1}{4}\frac{5}{5}$; whence we see that their ascending order of magnitude is $\frac{1}{3}$, $\frac{3}{5}$, $\frac{5}{7}$. Thus the ratio $\frac{3}{5}$ has been *increased* by the *addition* of 2 to each of the terms, and has been *diminished* by the *subtraction* of 2 from each. This result may be stated in a general form thus:—A ratio of **less inequality** is **increased** by **adding** equal numbers to the terms, and is **diminished** by **subtracting** equal numbers from the terms.

Similarly, a ratio of **greater** inequality is **diminished** by **adding** equal numbers to the terms, and is **increased** by **subtracting** equal numbers from both.

These properties can be established generally by algebraical methods, and may be expressed thus:

(i) If a is *greater* than b,

$$\frac{a+x}{b+x} \text{ is less than } \frac{a}{b}, \text{ and } \frac{a-x}{b-x} \text{ is greater than } \frac{a}{b}.$$

(ii) If a is *less* than b,

$$\frac{a+x}{b+x} \text{ is greater than } \frac{a}{b}, \text{ and } \frac{a-x}{b-x} \text{ is less than } \frac{a}{b}.$$

305. It must be remembered that ratio can exist only between quantities of the *same kind*. There cannot be a ratio between 3 hours and 5 shillings, or between 2 tons and 7 miles. There can, however, be a ratio between 3 oz. and 5 lb., for these are quantities of the *same kind*; but we cannot determine the numerical equivalent of the ratio until we have expressed them both as quantities of the *same name*. Thus, since 5 lb. = 80 oz., the ratio of 3 oz. to 5 lb. is represented by $\frac{3}{80}$.

306. To find the ratio between two compound quantities such as 2 cwt. 3 qrs. and 3 qrs. 14 lb., we reduce them to simple quantities of one and the same denomination. Thus,

2 cwt. 3 qrs. = 11 qrs., and 3 qrs. 14 lb. = $3\frac{1}{2}$ qrs.;

therefore $$\text{the ratio} = \frac{11}{3\frac{1}{2}} = \frac{22}{7}.$$

Or thus: 2 cwt. 3 qrs. = 308 lb., and 3 qrs. 14 lb. = 98 lb.;

$$\therefore \text{ the ratio} = \frac{308}{98} = \frac{22}{7}.$$

307. When the fractions which represent two or more simple ratios are multiplied together, the result will represent the ratio **compounded** of the simple ratios. Thus the ratio *compounded* of $\frac{3}{5}$ and $\frac{2}{7}$ is $\frac{6}{35}$.

Generally, the ratio compounded of $\frac{a}{b}$ and $\frac{c}{d}$ is $\frac{a}{b} \times \frac{c}{d}$, or $\frac{a \times c}{b \times d}$.

The **inverse**, or **reciprocal**, of a given ratio has the given antecedent for its consequent, and the given consequent for its antecedent. It is represented therefore by inverting the fraction which represents the given ratio. The inverse of the ratio $\frac{3}{5}$, for example, is $\frac{5}{3}$, and that of $\frac{a}{b}$ is $\frac{b}{a}$.

The **duplicate** ratio of two numbers is represented by the ratio of their squares. Thus the duplicate ratio of 3 to 5 is represented by $\frac{9}{25}$; and, generally, the *duplicate* ratio of a to b is represented by $\frac{a^2}{b^2}$.

EXAMPLES. 131. *Oral.*

Express in its simplest form the ratio of

1.	8 to 2,	**2.**	10 to 2,	**3.**	2 to 8,	**4.**	2 to 10,
5.	9 to 3,	**6.**	3 to 9,	**7.**	4 to 12,	**8.**	16 to 4,
9.	12 to 144,	**10.**	16 to 256,	**11.**	1 to $\frac{1}{2}$,	**12.**	2 to $\frac{1}{2}$,
13.	2 to $\frac{1}{4}$,	**14.**	$\frac{1}{2}$ to $\frac{1}{4}$,	**15.**	$\frac{3}{4}$ to $\frac{1}{2}$,	**16.**	$\frac{5}{7}$ to $\frac{1}{10}$,

17. 1 qr. to 1 lb., 18. 1 oz. to 1 dwt., 19. £1 to 15*s.*,
20. 3 yds. to 2 ft., 21. 6*s.* 8*d.* to £1, 22. 12*s.* 6*d.* to £1,
23. 100 lb. to $16\frac{2}{3}$ lb., 24. 9*s.* to £2,
25. 13 days to a common year, 26. 13 m. to 3 Km.

State the reciprocal of the ratio of

27. 2 to 3, 28. 4 to 7, 29. 8 to 11, 30. 7 to 12.

State the duplicate of the ratio of

31. 2 to 3, 32. 4 to 7, 33. 8 to 11, 34. 7 to 12.

EXAMPLES. 132.

Express in its simplest form the ratio of

1. $16\frac{2}{3}$ to 300, 2. $6\frac{1}{4}$ to 75, 3. $3\frac{1}{4}$ to 13,
4. 5 to ·75, 5. 2·25 to ·75, 6. 45 to 1980,
7. 28 to 910, 8. 39 to 110·5, 9. 6·5 to 7·75,
10. $\frac{5}{6}$ to $\cdot\dot{5}7142\dot{8}$, 11. £1. 8*s.* 6*d.* to £2. 6*s.* 3*d.*,
12. £3. 5*s.* to £1. 5*s.* 8*d.*, 13. £4. 7*s.* 6*d.* to £2. 3*s.* 4*d.*,
14. £6. 7*s.* 8*d.* to £4. 5*s.* 4*d.*,
15. 2 cwt. 1 qr. to 3 cwt. 1 qr. 14 lb.,
16. 3 ac. 3 ro. 4 per. to 5 ac. 1 ro.,
17. 6 hrs. 20 min. to 2 hrs.,
18. 5 gall. 1 qt. to 9 gall. 3 qts. 1 pt.,
19. 2 miles 3 fur. to 1 mile 2 fur.,
20. the *difference* to the *sum* of ·0231 of 21*s.* and ·19 of 2*s.* 6*d.*,
21. 29 m. 19 cm. to 37 m. 53 cm.,
22. 50 Hl. 49 l. to 22 Hl. 95 l.,
23. 29 Hg. 97 g. to 19 Kg. 98 Dg.,
24 43 Ha. 5 a. to 45 Ha. 92 a.

What is the ratio compounded of the ratios of

25. 8 to 2, 9 to 3, and 10 to 5? 26. 4 to 10 and 8 to 6?
27. 8 to 3 and 6 to 2? 28. 6 to 8, 16 to 10, and 12 to 9?
29. 4 to 9, 3 to 4, 5 to 6, and 12 to 7?

30. What is the inverse of the ratios of 65 to 15 and 99 to 11?
31. If 32 is the consequent, and $\frac{1}{3}$ the ratio, what is the antecedent?
32. If 13·5 is the antecedent, and 4 the ratio, what is the consequent?

33. If the consequent be $\frac{3}{4}\frac{2}{8}$ and the ratio $\frac{1}{6}$, what is the antecedent?

34. What has to £17. 14*s*. 6*d*. the ratio of 17 to 30?

35. What has to 3 cwt. 1 qr. 7 lb. the ratio of 13 to 28?

XXXIII. PROPORTION.

308. **Proportion** is an equality of ratios.

Thus the ratio of 12 to 20 is equal to the ratio of 3 to 5; for we may divide 12 and 20 by the same number 4 without affecting the ratio. This is expressed by saying that

12 is to 20 as 3 is to 5;

and is written $12 : 20 = 3 : 5$, or $\frac{1}{2}\frac{2}{0} = \frac{3}{5}$.

Each of these expressions indicates a *proportion*. The terms 12, 20, 3, 5 are called **proportionals**, and are distinguished as the 1st, 2nd, 3rd and 4th proportional respectively.

The first and fourth proportionals, 12 and 5, are called the **extremes**. The second and third proportionals, 20 and 3, are called the **means**.

309. Writing the proportion $12 : 20 = 3 : 5$ in the fractional form, it becomes $\frac{1}{2}\frac{2}{0} = \frac{3}{5}$; and if we express the two fractions with the same denominator, we get

$$\frac{12 \times 5}{20 \times 5} = \frac{3 \times 20}{5 \times 20};$$

whence we see that the **product of the extremes**, 12 and 5, is **equal to the product of the means**, 20 and 3.

310. The above statements may be put in a general form thus: If a, b, c and d represent four numbers, such that the ratio of a to b is equal to the ratio of c to d, the relation between them may be stated in either of the forms

$$a : b = c : d, \quad \text{or} \quad \frac{a}{b} = \frac{c}{d}.$$

The four numbers are said to be *in proportion*, and are called *proportionals*. If we multiply the two ratios by $b \times d$, or $d \times b$, we get

$$\frac{a}{b} \times b \times d = \frac{c}{d} \times d \times b, \quad \therefore \ a \times d = b \times c;$$

hence we see that when any four numbers are in proportion, *the product of the extremes* is equal to *the product of the means*.

The proportion can be written in any one of the forms

$$\frac{a}{b}=\frac{c}{d},\quad \frac{b}{a}=\frac{d}{c},\quad \frac{a}{c}=\frac{b}{d},\quad \frac{c}{a}=\frac{d}{b},$$

and each of these leads to the result, $a \times d = b \times c$.

311. When the 2nd and 3rd proportionals are equal to one another, each is called a **mean proportional** between the other two. For example, in the proportion, $24 : 12 = 12 : 6$, the term 12 is called a *mean proportional* between 24 and 6, and the three numbers 24, 12 and 6 are said to be in **continued proportion.** The term 6 is called the **third proportional** to 24 and 12.

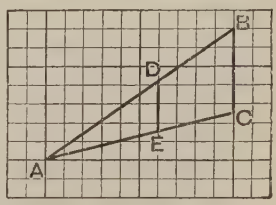
Generally, if a, b, c, d, ... be any numbers such that

$$\frac{a}{b}=\frac{b}{c}=\frac{c}{d}= \ \ldots,$$

then a, b, c, d, ... are said to be in *continued proportion*, and each is a mean proportional between the number before it and the number after it.

312. We can illustrate further the meaning of proportion, by using squared paper in the following manner.

If, through a point A, a straight line AB is drawn in any direction as in the figure, it will be seen to be divided into 10 equal parts at the points where it cuts the vertical lines on the paper. We see also that AD contains 6, and that BD contains 4, of these equal parts;



$$\therefore \ \frac{AD}{BD}=\frac{6}{4}=\frac{3}{2}.$$

If a second line AC be drawn from A in another direction so as to cut the same number of vertical lines; we find in the same way that

$$\frac{AE}{EC}=\frac{6}{4}=\frac{3}{2}.$$

Hence it follows that $\dfrac{AD}{BD}=\dfrac{AE}{EC}$.

It is clear that the argument is independent of the number of the vertical lines which are made use of, and we therefore conclude that, generally, a straight line drawn parallel to the base of a triangle divides the sides into parts which are proportionals.

Example i. *Find the unknown term* x *in the proportion* $\frac{1}{6} : x = 4 : 8$.

We have
$$\frac{x}{\frac{1}{6}} = \frac{8}{4};$$
$$\therefore\ 6x = 2;$$
$$\therefore\ x = \tfrac{2}{6} = \tfrac{1}{3}.$$

Example ii. *Find a mean proportional between* 8 *and* 32.

If x represent the mean proportional, we have
$$\frac{8}{x} = \frac{x}{32};$$
$$\therefore\ x^2 = 256;$$
$$\therefore\ x = 16;$$
$\therefore$ the mean proportional = 16.

EXAMPLES. 133.

In the following examples find the number represented by x.

1. $5 : 7 = 15 : x$.
2. $9 : 6 = 6 : x$.
3. $\frac{1}{5} : 3 = 7 : x$.
4. $3 : 4 = 9 : x$.
5. $8 : 6 = x : 3$.
6. $12 : x = 15 : 3$.
7. $x : 7 = 8 : 9$.
8. $27 : 3 = x : 1$.
9. $2 : 100 = 17 : x$.
10. $27 : 3 = 54 : x$.
11. $5\frac{1}{2} : x = 16 : 32$.
12. $9 : 150 = 105 : x$.
13. $x : \frac{2}{7} = \frac{7}{8} : \frac{3}{4}$.
14. $\frac{5}{16} : \frac{1}{4} = 15 : x$.
15. $648 : 243 = 24 : x$.
16. $65 : x = 75$ lb. $: 850$ lb.
17. 12 yds. : 4 yds. $= £9 : £x$.
18. 20 ac. : 25 ac. $= x$ lb. : 10 lb.
19. x days : 4 days $= £5 : £15$.
20. $1{\cdot}50 : 7{\cdot}50 = x$ pks. : 3 pks.
21. x men : 200 men $= £87\frac{1}{2} : £500$.
22. x hrs. : 9 hrs. = 60 men : 18 men.
23. 2 gall. 2 qts. : x gall. = 3 cwt. 1 qr. : 1 cwt.

Find a mean proportional between

24. 18 and 2.
25. 16 and 4.
26. 9 and 16.
27. 5 and 45.
28. 5 and 125.
29. 12 and 48.

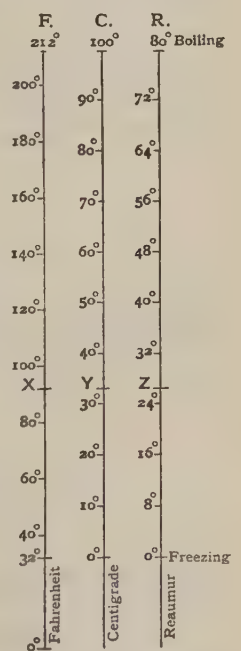
Find a third proportional to

30. 12 and 6.
31. 24 and 8.
32. 15 and 5.

313. The ratio which one quantity bears to another depends only on the *magnitudes* of the quantities and not on the *units* in terms of which they are measured; thus the ratio $a : b$ is always the same whatever unit may be chosen to measure both a and b. We can apply this principle to change the unit in terms of which

a quantity is measured, *i.e.* to express in terms of one unit a quantity which is already given in terms of another, the connection between the two units being known. As an illustration we will consider the different scales employed in the measurement of temperature.

Temperature is measured with an instrument called a **thermometer**, which generally consists of an air-tight glass tube partly filled with mercury and having a reservoir of mercury at one end. It is found that, as the temperature rises, the surface of the column of mercury rises in the tube; and that equal increases of temperature are accompanied by corresponding equal increases in the height of the mercury column. Two temperatures may therefore be compared by measuring the difference in the corresponding heights of the mercury column. Two fixed points are first determined; (i) the **freezing-point**, which is the situation of the surface of the mercury at the temperature of melting ice, and (ii) the **boiling-point**, which is the situation of the surface of the mercury at the temperature of steam from boiling water.



In **Fahrenheit's** thermometer, which is used for general purposes in the British Isles, the freezing-point is marked 32, denoting 32 degrees, or 32° Fahrenheit, and the boiling-point is marked 212 (denoting 212° F.), and the interval between the two points is divided into 180 equal parts. The zero-point in the Fahrenheit scale is thus 32 degrees below the freezing-point.

In the **Centigrade** thermometer the freezing-point is marked 0°, and the boiling-point 100°, and the interval between them is divided into 100 equal parts.

In **Réaumur's** thermometer the freezing-point is marked 0° and the boiling-point 80°, and the interval between them is divided into 80 equal parts.

Thus on the three different scales, the intervals between the freezing-point and the boiling-point denote, respectively, 180 degrees F., 100 degrees C. and 80 degrees R.

Suppose now that a certain temperature is denoted by F°, C° and R° on the three scales respectively.

If the three scales are placed side by side as in the diagram, with the freezing-points in a horizontal line, it is clear that the boiling-points will also be in a horizontal line, and that at any time the tops X, Y and Z of the mercury columns will be in a horizontal line also. It necessarily follows that the height of X on the Fahrenheit scale above the freezing-point bears the same ratio to the height of 212° above the freezing-point as the height of Y bears to that of 100° on the Centigrade scale, and as the height of Z bears to that of 80° on the Réaumur scale.

Hence $$\frac{F-32}{180}=\frac{C}{100}=\frac{R}{80},$$

or $$\frac{F-32}{9}=\frac{C}{5}=\frac{R}{4}.$$

NOTE.—Temperatures below zero are indicated by the sign (−) prefixed to the number of degrees.

Example i. *What temperature Fahrenheit is represented by 58° C.?*

$$\frac{F-32}{9}=\frac{58}{5};$$

$$\therefore\ F-32=\frac{522}{5}=104\tfrac{2}{5};$$

$$\therefore\ F=136\tfrac{2}{5};$$

$$\therefore\ 58°\text{ C.}=136{\cdot}4°\text{ F.}$$

Example ii. *What temperature Centigrade is represented by 118° F.?*

$$\frac{C}{5}=\frac{118-32}{9}=\frac{86}{9};$$

$$\therefore\ C=\frac{430}{9}=47\tfrac{7}{9};$$

$$\therefore\ 118°\text{ F.}=47\tfrac{7}{9}°\text{ C.}$$

EXAMPLES. 134.

1. What temperatures (i) Centigrade, (ii) Réaumur, are represented by 40° F., 73° F., 85° F., 12° F., −28° F.?

2. What temperatures (i) Fahrenheit, (ii) Réaumur, are represented by 20° C., 35° C., 92° C., −10° C., −36° C.?

3. What temperatures (i) Fahrenheit, (ii) Centigrade, are represented by 29° R., 35° R., 86° R., −8° R., −15° R.?

XXXIV. PROBLEMS INVOLVING PROPORTION.

(a). The Unitary Method.

314. In the solution of simple questions in which the idea of proportion is involved we will adopt first a mode of expression which is called the **UNITARY METHOD**. It will be seen from the following examples what the method is.

Example i. *If* 4 *yards of cloth cost* 12*s.*, *what will be the cost of* 96 *yards?*

The cost of 4 yards = 12*s.*,
∴ ..... 1 yard = 3*s.*,
∴ ..... 96 yards = 288*s.* = £14. 8*s.*

In the first line we place the quantity, in terms of which the answer is to be found, on the right-hand side of the equation.

In this *Unitary Method* we proceed from what is given to what is wanted, by passing through a *unit* common to both. In *Ex.* i, we have passed through the unit 1 yd., but we could have obtained more quickly what we wanted, by treating 4 yds. as the unit, and multiplying it by 24, thus,

the cost of 4 yards = 12*s.*,
∴ ..... 96 ..... = 24 × 12*s.* = 288*s.* = £14. 8*s.*

Example ii. *If* $11\frac{1}{4}$ *yards of calico cost* 5*s.* $7\frac{1}{2}$*d.*, *what is the cost of* 5 *dozen yards?*

The cost of $11\frac{1}{4}$ yards = 5*s.* $7\frac{1}{2}$*d.*,
∴ ..... 45 ..... = 22*s.* 6*d.*,
∴ ..... 15 ..... = 7*s.* 6*d.*,
∴ ..... 60 ..... = £1. 10*s.*,
∴ ..... 5 doz. yds. = £1. 10*s.*

Here we have passed from 45 yds. to 60 yds. by way of 15 yds., 15 being the G.C.M. of 45 and 60. As a general rule we pass from what is given to what is wanted by way of the *greatest* unit common.

Example iii. *How much must be given for* 13 *articles at the rate of* £3. 16*s.* 6*d. for* 6 *of them?*

The cost of 6 articles = £3. 16*s.* 6*d.*,
∴ ..... 1 article = 12*s.* 9*d.*,
∴ ..... 7 articles = £4. 9*s.* 3*d.*,
∴ ..... 13 ..... = £8. 5*s.* 9*d.* (by addition).

Here we see how to avoid the multiplication of 12*s.* 9*d.* by a number such as 13, which cannot be separated into simple factors.

Example iv. *If* $7\frac{3}{4}$ *acres can be mown in* 3 *days, how long will it take to mow* 54 *acres* 1 *rood?*

Time required to mow $7\frac{3}{4}$ acres = 3 days,
∴ ..... 31 ..... = 12 .....,
∴ ..... 1 ..... = $\frac{1}{31}\frac{2}{}$ .....,
∴ ..... $54\frac{1}{4}$ ..... = $\frac{1}{31}\frac{2}{} \times \frac{217}{4}$ days,
∴ the time required = 21 days.

Example v. *If* 19 *men can complete a work in* 437 *days, in what time will* 23 *men be able to do it?*

Time required by 19 men = 437 days,

$\therefore$ ..... 1 man = 19×437 days,

$\therefore$ ..... 23 men = $\frac{19 \times 437}{23}$ .....,

$= 19 \times 19$ .....,

$\therefore$ the time required = 361 days.

In questions such as this, which involve the time in which a varying number of workmen can do a given amount of work, it must be noticed that if we *increase* the number of workmen we *diminish* the time they require, and that we diminish it *at the same rate*.

And, conversely, if we *diminish* the number of men the time required by them becomes *greater* in the same proportion. For example, if we multiply the number of workmen by 2, 3, 4, $\frac{5}{8}$, etc., the time they require to do a piece of work is obtained by multiplying the time of the original number of men by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{8}{5}$, etc. respectively.

Example vi. *What weight of metal is worth* 106 *f.* 47 *c., if* 15 *Kg. be worth* 16 *f.* 90 *c.?*

The weight of metal worth 1690 c. = 15 Kg.,

$\therefore$ ..... 1 c. = $\frac{15}{1690}$ Kg.,

$\therefore$ ..... 10647 c. = $\frac{15 \times 10647}{1690}$ Kg.,

$\therefore$ the weight required = $\frac{15 \times 10647}{1690}$ Kg. = $\frac{189}{2}$ Kg.

= 94 Kg. 500 g.

Example vii. *If* 5 *cwt.* 2 *qrs.* 4 *lb. cost* £72. 6*s.* 8*d., how much can be bought for* £32. 13*s.* 4*d.?*

We see that 5 cwt. 2 qrs. 4 lb. = $5\frac{60}{112}$ cwt. = $5\frac{15}{28}$ cwt.

The weight which can be bought for £$72\frac{1}{3}$ = $5\frac{15}{28}$ cwt.,

$\therefore$ ..... £1 = $\frac{5\frac{15}{28}}{72\frac{1}{3}}$ cwt.,*

$\therefore$ ..... £$32\frac{2}{3}$ = $\frac{5\frac{15}{28} \times 32\frac{2}{3}}{72\frac{1}{3}}$ cwt.,

$\therefore$ the required weight = $\frac{155}{28} \times \frac{98}{217}$ cwt. = $\frac{5}{2}$ cwt. = 2 cwt. 2 qrs.

In cases such as this it is best not to attempt to simplify the result of the second line (*), but to reserve all cancelling till the end.

EXAMPLES. 135a. (*Oral.*)

1. If 1 copy of a book costs 5*s.*, what will 7 copies cost?
2. If 9 articles cost £36, what is the cost of 1 article?
3. If 12 yards of cloth cost 36*s.*, what will 1 yard cost?
4. If 7 articles cost 28*s.*, what is the cost of 1 article?
5. If 16 equal bags of flour weigh 96 lb., what will each weigh?

6. If 4 oranges cost 3 pence, what will 100 oranges cost?

7. If 4 horses eat as much as 3 oxen, how many oxen will eat as much as 12 horses?

8. If 15 yards cost 10 shillings, what will 3 yards cost?

9. If 10 eggs cost 15 pence, what will 5 eggs cost?

10. If 12 horses cost £156, what will 3 horses cost?

11. If 4 lb. of sugar cost 1*s.*, how much will 72 lb. cost?

12. If 6 yards of cloth cost 24*s.*, what will 18 yards cost?

13. If a servant's wages are 8*s.* per week, how much should he receive for 10 weeks?

14. If 72 gallons cost 360*s.*, how much will 36 gallons cost?

15. A ship sails 400 miles in 16 days; how many days should she take to sail 1200 miles?

16. If 25 eggs cost 3*s.*, how much will 225 eggs cost?

17. A servant's wages being 208*s.* for 52 weeks, how much ought she to receive for 7 weeks?

18. If 50 packets of envelopes cost 150 pence, what will 75 packets cost?

19. If 13 tons of coal cost 273 shillings, what will $6\frac{1}{2}$ tons cost?

20. If 10 men can do a piece of work in 4 days, in how many days would 1 man do it?

21. If 1 man can reap a field in 12 days, how long will it take 6 men?

22. If 8 men can do a piece of work in 3 days, in how many days should 1 man do it?

23. If 6 men can do a piece of work in 4 days, how long would it take 1 man?

24. If 1 pair of horses can plough a field in 12 days, in how many days would 3 pair of horses do it?

25. If 6 cwt. is enough to feed 30 horses for 4 days, how many horses would it feed for 1 day?

26. How many books at 4 shillings each might be exchanged for 20 books at 3 shillings each?

27. If a train travel for 3 hours at 20 miles an hour, how long would it take another train to travel the same distance at 10 miles an hour?

28. If a bankrupt owes £1000, and has property amounting to £250, how much can he pay for each £ that he owes?

29. If 25 men can do a piece of work in 24 days, in how many days will 5 men do it?

28. If 5 cwt. cost £5. 16*s.* 9*d.*, what is the cost of 7 tons 5 cwt.?

29. Find the cost of 37 yds. of silk when 25 yds. cost £4. 7*s.* 6*d.*

30. If $3\frac{5}{7}$ lb. cost 8*s.* 8*d.*, what will $2\frac{1}{4}$ cwt. cost?

31. If a man's daily wage is 5*s.*, how much will 30 men earn in 52 weeks, excluding Sundays?

32. If 4 Hl. 55 l. of wine can be bought for 520 f., how much can be bought for 93 f. 60 c.?

33. If $\frac{1}{8}$ of a ship is worth £70. 19*s.* 6*d.*, what part of her is worth £250. 10*s.*?

34. If 37 articles cost £8. 11*s.* $1\frac{1}{2}d.$, how many can be bought for £35. 3*s.*?

35. If 87 yards of cloth cost £6. 0*s.* 10*d.*, how many can be obtained for £60?

36. If $3\frac{3}{5}$ lb. cost 8*s.* 8*d.*, what will $\frac{9}{16}$ cwt. cost?

37. A brewer receives 12 doz. of brandy in exchange for 5 barrels of ale, worth 78*s.* a barrel; what does the brandy cost him per bottle?

38. If $2\frac{1}{2}$ cwt. cost £1. 9*s.* 2*d.*, what will 42 lb. cost?

39. Find the cost of 42 cwt. of sugar, if 56 lb. cost £1. 17*s.* 4*d.*

40. If $10\frac{1}{3}$ cwt. cost £5. 12*s.*, what will $7\frac{3}{4}$ lb. cost?

41. Find the cost of $1\frac{5}{8}$ cwt. of sugar at $2\frac{1}{4}d.$ per lb.

42. If $5\frac{1}{4}$ cwt. cost 23 guineas, what is the cost of 196 lb.?

43. If 16 Kg. 1 Hg. cost 25 f. 30 c., how much can be bought for 71 f. 50 c.?

44. If 19 yards of cloth cost £3. 1*s.* 9*d.*, how much can be bought for 1*s.* $7\frac{1}{2}d.$?

45. If a person travel 3 miles 7 fur. 5 ch. in 1 hr., how far will he travel in 3 hrs. 7 min. 20 sec.?

46. If £37 be equivalent to 179 dol. 8 c., what is 30 dol. 25 c. equivalent to in British money?

47. If the rent of 5 Ha. 35 a. be 8400 f., what is the rent of 22 Ha. 47 a.?

48. If a certain weight can be carried 184 Km. for 23 f. 80 c., what will it cost to carry it 82 Km. 8 Hm.?

49. If 8 c.cm. weigh 84 g., what is the volume of 273 g.?

50. If 28 Kg. 490 g. be carried 481 Km. for 12 f. 45 c., how far will 100 Kg. 100 g. be carried for the same money?

51. If 4 dol. 83 c. = £1, and 25 f. 22 c. = £1, find in francs and centimes, to the nearest centime, the value of 50 dollars.

52. A sack of potatoes weighs 178 lb.; if 6 such sacks cost £2. 4*s*. 6*d*., what will be the cost of 44 lb.?

53. If $\frac{3}{4}$ yd. cost £3. 16*s*. 6*d*., what will 36·6875 yds. cost?

54. If 4 oz. cost ·5625*s*., what is the value of ·15625 cwt.?

(b). Proportion and Equations.

315. Problems involving proportion may also be solved by introducing a symbol to represent the unknown quantity, and stating the proportion in the form of an equation.

Example i. *If* 18 *gallons of beer cost* £1. 1*s*., *what is the cost of* 11 *pints?*

The cost of the beer is clearly proportional to the quantity bought; thus, if x pence be the cost of 11 pints, it follows that x pence must bear the same ratio to 21*s*., or 252*d*., that 11 pints bear to 18 gallons.

Hence
$$\frac{x}{252}=\frac{11 \text{ pints}}{18 \text{ gallons}}=\frac{11}{144};$$
$$\therefore\ x=\tfrac{11}{144}\times 252=\tfrac{11}{12}\times 21=\tfrac{77}{4}=19\tfrac{1}{4};$$
$$\therefore \text{ cost of 11 pints}=1s.\ 7\tfrac{1}{4}d.$$

Example ii. *If* 27 *men can complete a piece of work in* 204 *days, in what time should* 51 *men be able to do it?*

The time taken to do the work *diminishes* at the same rate as the number of men employed *increases*; the proportion in such a case is called **inverse**. Thus, if x days be the time required by 51 men, we have
$$\frac{x}{204}=\frac{27 \text{ men}}{51 \text{ men}}=\frac{27}{51}=\frac{9}{17};$$
$$\therefore\ x=\frac{9}{17}\times 204=9\times 12=108;$$
$$\therefore \text{ the time required}=108 \text{ days}.$$

316. We may also form an equation by using the same common-sense reasoning which is the characteristic of the *Unitary Method*. We do this in the following examples.

Example i. *If* 46 *yds. of cloth cost* £13. 11*s*. 2*d*., *how much would* $57\frac{1}{2}$ *yds. cost at the same rate?*

Let the cost of $57\frac{1}{2}$ yards $=£x$; then the cost of 1 yard $=£\frac{x}{57\frac{1}{2}}$.

But the cost of 46 yards $=£13.\ 11s.\ 2d.=£13\frac{134}{240}=£13\frac{67}{120}$;

$\therefore$ ..... 1 yard $=£\frac{13\frac{67}{120}}{46}$;

$$\therefore\ \frac{x}{57\frac{1}{2}}=\frac{13\frac{67}{120}}{46};$$
$$\therefore\ x=\frac{1627}{46\times 120}\times 57\tfrac{1}{2}=\frac{1627}{46\times 120}\times\frac{115}{2}=\frac{8135}{480};$$
$$\therefore \text{ the cost of } 57\tfrac{1}{2} \text{ yards}=£\tfrac{8135}{480}=\tfrac{8135}{2}d.=4067\tfrac{1}{2}d.=338s.\ 11\tfrac{1}{2}d.$$
$$=£16.\ 18s.\ 11\tfrac{1}{2}d.$$

Example ii. *If 3 tons 2 qrs. 12 lb. can be carried 105 miles for a certain sum, how far can 5 tons 6 cwt. 7 lb. be carried for the same money?*

Let the distance required $= x$ miles; then the weight which could be carried 1 mile $= 106$ cwt. 7 lb. $\times x = 106\frac{1}{16}$ cwt. $\times x$;

but the weight which could be carried 1 mile

$= 60$ cwt. 68 lb. $\times 105 = 60\frac{17}{28}$ cwt. $\times 105$;

$\therefore\ 106\frac{1}{16} \times x = 60\frac{17}{28} \times 105$;

$$\therefore\ x = \frac{60\frac{17}{28} \times 105}{106\frac{1}{16}}$$

$$= \frac{1689}{28} \times \frac{16}{1697} \times 105 = \frac{4}{7} \times 105 = 60;$$

$\therefore$ the distance required $= 60$ miles.

EXAMPLES. 136.

1. If 12 g. 68 cg. of a substance cost £1. 6*s*. 5*d*., what will 15 g. 20 cg. cost?

2. If 1 cwt. 1 qr. 21 lb. of copper cost £4. 6*s*. 3*d*., what is the price per ton?

3. If 5 Ha. 35 a. of land is sold for £25. 15*s*., what would 9 Ha. 63 a. be sold for at the same rate?

4. If 182 Kg. can be carried $346\frac{1}{2}$ Km. for £2, how far can 100 Kg. 1 Hg. be carried for the same money?

5. If 9 Hl. 31 l. of wine cost £21, what would 66·5 Hl. cost?

6. What should a person's salary be from April 5th to July 12th inclusive, at the rate of £136. 17*s*. 6*d*. a year?

7. If a railway train travels at the rate of 22 Km. an hour, how far can it go between 6.15 a.m. and 2.35 p.m.?

8. A coach travels $7\frac{1}{2}$ miles an hour; how far can it go between 10.15 a.m. and 5.45 p.m.?

9. A contractor agrees to provide dinner for 17 persons for £5. 2*s*.; what should he charge if 3 more than the specified number were present?

10. The railway fare for 57 Km. is 7 f. 60 c.; what is the fare for 90 Km.?

11. If a quantity of cloth cost 378 f., for how much must it be sold in order to gain 15 f. for each 100 f. expended?

12. If 8 men can mow a field in 12 days, in how many days should 6 men mow another field which is 3 times as big?

13. A clock is set correctly at midnight. On the following day, when the time by the clock is noon, the true time is 12.30 p.m. How much does the clock lose in 12 hours true time?

14. If 49 Kg. 140 g. cost £187. 4*s*., how much may be bought for £13. 17*s*. 4*d*.?

15. A train travels at the rate of 60 miles per hour; how many metres per minute does it go, if 5 centimetres = 2 inches?

16. If 17 ac. 2 ro. 38 per. supply 3 horses, how many acres will supply 12 cows, if 3 cows eat as much as 4 horses?

17. If 12 cwt. 2 qrs. 20 lb. of sugar cost £25, what is the value to the nearest penny of 35 Kg., if $2\frac{1}{5}$ lb. = 1 Kg.?

18. If the rent of 42 ac. 1 ro. 2 per. be £91. 17*s*. 6*d*., find the rent of 3 fields, each 5 ac. 3 ro. in area.

19. If 3 t. 7 q. 60 Kg. can be bought for £39. 19*s*., how much can be bought for 11*s*. $8\frac{1}{4}d$.?

20. If the yard is to the metre as 54 is to 59, find to the nearest integer the number of kilometres in 250 miles.

21. A picture 5 m. 1 dm. long, and 4 m. 8 cm. broad, is photographed on a reduced scale. The distance between two points on the picture is $8\frac{1}{2}$ cm.; and the distance between the corresponding points on the photograph is 5 mm. Find the dimensions of the photograph.

22. If 1 metre = 39·37079 in., find, to the nearest are, the equivalent of 1 acre in metric land measure.

23. Find, to the nearest shilling, the cost of 555 lb. of copper, the price of copper being £40 per ton.

24. If 31 metres of cable weigh 62·346 Kg., find, to the nearest metre, the length of cable which weighs 3250 Kg.

25. If 75·346 metres cost £57. 14*s*. $7\frac{1}{4}d$., find, to the nearest farthing, the cost of 115 metres 84 cm.

26. If 78 t. 945 Kg. cost £846, find, to the nearest penny, the cost of 500 Kg.

27. A bicycle wheel is 3·625 yards in circumference and travels at the rate of 43·5 miles in 4 hours. How many times will it revolve in a minute?

28. Silver is 10·5 times, and gold is 19·3 times, as heavy as water. Find, to the nearest grain, the weight of a gold cup, if a silver one of the same size weighs 8 oz. 14 dwt. 12 grs.

29. If 1 metre = 3·281 ft. and 1 Kg. = 2·205 lb., find, to the nearest ounce, the number of ounces in a cubic foot of water.

30. Lead is 11·4 times, and iron is 7·8 times, as heavy as water. How many cubic centimetres of lead will weigh as much as a bar of iron, 117 cm. long, whose cross section is a square of 9 mm. side? (Answer to nearest integer.)

(c). Fractional Multipliers and Equations.

317. In the preceding examples we have found the change in *one* quantity corresponding to the change in *one* other. Sometimes, however, *three* quantities are involved; two of them change in a given way, and we have to find the corresponding change in the third.

Example i. *If* 16 *horses can plough* 1280 *acres in* 8 *days, how many acres can* 12 *horses plough in* 5 *days?*

Here we have to find the change in the number of acres which corresponds to a change in the number of horses from 16 to 12, and in the time from 8 days to 5 days.

In 8 days 16 horses can plough 1280 acres,
∴ ... 1 day 16 ... . 160 ..... ,
∴ ... 5 days 16 ..... 800 ..... ,
∴ ... 5 ..... 4 ..... 200 ..... ,
∴ ... 5 ..... 12 ..... 600 ..... ;
∴ the number of acres = 600.

In the first three lines we have changed the time from 8 days to 5 days, without changing the number of horses; and in the last two lines we have changed the number of horses from 16 to 12.

Example ii. *The price of* 13 *sheep and* 11 *pigs is* £49, *and that of* 7 *sheep and* 33 *pigs is* £67 : *find the price of a sheep and of a pig.*

We will suppose £x and £y to represent the price of a sheep and of a pig, respectively. Then

$$13x+11y=49, \quad \text{.....(i)}$$

and

$$7x+33y=67. \quad \text{.....(ii)}$$

Multiplying the line (i) by 3 we get

$$39x+33y=147;$$

∴ by subtraction, $32x=80$;

$$\therefore\ x=\tfrac{80}{32}=2\tfrac{1}{2};$$

∴ from (ii), $7\times 2\tfrac{1}{2}+33y=67$;

$$\therefore\ 33y=67-17\tfrac{1}{2}=49\tfrac{1}{2};$$

$$\therefore\ y=\frac{49\frac{1}{2}}{33}=\tfrac{99}{66}=1\tfrac{1}{2};$$

∴ the price of a sheep = £2. 10*s*., and the price of a pig = £1. 10*s*.

Example iii. *If* 15 *pumps working* 8 *hours a day can raise* 1260 *tons of water in* 7 *days, how many pumps working* 12 *hours a day will be required in order to raise* 7560 *tons of water in* 14 *days?*

1*st method.* We will suppose x to represent the number of pumps required. It will be clear that what is done by x pumps in 14 days of 12 hours, will be $x\times 14\times 12$ *units*, a unit being what is done by 1 pump in 1 hour. Similarly, what is done by 15 pumps in 7 days of 8 hours, will be $15\times 7\times 8$ of the same units.

But what is done by the x pumps and the 15 pumps is the lifting of 7560 tons of water and of 1260 tons of water, respectively. Hence $x \times 14 \times 12$ must bear the same ratio to $15 \times 7 \times 8$ that 7560 bears to 1260;

$$\therefore \frac{x \times 12 \times 14}{15 \times 7 \times 8} = \frac{7560}{1260};$$

$$\therefore x = \frac{7560}{1260} \times \frac{15 \times 7 \times 8}{12 \times 14} = 30;$$

$\therefore$ the number of pumps required $= \underline{30}$.

2nd method. The required number of pumps is equal to the given number, multiplied by three correcting fractions; (i) for the change from 7 days to 14 days; (ii) for the change from 8 hours to 12 hours, and (iii) for the change from 1260 to 7560 tons.

(i) Owing to the change in *days*, fewer pumps will be wanted;

$\therefore$ we *diminish* the number by multiplying it by the *proper* fraction $\frac{7}{14}$.

(ii) Owing to the change in *hours*, fewer pumps will be needed;

$\therefore$ we *diminish* the number by multiplying it by the *proper* fraction $\frac{8}{12}$.

(iii) Owing to the change in *weight*, more pumps will be needed;

$\therefore$ we *increase* the number by multiplying it by the *improper* fraction $\frac{7560}{1260}$.

Hence the required number of pumps $= 15 \times \frac{7}{14} \times \frac{8}{12} \times \frac{7560}{1260}$

$$= 15 \times \tfrac{1}{2} \times \tfrac{2}{3} \times 6$$

$$= \underline{30}.$$

NOTE.—In applying the 2nd method a beginner may find it helpful to tabulate the given conditions and the altered conditions, side by side, thus:

Given conditions	*New conditions*
15 pumps	Required number of pumps
8 hours	12 hours
1260 tons	7560 tons
7 days	14 days

The changes of conditions are thus clearly stated, and the corresponding multiplying fractions can be readily determined.

EXAMPLES. 137.

1. The fare for a certain railway journey is 18 f. 70 c.; for another journey, 87 Km. longer than the former, the fare is 24 f. 20 c.; find the length of each journey.

2. One metre is equal to 3·28 English feet, and one old French foot is equal to 1·066 of an English foot. Express 40 French feet in metres.

3. What is a quarter's rent of 120 Ha. of land, if 270 f. 90 c. per annum be given for 3 Ha. 60 a.?

4. If 17 men can earn £6. 7*s*. 6*d*. in 2 days, how many men can earn £11. 5*s*. in 4 days, at the same rate?

5. If 32 horses eat 9 bushels of corn in 21 days, for how many days will 66 bushels feed 7 horses?

6. If 15 men earn £68. 5*s.* in 26 days, in how many days will 20 men earn £35?

7. If the wages of 4 men for 12 days be 153 f. 60 c., what would be the wages of 6 men for 10 days?

8. If a family of 7 persons can live on £140 for 28 weeks, how long can a family of 9 persons live on £135?

9. If 9 men reap a field of 8 acres in 12 hours, how many men should reap a field of 28 acres in 18 hours?

10. If the wages of 29 men for 54 days amount to £80. 9*s.* 6*d.*, how many men must work 12 days to receive £407?

11. If 3 fires consume 65 cwt. of coal in 26 days, in how many days will 12 fires consume 4 tons?

12. If 25 horses can be kept 13 days for £32. 10*s.*, how many can be kept 6 weeks for 20 guineas?

13. If pasturage for 28 cows for 11 days can be obtained for £7. 10*s.*, what should be paid for 33 cows for 63 days?

14. If 12 horses are fed for 17 days at a cost of £11. 1*s.*, for how many days can 4 horses be fed for £11. 14*s*, the price of food and the rate of consumption being the same in both cases?

15. If the carriage of 620 Kg. for 108 Km. be 11 f. 20 c., how far should 3 t. 720 Kg. be carried for 123 f. 20 c.?

16. If the carriage of 3 cwt. 3 qrs. 14 lb. for 104½ miles costs £1. 14*s.* 10*d.*, what should be charged for carrying 5 tons 93 miles?

17. If 2 horses can plough 7 acres of land in a day, how many horses will be required to plough 161 acres in 11½ days?

18. If 5 horses in 3 days can plough 21 Ha. 15 a. of land, how much should 16 horses plough in 8 days?

19. If 40 men can mow 19 ac. in 8½ days of 10 hours each, how many acres can 17 men mow in 50 days of 8 hours each?

20. If 120 men can build a house 60 ft. high in 15 days, how many men will it take to build one 55 ft. high in 10 days?

21. If 85 men can build a wall, 510 metres long, in 8 days, how many metres of a similar wall can 72 men build in 5 days?

22. How many hours a day must 24 men work, in order to accomplish as much in 5 days as 25 men, working 6 hours a day, could do in 4 days?

23. If 6 men can do a piece of work in 30 days of 9 hours each, how many men will it take to do 10 times the amount, if they work 25 days of 8 hours each?

24. If 7 men in 25 days of 10 hours earn £45. 12*s.* 6*d.*, how many hours a day should 12 men work to earn £54. 15*s.* in 21 days, at the same rate?

25. A person completes a journey of 1056 Km. in 12 days, travelling 11 hours a day: in how many days would he travel 480 Km. going 6 hours a day at the same rate?

26. A can do a piece of work in 2 days, and B can do it in $3\frac{1}{2}$ days. A's wages are 38*s.* 6*d.* a week, and B's wages are 27*s.* 6*d.* a week. What would A receive for doing a piece of work for which B gets £5. 10*s.*?

27. If 15 men can do a certain quantity of work in $12\frac{3}{8}$ days of $9\frac{4}{5}$ hours each, how many days of 11 hours each will 9 men require in order to do $4\frac{4}{7}$ times as much work?

28. If 12 men working 8 hours a day take 15 days to complete a piece of work, how long will 16 boys working 10 hours a day take to do the same work, the work of one man being equal to that of two boys in the same time?

29. If 33 men earn £288. 15*s.* in 25 days, how much will 17 women earn in 12 days, a woman's wages being $\frac{5}{7}$ of a man's?

30. If 3 ducks are worth 4 chickens, and 3 geese are worth 10 ducks, find the value of a goose; a pair of chickens being worth 4*s.* 6*d.*

31. If 3 Kg. of tea be worth 4 Kg. of coffee, and 6 Kg. of coffee be worth 20 Kg. of sugar, how many Kg. of sugar should be obtained in exchange for 9 Kg. of tea?

32. The price of a horse is as many times that of a cow as the price of a cow is that of a sheep; the price of a horse is £40. 10*s.*, while that of a sheep is £3. 2*s.* 6*d.*; what is the price of a cow?

33. 11 c.cm. of iron weigh as much as 7 c.cm. of lead. The price per metric ton of lead is £16, and of iron £5. Find the value of a block of lead, which is half the size of a block of iron worth £4. 4*s.* 7*d.*

34. The railway fare from Calais to Paris (240 kilometres) is 38 f. 16 c.; from London to Dover (70 miles) it is 14*s.* 7*d.* Compare the cost of travelling on the English and French railways; given that 1 Km. $= \frac{5}{8}$ mi., and 25·44 f. = £1.

35. A contractor undertook to make a railway $97\frac{1}{2}$ Km. long in 40 weeks. He employed 2160 men, and at the end of 13 weeks $32\frac{1}{2}$ Km. were finished. How many men had he then to pay off, so that the work might not be completed sooner than required?

36. When wheat is 5*s.* a bushel, 8 men can be fed for 12 days at a certain cost. For how many days can 6 men be fed for the same cost, when wheat is 4*s.* a bushel?

37. If a sixpenny loaf weigh 4·35 lb., when wheat is 5·75*s.* a bushel, what should be paid for 98·6 lb. of bread when wheat is 9·2*s.* a bushel?

38. If a quartern loaf (4 lb.) cost $5\frac{1}{2}d.$ when wheat is 38*s.* a quarter, find, to the nearest farthing, the cost of a 2 Kg. loaf when wheat is 16*s.* per Hl. [5 Kg. = 11 lb.; 4 Hl. = 11 bush.]

39. The cost of keeping 25 horses being at the rate of £11. 6*s.* $0\frac{1}{2}d.$ per week, what will be the cost of keeping 13 horses from September 1 to October 31, including both days?

40. 600 men were besieged in a town with provisions for 35 days, allowing each man 18 oz. a day. At the end of a fortnight, the garrison is reinforced by 300 men and the daily allowance is reduced to 14 oz. a day. How long can they hold out after the reinforcement?

41. If a pair of blankets each measuring 6 ft. 9 in. by 5 ft. 3 in. cost 10*s.* 6*d.*, what will be the cost of a pair of like thickness and quality, each measuring 9 ft. 9 in. by 8 ft. 3 in.?

42. If 15 horses and 148 sheep can be kept for 9 days for £75. 15*s.*, what sum will keep 10 horses and 132 sheep for 8 days, supposing that 5 horses eat as much as 84 sheep?

43. If 5 men and 7 boys can reap a field of 125 acres in 15 days, in how many days will 10 men and 3 boys reap a field of 75 acres, each boy's work being one-third of a man's?

44. If 200 men can make an embankment 5 miles long in 25 days, how much overtime must 60 men work to finish an embankment 2 miles long in 32 days, 12 hours being a day's work?

45. If the cost of printing a book of 320 leaves, with 21 lines on each page, and on an average 11 words in each line, be £19, find that of printing a book with 297 leaves, 28 lines on each page, and 10 words in each line.

46. If 5 men working 8 hours a day dig a trench, 105 ft. long, 6 ft. wide and 1 foot deep, in 3 days, how many hours a day must 32 men work in order to complete a trench, 126 ft. long, 22 ft. wide and 10 ft. deep, in 15 days?

47. If 200 men in 12 days of 8 hours each can dig a trench 160 m. long, 6 m. wide and 4 m. deep, in how many days of 10 hours will 90 men dig a trench, 450 m. long, 4 m. wide and 3 m. deep?

48. If 15 men be necessary to excavate 966 cu. yds. in 8 days, working $10\frac{1}{2}$ hours daily, how many men would be required to excavate 575 cu. yds. in 12 days, working $7\frac{1}{2}$ hours daily, 4 extra men being taken on for the last 4 days?

49. Four reapers begin to cut a field of barley; after working for 5 days they have cut 10 acres: two additional men who work at the same rate are then employed, and the whole is cut in 3 days more. How many acres does the field contain?

50. A man contracts to do some work in a certain time, and employs 15 men, who work 9 hours a day. When $\frac{2}{3}$ of the time has passed, it is found that only $\frac{3}{5}$ of the work is done. If 3 more men be then employed, how many hours a day must all work in order that the whole may be finished in the required time?

51. If 11 men and 4 boys earn £25 in one week, and 8 men and 7 boys earn £85 in 4 weeks, how much will 9 men and 12 boys earn in 7 weeks?

52. If 35 men can do a piece of work in 45 days, and if 7 men drop off every 15 days, find the time that will elapse before the work is finished.

53. If 12 men could do a certain work in 9 days of 10 hours each, how many men should be set to do the same work in 15 days of 8 hours each, if 3 men join them at the beginning of the fourth day, and 4 more at the beginning of the tenth day?

54. A farmer engaged a number of men and boys to reap corn. The men were to receive 4*s.* a day, and the boys 2*s.* 6*d.*; if the work of three boys was equal to that of 2 men, and 4 men and 5 boys could reap 22 acres in 4 days, what sum should 6 men and 7 boys receive for reaping 48 acres?

55. The railway between A and B ascends at a gradient of 1 in 275 from A to C, distant $16\frac{1}{4}$ miles from A. It then descends at a gradient of 1 in 165 from C to B. If the station at B is 28 feet lower than that at A, find the distance from A to B.

56. If 10 c.cm. of gold and 16 c.cm. of silver weigh 361 g., and 7 c.cm. of gold and 22 c.cm. of silver weigh 366·1 g., find the weight of a rectangular silver plate, 12 cm. long, 7 cm. wide and ·5 mm. thick, coated on one side with gold of a uniform thickness of ·04 mm.

57. If the corn eaten by 30 horses in 20 days cost £60 when corn is 5*s.* per bushel, how much will the corn eaten by 26

ponies in 8 weeks 4 days cost when corn is 5*s.* 6*d.* per bushel, assuming that 8 horses eat as much as 13 ponies?

58. If 3 engines, each of 520 horse-power, can raise $60\frac{1}{2}$ million gallons of water to the height of 260 ft., by working $57\frac{7}{24}$ days of 8 hours each; in how many days of 11 hours each would 2 engines, each of 832 horse-power, lift $49\frac{1}{200}$ million gallons of water to the height of 104 ft.?

XXXV. PERCENTAGES.

318. In many cases we express the fraction which one quantity is of another, or which a part is of the whole, by a symbol which has 100 for its denominator; the numerator being an integer, or a fraction, or a mixed number. The first quantity is thus estimated *in hundredths*, or as a **percentage**, of the other.

For example, $3s. = \frac{3}{20}$ of £1 $= \frac{15}{100}$ of £1;

i.e. 3*s.* is 15 *hundredths*, or 15 **per cent.**, of £1.

Also $1s.\ 7d. = \frac{19}{50}$ of $4s.\ 2d. = \frac{38}{100}$ of $4s.\ 2d.$;

i.e. $1s.\ 7d.$ = 38 **per cent.** of $4s.\ 2d.$

Similarly, 6 pence $= \frac{1}{40}$ of £1 $= \frac{5}{200}$ of £1 $= \frac{2\frac{1}{2}}{100}$ of £1;

i.e. 6 pence $= 2\frac{1}{2}$ **per cent.** of £1.

319. The *numerator*, which expresses the number of hundredths, is called the **rate per cent.** For example, in the cases considered in Art. 318 the *rates per cent.* are 15, 38 and $2\frac{1}{2}$. The words *per cent.* are briefly expressed by the symbol %, or by p. c. Thus $3\frac{1}{4}$ *per cent.* may be written $3\frac{1}{4}\%$ or $3\frac{1}{4}$ p. c.

EXAMPLES. 138. *Oral.*

What fractions are denoted by the following rates per cent.?

1.	1.	2.	2.	3.	3.	4.	4.	5.	5.
6.	6.	7.	7.	8.	8.	9.	10.	10.	20.
11.	25.	12.	40.	13.	50.	14.	75.	15.	100.
16.	125.	17.	150.	18.	200.	19.	500.	20.	$2\frac{1}{2}$.
21.	$8\frac{1}{3}$.	22.	$12\frac{1}{2}$.	23.	$33\frac{1}{3}$.	24.	$66\frac{2}{3}$.	25.	$\frac{1}{2}$.
26.	$\frac{1}{4}$.	27.	$\frac{1}{3}$.	28.	$\frac{1}{5}$.	29.	$\frac{3}{5}$.	30.	$\frac{3}{10}$.

EXAMPLES. 139. *Oral.*

What rates per cent. are denoted by the following fractions?

1. $\frac{1}{2}$.	2. $\frac{1}{4}$.	3. $\frac{1}{5}$.	4. $\frac{1}{10}$.	5. $\frac{1}{20}$.
6. $\frac{3}{20}$.	7. $\frac{1}{25}$.	8. $\frac{7}{25}$.	9. $\frac{1}{50}$.	10. $\frac{13}{50}$.
11. $\frac{1}{3}$.	12. $\frac{1}{6}$.	13. $\frac{1}{7}$.	14. $\frac{1}{8}$.	15. $\frac{1}{9}$.
16. $\frac{1}{11}$.	17. $\frac{1}{12}$.	18. $\frac{1}{80}$.	19. $\frac{1}{40}$.	20. $\frac{3}{40}$.
21. $\frac{3}{5}$.	22. $\frac{7}{10}$.	23. $\frac{3}{4}$.	24. $\frac{5}{4}$.	25. $\frac{101}{100}$.
26. $\frac{103}{50}$.	27. $\frac{5}{2}$.	28. $\frac{2}{3}$.	29. $\frac{1}{75}$.	30. $\frac{11}{400}$.

To Find a Percentage.

320. To find a percentage of a number or quantity, we first express the percentage as a vulgar or decimal fraction.

Thus $16\frac{2}{3}$ per cent. of a day $= \frac{16\frac{2}{3}}{100}$ of a day $= \frac{50}{300}$ of a day

$= \frac{1}{6}$ of a day $= 4$ hrs.

EXAMPLES. 140. *Oral.*

Find the value of the following percentages:—

1. 5 per cent. of £20, £40, £100, £70, £150.
2. 3 per cent. of £50, £100, £150, £200.
3. $2\frac{1}{2}$ per cent. of £10, £40, £60, £100.
4. 8 per cent. of £50, £100, £75, 4 f., 2 dol.
5. 10 per cent. of £20, £50, £80, £25, £175, 1 dol.
6. 20 per cent. of 10 yds., £50, 5 Kg., 1 Km.
7. 75 per cent. of 1 ton, 1 cwt., 1 acre, 1 rood, £1, 1 f.

EXAMPLES. 141.

Find the value of

1. $12\frac{1}{2}$ per cent. of 144 mi.	2. $16\frac{2}{3}$ per cent. of a foot.
3. 5 per cent. of an acre.	4. 20 per cent. of a cu. ft.
5. $66\frac{2}{3}$ per cent. of a sq. ft.	6. 3 per cent. of £$123\frac{1}{3}$.
7. $\frac{3}{4}$ per cent. of a ton.	8. 9·75 % of 5000 tons.
9. 35 per cent. of 6940 men.	10. ·875 % of 5168 miles.
11. 7 per cent. of 1345 f.	12. $8\frac{1}{3}$ % of 89 Km. 58 Dm.
13. $4\frac{1}{6}$ % of 202 Kg. 944 g.	14. $12\frac{1}{2}$ % of 16 gal. 2 qts. 1 pt.
15. $33\frac{1}{3}$ % of 1482 ac. 2 ro. 10 per.	16. 28 % of £8569. 17*s.* 6*d.*
17. 6 % of 15 cwt. 1 qr. 12 lb.	18. $7\frac{1}{2}$ % of 3 mi. 4 fur.

19. A man has an income of £3780. If he spends $66\frac{2}{3}$ per cent. of it, how much does he save?

20. A man left £4275 to his wife and two children. His wife received $33\frac{1}{3}\%$ of the whole; what amount did each child get?

21. A house is bought for 2425 dol. Twenty per cent. of the money is paid at once; how much remains?

22. How many persons are engaged in agriculture, when they constitute 24 per cent. of a population of 30725?

23. A man has 5150 f. If he should increase it by 40 per cent., how much would he have then?

24. Goods cost £9675. 15*s.*, and are sold for 16 per cent. more; how much were they sold for?

25. What is the difference in square yards between $\frac{1}{2}$ an acre and $\frac{1}{2}$ per cent. of an acre?

To Find the Rate Per Cent.

321. To find the rate per cent. we express the one quantity as a vulgar fraction of the other, and we convert this fraction into a fraction having 100 for its denominator.

Example i. *What per cent. of* $62\frac{1}{2}$ *bushels is* 16 *bushels?*

$$\text{The fraction} = \frac{16}{62\frac{1}{2}} = \frac{32}{125} = \frac{128}{500} = \frac{25\frac{3}{5}}{100},$$

$$\therefore \text{ rate per cent.} = 25\tfrac{3}{5} \text{ or } 25{\cdot}6.$$

Example ii. *What per cent. of a ton is* 14 *lb.?*

$$\text{The fraction} = \frac{14}{2240} = \frac{1}{160} = \frac{\frac{1}{16}}{10} = \frac{\frac{5}{8}}{100},$$

$$\therefore \text{ rate per cent.} = \tfrac{5}{8}.$$

EXAMPLES. 142.

What per cent.

1. of £28 is £14?
2. of £35 is £7?
3. of 75 lb. is 45 lb.?
4. of £7. 10*s.* is £5?
5. of 1 bush. is 4 qts.?
6. of ·625 is $\frac{1}{8}$?
7. of 12 cwt. 2 qrs. 20 lb. is 5 cwt. 2 qrs. 23 lb.?
8. of 3 pks. 2 qts. is 1·04 qts.?
9. of 4 hrs. 25 min. 12 sec. is 2 hrs. 12 min. 36 sec.?
10. A cask contained 42 gallons of which 14 gallons escaped; what per cent. was this loss?

11. A car proprietor had 160 horses in 1903, and 172 in 1904; what is the increase per cent.?

12. On a plantation containing 4000 trees, 20 trees died; what per cent. remained?

13. The population of a town was 5600 in 1891 and 4802 in 1901; what per cent. was the decrease?

14. Of a farm containing 1640 acres, there are 246 acres in meadow; what per cent. remains?

15. One man owed £235 to another, and has paid £110; what per cent. of the debt remains unpaid?

To Find the Quantity, given the Percentage and the Rate Per Cent.

322. The student must notice the distinction between *percentage* and *rate per cent.* If, for example, £20 be the given quantity and 10 the *rate per cent.*, the *percentage* is £2.

Example i. *Find the number of which* 16 *is* 24 *per cent.*

Suppose x to be the required number; then

$$\tfrac{24}{100} \text{ of } x = 16;$$

$$\therefore \text{ the required number} = \tfrac{100}{24} \times 16 = \tfrac{200}{3} = \underline{66\tfrac{2}{3}}.$$

Example ii. £235. 10*s.* *is* $15\tfrac{7}{10}$ *per cent. of what amount?*

$$\frac{15\tfrac{7}{10}}{100} \text{ of the required amount} = £235.\ 10s.;$$

$$\therefore \tfrac{157}{1000} \text{ of the required amount} = £235\tfrac{1}{2};$$

$$\therefore \text{ the required amount} = \tfrac{1000}{157} \times £235\tfrac{1}{2} = \frac{£1000 \times 471}{157 \times 2} = \underline{£1500.}$$

EXAMPLES. 143.

Of what number is

1. 96, 6 per cent.?
2. 960, $33\tfrac{1}{3}$ per cent.?
3. 114, $4\tfrac{3}{4}$ per cent.?
4. 1·04, 4 per cent.?
5. 7·80, $\tfrac{3}{4}$ per cent.?
6. $235\tfrac{1}{2}$, $15\tfrac{7}{10}$ per cent.?

7. A man owes £67. 10*s.*, which is 75 per cent. of what he possesses; how much does he possess?

8. How much did a barrel contain, if 10·08 gallons was 16 per cent. of it?

9 £393 is $13\tfrac{1}{10}$ per cent. of a man's property; how much does he possess?

10. £16 is $\frac{2}{5}$ per cent. of the value of a house; what is the house worth?

11. A man spends 65 per cent. of his income, and saves 4340 f.; what is the amount of his income?

12. The population of a certain town is 12175, and is 25 per cent. more than it was five years ago; what was it then?

Miscellaneous Examples on Percentages.

323. The following examples are somewhat more complicated than those which we have considered hitherto, but the principles involved are the same. They should be studied carefully, so that the method of treatment may be clearly understood.

Example i. *Two sides of a rectangle are measured. The measure of one side is too long by 4 % of its true length, and that of the other is too short by 5 % of its true length. Find the error per cent. in the measure obtained for the area of the rectangle.*

We will suppose x *units* and y *units* to be the true lengths of the two sides. The wrong measurements are therefore $\frac{104}{100}x$ units and $\frac{95}{100}y$ units, respectively. Consequently, as obtained from these measurements,

the area of the rectangle $= \frac{104}{100}x \times \frac{95}{100}y$ units of area

$= \frac{26}{25}x \times \frac{19}{20}y$ units of area

$= \frac{494}{500} \times x \times y$ units of area

$= \frac{98{\cdot}8}{100}$ of the true area.

The calculated area is therefore too small by 1·2 per cent. of the true area.

Example ii. *If the duty on an article be reduced by 40 % of its present amount, by how much per cent. must the consumption increase in order that,* (i) *the revenue may be unaltered;* (ii) *the revenue may be increased by 10 %?*

We will suppose that x is the number of articles consumed originally, and that y shillings is the duty on each article. The present revenue derived from them is therefore $x \times y$ shillings.

The new duty on each article $= \frac{60}{100}$ of y shillings $= \frac{3}{5}$ of y shillings.

(i) If unaltered, the new revenue will be $x \times y$ shillings, as before;

$\therefore$ number of articles to be consumed $= xy \div \frac{3}{5}y = \frac{5}{3}x = \frac{500}{300}x = \frac{166\frac{2}{3}}{100}x$

$= \frac{166\frac{2}{3}}{100}$ of the present number;

$\therefore$ the consumption must be increased by $66\frac{2}{3}$ %.

(ii) If increased by 10 %, the new revenue will be $\frac{110}{100}xy$ shillings;

$\therefore$ number of articles to be consumed $= \frac{110}{100}xy \div \frac{3}{5}y = \frac{550}{300}x = \frac{183\frac{1}{3}}{100}x$

$= \frac{183\frac{1}{3}}{100}$ of the present number;

$\therefore$ the consumption must be increased by $\underline{83\frac{1}{3}\ \%}$.

EXAMPLES. 144.

1. After paying 12 per cent. of his income in rates and taxes, a man has £299. 4*s*. left. Find his whole income.

2. For a monthly ticket 25 per cent. more is paid than for a single ticket. The holder obtains an extension of time for a week by paying 5 per cent. extra on the monthly ticket. If he pays £2. 12*s*. 6*d*. in all, find the price of a single ticket.

3. Ten per cent. of a battalion were absent from drill on a certain day, and of those 75 % were sick. If the number present at drill was 576, find how many of those absent were not sick.

4. In measuring the side of a square, an error of 5 % in excess is made. Find the error per cent. in the calculated area.

5. In measuring the sides of a rectangle, one side is taken 5 per cent. in excess, and the other 4 per cent. in defect. Find the error per cent. in the area calculated from these measurements.

6. What is the total number of candidates at an examination, if 31 per cent. fail, and the number of those who pass exceeds the number of those who fail by 247?

7. An army loses $12\frac{1}{2}$ per cent. of its number through disease, and 15 per cent. of the rest in battle. If 108,290 men remain, how many were there originally?

8. If a side of a square be measured 2 per cent. in defect, and the calculated area be 72 Ha. 3 a., find the true area.

9. The difference between the number obtained by increasing a certain number by 8 per cent., and that obtained by diminishing it by 3 per cent., is 407. Find the original number.

10. If 6 per cent. of an army of 40,850 men desert to the enemy, who, after the increase, number 84,151 men; by how much per cent. was the number of the enemy increased?

11. If the numerator of a fraction be increased by 15 per cent., and its denominator be diminished by 8 per cent., the value of the fraction is $\frac{15}{16}$. Find the original fraction.

12. If a man's wages are increased by 10 % and afterwards decreased by 10 %, what is the total change per cent.?

13. A's income was to B's income as 5 is to 4. A's income was increased by 5 per cent., and B's income at a different rate. Later on, their incomes were again increased at the same rates as before, and A's income then was to B's as 1125 is to 1024. At what rate per cent. was B's income increased?

14. In 1881 the population of a certain town had increased 5 % on that of 1871. In 1891 the increase was 10 % on the population in 1881, and in 1901 it was 15 % on that of 1891. What was the increase % in the 30 years from 1871 to 1901?

15. If the tax on tea be decreased by 25 per cent., by how much per cent. must the consumption be increased in order that the total revenue, derived from the tax, may be the same?

16. If the tax on sugar be decreased by 5 %, by how much per cent. must the consumption be increased, in order that there may be an increase of 14 per cent. in the total amount received?

17. If the price of an article be lowered by 4 per cent., 5 more articles can be bought for £1. Find the price of an article before the reduction.

In working the following questions contracted methods should be used.

18. If the total revenue for a year be £143,581,500, and the tax on tea produces £5,793,000, what per cent. of the whole are the proceeds of the tax on tea, to 2 decimal places?

19. If the total expenditure for a year be £183,592,264, and the army cost £91,710,000, and the navy £29,520,000, express the cost of the navy and army as a percentage of the whole, to one decimal place.

20. If the population of England and Wales in 1861 was 20,066,224, and in 1891, 29,002,165, find the increase per cent in the 30 years, to 2 decimal places.

21. For 1896 the gross receipts of the L. & N.W.R. were £12,333,655, and the working expenses were £6,754,187. Find, to 2 decimal places, the profit per cent. on the gross receipts.

XXXVI. COMMISSION, RATES AND TAXES, ETC.

324. An **agent** is a man who acts for another.

When a person employs an agent to buy or sell a house, a horse, or property of any kind, he pays the agent for his trouble by giving him a *percentage* on the money paid or received, or upon the amount of property bought or sold. The money thus paid

to the agent is called his **commission**. The agent is sometimes called a **broker**, and the commission **brokerage**.

An insurance company, in return for certain payments, undertakes to make good a loss incurred through fire or shipwreck, or to pay a certain sum of money after a man's death to his relatives or friends. The person who *insures* his property, or his life, pays annually to the company a certain percentage of the sum which he or his friends are to receive if fire, shipwreck or death should occur. The percentage is in this case called a **premium**.

325. Tradesmen frequently encourage customers to pay ready money by allowing a reduction on the amount of the bill, if it is paid within a certain short time. This reduction is usually a percentage on the amount of the bill, and is called **Tradesman's discount**.

Example i. *An estate agent buys a house for £5250, and receives a commission of 5s. per cent.; what does he get?*

$$\text{Commission} = \frac{\frac{1}{4}}{100} \times £5250 = £\frac{5250}{400} = £13\tfrac{1}{8} = \underline{£13.\ 2s.\ 6d.}$$

Example ii. *An annual premium of £2. 14s. per cent. is paid on an insurance of £12550; what does the payment amount to?*

$$\text{Annual payment} = \frac{2\frac{7}{10}}{100} \times £12550 = £\frac{27}{1000} \times 1255$$

$$= £\frac{27 \times 251}{20} = 6777 \text{ shillings}$$

$$= \underline{£338.\ 17s.}$$

Example iii. *If a tradesman's bill for £25. 15s. 6d. be subject to 2½% discount for ready money, what money should be sent in payment of it?*

$$\text{The discount} = \tfrac{1}{40} \text{ of } £25{\cdot}775 = £{\cdot}644 = 12s.\ 10\tfrac{1}{2}d.;$$

$$\therefore \text{ amount to be paid} = \underline{£25.\ 2s.\ 7\tfrac{1}{2}d.}$$

326. A man is **bankrupt**, or **insolvent**, when the money which he owes is more than that which he can use in payment. What he owes is called his **liabilities**, or **debts**; what he possesses is called his **assets**. He is the **debtor**; those to whom he owes anything are his **creditors**. Each creditor is supposed to receive the same fraction of the assets that the money due to him is of the debtor's whole liabilities. If the assets amount, for example, to $\frac{2}{3}$ of the debts, each creditor receives $£\frac{2}{3}$ for each £1 that is due to him; and the debtor is said to pay **a dividend** of 13*s.* 4*d.* in the £.

Example i. *A bankrupt pays* 12*s.* 8*d. in the pound, and his assets amount to* £950*; what is the amount of his debts?*

To each 12*s.* 8*d.* in the assets corresponds a debt which = £1,

$\therefore$ ..... 4*d.* ..... $= £\frac{1}{3\,8}$,

$\therefore$ ..... £1 ..... $= £\frac{6\,0}{3\,8} = £\frac{3\,0}{1\,9}$,

$\therefore$ ..... £950 ..... $= £\frac{3\,0}{1\,9} \times 950$;

$\therefore$ total debts = £1500.

327. Rates are levied by a local authority on property situated in its district. The sum upon which it is levied is called the **rateable value** or **assessment** of the property, and usually bears a certain ratio to the annual rent. **Taxes** are levied by the Imperial Parliament. Rates and taxes are usually calculated at so much in the £. The amount payable can readily be ascertained by expressing the sum, on which the rate or tax is levied, as the decimal of a £.

Example i. *Find the amount of a poor-rate at* 2*s.* 4*d. in the* £*, when the sum upon which it is levied is* £567. 13*s.* 6*d.*

10 | £567·675 = rateable value;

56·7675 = amount at 2*s.* in the £;

4*d.* = $\frac{1}{6}$ of 2*s.*; $\therefore$ 9·4613 = ... 4*d.* .....£;

Amt. of rate = £66·229 = £66. 4*s.* 7*d.*

Example ii. *After paying income-tax at the rate of* 11*d. in the* £*, a man has* £629. 15*s. left. What was his original income?*

Out of every 240*d.* of income, he pays 11*d.* in income-tax and has 229*d.* left. If £x be his income, it follows that $\frac{2\,2\,9}{2\,4\,0}$ of £x is the amount of what he has left;

$\therefore \frac{2\,2\,9}{2\,4\,0}x = 629\frac{3}{4}$;

$\therefore x = \frac{2\,4\,0}{2\,2\,9} \times 629\frac{3}{4} = \frac{2\,4\,0}{2\,2\,9} \times \frac{2\,5\,1\,9}{4} = 660$;

$\therefore$ his original income = £660.

EXAMPLES. 145.

1. A broker purchases £20000 worth of goods; what is his commission at $\frac{1}{4}$ per cent.?

2. An agent collects debts to the amount of £481. 10*s.*; what is his commission at $2\frac{1}{2}$ per cent.?

3. A broker buys £11000 worth of goods; what is his commission at $\frac{7}{8}$ per cent.?

4. What is the cost of insuring a cargo valued at £6750, the premium being 3 per cent.?

5. Find the commission on the sale of 736 barrels of flour for £1. 5*s.* per barrel, the rate of commission being $2\frac{1}{2}$ per cent.

6. An agent buys land for £10500, and receives a commission of $\frac{1}{4}$ per cent.; what has his employer to pay altogether?

7. If a tradesman allow a reduction of 5 per cent. for cash, what may a customer pay whose bill amounts to £15. 16*s.* 6*d.*?

8. A tax-collector collects £5692. 15*s.* at $1\frac{1}{2}$ per cent.; what commission does he retain, and how much money does he pay over?

9. A house is insured for £7680 at a premium of $\frac{1}{10}$ per cent.; what is the annual payment?

10. An agent received 5 per cent. for buying cotton. His commission amounted to £432. 10*s.*; what was the value of the cotton bought?

11. Find the premium on a policy of assurance for £1258. 5*s.*, at $6\frac{1}{4}$ per cent.

12. A ship is worth £34000, and is insured for $\frac{3}{4}$ of its value; what does the premium amount to at 3 per cent.?

13. A house is insured for $\frac{3}{4}$ of its value at $1\frac{1}{2}$ per cent., and the premium is £18; what is the value of the house?

14. If a tradesman allow $2\frac{1}{2}$ % discount for cash, what would he receive in payment of a bill amounting to £67. 19*s.* 3*d.*?

15. Find the income-tax on £16241. 5*s.* at 13*d.* in the £.

16. Find the dividend on £274. 10*s.* at 8*s.* 3*d.* in the pound.

17. What is the amount to be paid on a rental of £74. 10*s.* for rates amounting to 3*s.* 4*d.* in the pound?

18. A bankrupt's debts are £4264, and he pays 7*s.* 6*d.* in the pound; what are his assets?

19. If the income-tax be at the rate of 5 pence in the pound, what will have to be paid on an income of £980. 10*s.*?

20. How much will a poor-rate of 2*s.* 8*d.* in the pound produce in a parish where the whole property is rated at £4736. 5*s.*?

21. If the income-tax be at the rate of 6*d.* in the £, and a man has to pay £13. 6*s.* 8*d.*, what is the amount of his income?

22. If I pay £19. 13*s.* 9*d.* per annum for income-tax, at the rate of 5 pence in the pound, what is my income?

23. What is the income of a man whose income-tax, at 4 pence in the pound, amounts to 23 guineas?

24. If I am promised a discount of $12\frac{1}{2}$ % off a bill for £16. 16*s.* 8*d.*, and find that only 10 % has been deducted, how much more am I entitled to deduct?

25. A bankrupt's effects amount to £1980, and he pays his creditors 13*s.* 6*d.* in the pound; what do his debts amount to?

26. A bankrupt's assets are £613. 2*s.* 8*d.*, and his debts are £3678. 16*s.*; how much can he pay in the pound?

27. A person, after paying income-tax on £515, has £484. 19*s.* 2*d.* left; how much did he pay in the pound?

28. If a man has to pay £18. 15*s.* for income-tax on an income of £1500, what is the rate of tax per pound?

29. A man pays a house-rate of 1*s.* 6*d.* per £1 on his rental; a water-rate of 1*s.* per £1; a poor-rate of 1*s.* $10\frac{1}{2}d.$ per £1. If the rent and rates amount to £85. 6*s.* 3*d.*, what is the rent?

30. A person after paying 7 pence in the £1 for income-tax has £349. 10*s.* left. What was his gross income?

31. A person after paying 3 pence in the pound for income-tax has £1007. 14*s.* $10\frac{1}{2}d.$ remaining; what had he at first?

XXXVII. PROFIT AND LOSS.

328. When a thing is sold for more than it cost, it is said to be sold **at a profit**: when it is sold for less, it is said to be sold **at a loss**. The amount of *profit* in the one case, and of *loss* in the other, is the difference between the amount for which the thing is bought and the amount for which it is sold.

329. Profit or loss may be reckoned in general terms as a percentage **of the money originally spent**. It must be noticed carefully that **it is reckoned on the outlay, and not on the amount for which the goods are sold.**

For example, a man, who buys goods for £100 and sells them for £110, makes a *profit* of £10 on an outlay of £100, *i.e.* a profit of 10 per cent. And if he buys them for £100 and sells them for £90, he *loses* £10 on an outlay of £100, *i.e.* he loses 10 per cent.

330. CASE I. **To find the gain or loss per cent.**

Example i. *A grocer buys coffee at* 10*d. per lb., and sells it at* 1*s.* 4*d. per lb.; what is his gain per cent.?*

Gain on 10*d.* = 6*d.*;
∴ gain on 100*d.* = 60*d.*;
∴ gain per cent. = 60.

Example ii. *A grocer sells tea at* 1*s.* 6*d. per lb. If it cost him* 2*s. per lb., what is his loss per cent.?*

Loss on 24*d.* = 6*d.*;
∴ loss on 4*d.* = 1*d.*;
∴ loss on 100*d.* = 25*d.*;
∴ loss per cent. = 25.

EXAMPLES. **146.**

What is the gain or loss per cent. in the following cases:

1. If a brooch which cost £3 is sold for £3. 15*s.*?

2. If a dealer bought a gross of toys for 10*s.*, and sold them at a penny each?

3. If coal cost 9*d.* per cwt., and is sold at 22*s.* 6*d.* per ton?

4. If a newspaper boy buys papers at 9*d.* per dozen (13 to the dozen), and sells them at a penny each?

5. If sugar, costing 80 f. per quintal, is sold at 60 c. per Kg.?

6. If sugar, bought at £18. 13*s.* 4*d.* per ton, is sold at $1\frac{1}{4}d.$ per lb.?

7. If I buy 6 ponies for £170 and 4 cows for £38, and sell the ponies at £38 each and each cow for £8?

8. If tea cost 3 f. 50 c. per Kg., and is sold at 4 f. 20 c. per Kg.?

331. Case II. **To find the selling price,** when the cost price and the gain or loss per cent. are given.

Example i. *A house, bought for* £9600, *is sold at a profit of* $12\frac{1}{2}$ *per cent.; what does the profit amount to, and for how much is the house sold?*

$$\text{Profit} = \frac{12\frac{1}{2}}{100} \text{ of } £9600 = £25 \times 48 = £1200;$$

$\therefore$ selling price of house $= £9600 + £1200 = \underline{£10800}.$

Otherwise, more directly:

$$\text{selling price of house} = \frac{112\frac{1}{2}}{100} \text{ of } £9600 = \tfrac{9}{8} \text{ of } £9600 = \underline{£10800}.$$

Example ii. *A house, bought for* £1350, *is sold at a loss of* 20 *per cent.; what does the loss amount to, and for how much is the house sold?*

$$\text{Loss} = \tfrac{20}{100} \text{ of } £1350 = £270;$$

$\therefore$ selling price of house $= £1350 - £270 = \underline{£1080}.$

EXAMPLES. **147.**

1. An estate is bought for £8750, and is sold at a profit of 8 per cent.; what was the amount of profit, and for how much was the estate sold?

2. At what price must spice, costing 18*s.* 9*d.* per lb., be sold to gain 12 per cent.?

3. A man bought a horse for £45, and sold it at a loss of 7 per cent.; for how much was the horse sold?

4. If 128 metres of silk cost £28. 15*s*, for how much must the whole be sold so as to gain 9 per cent.?

5. A man buys goods for £54. 16*s*. 8*d*., and sells them at a loss of 30 per cent.; for what amount does he sell them?

6. If a grocer buys sugar at £1. 12*s*. 8*d*. per cwt., at what price per lb. must he retail it to gain $7\frac{1}{7}$ per cent.?

7. A man bought 100 gallons of wine for £125, and sold it at a profit of 6 %. At what price did he sell it per gallon?

8. A plumber bought 4 tons 16 cwt. of lead at £1. 0*s*. $2\frac{1}{2}d$. per cwt., and he sold the whole to clear $12\frac{1}{2}$ per cent.; what did he receive from the sale?

9. A farmer rents $305\frac{1}{2}$ Ha. of land at 390 f. per Ha., but the landlord allows him an abatement of 20 per cent. What rent altogether does the farmer actually pay?

332. Case III. **To find the cost price,** when the selling price and the gain or loss per cent. are given.

Example i. *Goods are sold for* £11500, *at a profit of* 15 *per cent. on the cost price. What did they cost?*

£11500 = cost price + $\frac{15}{100}$ of cost price

$= \frac{115}{100}$ of cost price

$= \frac{23}{20}$ of cost price;

∴ cost price = $\frac{20}{23}$ of £11500 = £10000.

Example ii. *A cow is sold for* £19. 4*s*., *at a loss of* 20 *per cent. on the cost price. What did it cost?*

£19. 4*s*. = cost price − $\frac{1}{5}$ of cost price = $\frac{4}{5}$ of cost price;

∴ cost price = $\frac{5}{4}$ of £19. 4*s*. = £24.

EXAMPLES. 148.

Find the cost price in the five following cases:—

1. When a ring is sold for £31. 10*s*., at a gain of 5 per cent.
2. When a horse is sold for £90, at a loss of 28 per cent.
3. When a clock is sold for £26, at a gain of $8\frac{1}{3}$ per cent.
4. When a set of china is sold for £63, at a profit of 8 per cent.
5. When goods are sold for £512, at a loss of 4 per cent.
6. A grocer by selling sugar at $3\frac{1}{2}d$. per lb. loses 2 per cent. on his outlay; what did the sugar cost him per cwt.?
7. A tobacconist sells cigars at 75 c. each, and makes a profit of $42\frac{6}{7}$ per cent. on his outlay; what was the cost price of 100?

8. By selling ten houses for £3059. 10*s.* a man gained $5\frac{1}{2}$ per cent. What did each house cost him originally?

9. Find the cost of goods per ton, if the sale of 16 tons for £21. 8*s.* 9*d.* yields a profit at the rate of 5 per cent.

10. Goods are sold at the rate of 12 f. 85 c. per 50 Kg., and a profit of 20 per cent. is made. Find the cost of 3 metric tons.

11. I sell 512 metres of silk for £89. 16*s.* 8*d.*, and realise a profit of $9\frac{3}{8}$ %. At what price per metre did I buy the silk?

333. CASE IV. **To compare the gain or loss per cent., when two selling prices are given.**

Example. *By selling goods for* £9. 10*s.*, *a man loses* 5 %. *What would be his loss or gain per cent., if he had sold them for* £11. 17*s.* 6*d.*?

£9. 10*s.* $= \frac{95}{100} \times$ cost price;

$\therefore$ cost price $= £\frac{20}{19} \times 9\frac{1}{2} = £10$;

$\therefore$ by selling at £11. 17*s.* 6*d.*, the gain on £10 $= £1\frac{7}{8}$;

$\therefore$ the gain on £100 $= £18\frac{3}{4}$;

$\therefore$ required gain per cent. $= \underline{18\frac{3}{4}}$.

EXAMPLES. 149.

1. By selling silk at 17*s.* 3*d.* per yard, a mercer gains 15 %; what would he gain per cent. if he sold it at 18*s.* per yd.?

2. A person selling an article for £2. 12*s.* 6*d.* gains 5 per cent.; what would he have gained or lost per cent. by selling it for £2. 7*s.* 6*d.*?

3. By selling a horse for £32. 10*s.*, a person lost $2\frac{1}{2}$ per cent.; how much per cent. would he have lost or gained if he had sold it for £45?

4. If I gain $12\frac{1}{2}$ per cent. by selling a watch for £5. 12*s.* 6*d.*, how much shall I gain or lose per cent. by selling it for £4. 14*s.* 6*d.*?

5. If 3 per cent. be gained by selling cloth at 15 f. 45 c. per metre, what would have been the gain per cent. if it had been sold at 16 f. 20 c. per metre?

6. If $3\frac{1}{2}$ per cent. be gained by selling butter at £5. 3*s.* 6*d.* per cwt.; what would be the gain or loss per cent., if it were sold at £7 per cwt.?

7. By selling goods for £116. 17*s.* I lose 5 per cent.; what should I have gained per cent. by selling them for £127. 12*s.* 3*d.*?

8. A draper, by selling cloth at 7*s.* per yard, gains 5 per cent.; what would be his rate of profit or loss if he were to sell the same cloth at 5*s.* 7½*d.* per yard?

9. By selling 6 Kg. of tea for 37 f. 80 c., a grocer gains 5 per cent.; how much will he gain per cent. if he sells 25 Kg. of the same tea for 155 f.?

10. If, by selling cloth at £2. 17*s.* for 10 yards, the gain would be $6\frac{2}{5}$ per cent., what would be gained or lost per cent. if it were sold at the rate of £3. 15*s.* for 14 yards?

334. CASE V. **To compare selling prices,** when the gain or loss per cent. varies.

Example. *By selling hay at £3 a ton, there is a loss of 10 per cent.: at what price must it be sold to gain 15 per cent.?*

$\frac{90}{100}$ × cost price = £3;

∴ cost price = $£\frac{10}{9} \times 3$ = £3. 6*s.* 8*d.*;

the gain on £3. 6*s.* 8*d.* at 15 % = $\frac{15}{100}$ of £3. 6*s.* 8*d.* = $£\frac{3}{20} \times \frac{10}{3}$ = 10*s.*;

∴ price at which hay should be sold = £3. 6*s.* 8*d.* + 10*s.* = £3. 16*s.* 8*d.*

Otherwise, more directly:

Required selling price = $\frac{115}{90}$ of £3 = ... = £3. 16*s.* 8*d.*

EXAMPLES. 150.

1. By selling an article for 38*s.*, the seller loses 5 per cent.; for how much must he sell it so as to gain 10 per cent.?

2. If by selling silk at 5 f. 40 c. per metre I gain 8 per cent., at what price must I sell it to gain 16 per cent.?

3. If by selling cloth at 8*s.* per yard there is a loss of 4 %, for how much per yard must it be sold so as to gain 11 per cent.?

4. If 4 per cent. be lost by selling silk at 10*s.* per yard, at what price per yard should it be sold in order to gain 5 %?

5. By selling an article for £8. 2*s.* 6*d.* a tradesman loses $2\frac{1}{2}$ per cent.; at what price should he sell it to gain $3\frac{1}{2}$ per cent.?

6. If 8 % be gained by selling 109 yds. of cloth for £46. 6*s.* 6*d.*, at what price per yard must it be sold to gain 17 %?

7. A grocer, by selling cheese at £3. 13*s.* 4*d.* per cwt., gains 10 %; at what price per lb. must he sell it so as to gain 12 %?

8. By selling 12 Kg. of tea for 75 f. 60 c., a grocer gains 5 per cent.; for how much must he sell 50 Kg. of it to gain $3\frac{1}{3}$ %?

Miscellaneous Examples on Profit and Loss.

335. If the letters C, r and S represent the *Cost Price*, the *rate per cent.* and the *Selling Price* respectively, the relation between them may be expressed in the general form

$$S = C + \frac{r}{100} \times C, \quad \text{or} \quad S = \left(\frac{100 + r}{100}\right) \times C,$$

when the goods are sold **at a profit**; and in the form

$$S = C - \frac{r}{100} \times C, \quad \text{or} \quad S = \left(\frac{100 - r}{100}\right) \times C,$$

when the goods are sold **at a loss**. These results may be put together in the form

$$S = \frac{100 \pm r}{100} \times C.$$

336. The following examples will illustrate the method of dealing with questions on profit and loss when they involve the principles in a less simple manner.

Example i. *A man marks his goods at a price from which he can deduct $7\frac{1}{2}\%$ for cash, and still get a profit of 10% on his outlay. Find the cost price of an article which he marks at £2. 4s.*

We will suppose that the cost price of the article $= x$ shillings. Then the money received by the dealer must be $\frac{110}{100}$ of x shillings, if he still makes a profit of 10%; and it must also be $\frac{92\frac{1}{2}}{100}$ of 44 shillings, if it is the remainder when $7\frac{1}{2}\%$ has been deducted from the marked price;

$$\therefore \ \tfrac{110}{100} x = \frac{92\frac{1}{2}}{100} \times 44;$$

$$\therefore \ \tfrac{11}{10} x = \tfrac{185}{100} \times 22;$$

$$\therefore \ x = \tfrac{10}{11} \times \tfrac{37}{20} \times 22 = 37;$$

$\therefore$ the cost price $=$ £1. 17*s.*

Example ii. *The cost of an article is 56% of the price at which it is marked for sale; and the article is sold subject to a trade discount of $12\frac{1}{2}\%$. How much per cent. does the seller gain?*

We will suppose the marked price $=$ £x;

then the money received on the sale $=$ £$\frac{87\frac{1}{2}}{100}$ of x $=$ £$\frac{7}{8}x$,

and the cost of the article $=$ £$\frac{56}{100}x$;

$$\therefore \ \frac{\text{money received}}{\text{cost of article}} = \tfrac{7}{8} \div \tfrac{56}{100} = \tfrac{7}{8} \times \tfrac{100}{56} = \tfrac{25}{10}$$

$$= \frac{125}{80} = \frac{625}{400} = 156\tfrac{1}{4};$$

$\therefore$ rate of profit $= 56\frac{1}{4}\%$.

EXAMPLES. 151.

1. At what rate per cent. is the profit on selling for 24*s.* what cost 18*s.*?

2. Cloth is bought at 1 f. 80 c. per metre; at what price per metre must it be sold to gain 25 per cent.?

3. A certain kind of tea cost 1 f. 75 c. per Kg.; at what price must it be sold to gain 12 per cent.?

4. If eggs be bought at the rate of 3 a penny, how must they be sold to gain 40 per cent.?

5. A tradesman, selling off, reduces his prices to 10 % below cost price; what did he get for goods for which he gave £65?

6. Goods are sold at a loss of £30, which is at the rate of $1\frac{1}{4}$ per cent.; what did they cost?

7. Goods were sold for £216, at a loss of 10 per cent.; what did they cost?

8. A horse was sold for 700 f., at a gain of $16\frac{2}{3}$ per cent.; what did it cost?

9. By selling 10 acres of land for £4699. 8*s.* 3*d.*, a man gained $5\frac{1}{4}$ per cent.; what was the original price per acre?

10. A merchant buys certain goods at £35 per ton; at what price per lb. must he sell them to gain 10 p. c. on his outlay?

11. The cost price of a book is 8*s.* 4*d.* If the expenses of sale be 15 per cent. upon this, and the net profit be 35 per cent. upon the same, what would be the retail price?

12. If a person sells 22 articles for the same money which he paid for 36, what does he gain per cent. on his outlay?

13. 126 sheep are bought for £409. 10*s.*, and sold for £488. 5*s.*; what is the profit on each, and the profit per cent.?

14. A tradesman sold $\frac{3}{4}$ of his stock for the money that he gave for the whole; at what rate per cent. does he gain?

15. If oranges are bought at the rate of 20 a shilling, how many must be sold for £1. 8*s.* in order to gain 40 per cent.?

16. Sugar costing £1. 6*s.* per cwt. is sold at $3\frac{3}{4}d.$ per lb.; what is the gain per cent.?

17. A man buys goods for 735 f., and sells them again for 565 f. 95 c.; how much does he lose per cent.?

18. A draper pays 10 guineas for a piece of silk containing 46 yards, and sells it at the rate of £2. 7*s.* 3*d.* for 8 yards; what does he gain per cent.?

19. Ten tons of goods, bought at 27*s.* 6*d.* per cwt., are sold at £30 per ton; what is the gain per cent.?

20. A woman bought eggs at the rate of 8 for 5*d.*, and sold them so as to gain $1\frac{1}{2}d.$ on a dozen; what did she charge for each, and what did she gain per cent.?

21. A house was sold for £5000, at a profit of £500; what would have been the loss per cent., if it had been sold for £4000?

22. Of a load of fruit 25 per cent. is spoiled. The remainder is sold at a profit of $16\frac{2}{3}$ per cent. on its cost. If the whole load cost £80, what is gained or lost?

23. A man buys goods at the rate of £24 per cwt., and sells 3 tons 13 cwt. 1 qr. for £2197. 10*s.*; how much has he gained or lost per cent. on his outlay?

24. A quantity of tea is sold for 4*s.* 7*d.* per Kg.; the gain is 10 per cent., and the total gain is £12: what is the quantity?

25. What was the cost of lead per cwt., if the sale of 48 cwt. for £45. 11*s.* 3*d.* gives a profit of £12. 10*s.* per cent.?

26. An article which cost 19 guineas per cwt. is retailed at 4*s.* 6*d.* per lb., and there is a waste of 5 per cent.; what is the rate of profit per cent.?

27. A merchant bought 240 Kg. of sugar at 2*d.* per Kg., and sold $\frac{1}{3}$ of it at $3\frac{1}{2}d.$, $\frac{1}{4}$ of it at 3*d.*, and the remainder at $2\frac{1}{2}d.$ per Kg.; what does he gain per cent.?

28. Goods worth £1 per ton absorb moisture and become $3\frac{1}{3}$ per cent. heavier; what is then the worth per ton?

29. If eggs are sold at the rate of 175 for 12 f. and the gain at this rate is 20 per cent., at what rate were they purchased?

30. A grocer buys 21 cwt. 3 qrs. 14 lb. of sugar for £30, and pays 12*s.* 6*d.* for expenses; at what rate must he sell it per lb. to gain not less than 25 per cent.?

31. If I gain 5 per cent. by selling an article for £5. 5*s.*, how much shall I gain or lose per cent. by selling it for $4\frac{1}{2}$ guineas?

32. A silver cup was sold for 12 guineas, at a loss of $5\frac{1}{2}$ per cent.; for what should it have been sold to gain 26 per cent.?

33. If by selling wine at 3 f. per litre I lose 6 per cent., at what price must I sell it to gain $17\frac{1}{2}$ per cent.?

34. If $5\frac{1}{2}$ per cent. be lost by selling goods for £39. 12*s.* 9*d.*, for what sum must they be sold in order to gain 5 per cent.?

35. By selling a horse for £42. 10*s.* a person lost $7\frac{1}{2}$ per cent.; find the gain or loss per cent. by selling it for £48. 5*s.*

36. By selling hay at £3 a ton there is a loss of 10 per cent.; at what price must it be sold to gain 15 per cent.?

37. Goods were sold for £30 at a loss of 10 per cent. They ought to have been sold to gain 30 per cent.; how much were they sold under their proper price?

38. If, by selling sugar at $2\frac{1}{2}d.$ per 2 lb., one loses 16 per cent., at what price per cwt. should one sell it to gain 8 per cent.?

39. I bought 56 gallons of brandy at 22*s.* 6*d.* per gallon, but 7 gallons were lost; at what price per gallon must I sell the remainder to obtain 15 per cent. profit on the whole outlay?

40. By selling tea at 6 f. 40 c. per Kg., a merchant gains $\frac{1}{8}$ of his outlay. He then raises the price to 7 f. 40 c. per Kg.; what does he gain per cent. on the original outlay?

41. A merchant sells tea to a tradesman at a profit of 60 per cent.; but the tradesman, becoming bankrupt, pays only 2*s.* 6*d.* in the £; how much per cent. does the merchant gain or lose?

42. A tradesman's prices are 20 per cent. above cost; if he allow a customer 10 per cent. discount, what profit does he make?

43. How much per cent. must a tradesman add on to the cost price of his goods in order that he may make 10 per cent. profit after allowing a customer 4 per cent. off his bill?

44. A piece of cloth is sold for £4. 1*s.* 3*d.* at a profit of 30 per cent. If it had been sold at 3*s.* 6*d.* per metre, the profit would have been £1. 5*s.*; how many metres of cloth are there?

45. A man embarks his property in four successive ventures. In the first he clears 100 per cent., and in each of the others he loses 20 per cent.; find the profit per cent. on his original outlay.

46. A sells goods to B for £115. 19*s.* 2*d.*, and gains 10 per cent. on the price he originally paid for them. B sells the same goods again, and loses 10 per cent. on the price at which he bought them. At what price did A buy the goods, and at what price did B sell them?

47. If 3 per cent. more be gained by selling a horse for £83. 5*s.* than by selling it for £81, what was its original price?

48. A person buys an article, and sells it so as to gain 5 per cent. If he had bought it at 5 per cent. less, and sold it for 1*s.* less, he would have gained 10 per cent.; find the cost price.

49. A certain number of apples were bought at four a penny, and an equal number at three a penny. The whole were sold at seven for two pence; find the gain or loss per cent.

XXXVIII. (a). SIMPLE INTEREST.

337. Some people having *more* money than they are able to use in their own affairs, and others having *less* money than they need to carry on their business, it is natural for the latter to borrow from the former, and for the former to lend to the latter: and it is fair for the former to demand, and for the latter to pay, a *proportionate* fee for the use of the money. This charge is called **Interest**, and should clearly be proportional to the *amount* of the money, and also to the *time* for which it is lent. It is customary to reckon the charge at so much per £100 for one year, *i.e.* at a certain **Rate per cent. per annum**. If the rate be 5 per cent. per annum, it follows that

the interest on £100 for 1 year $= £5$;
..... £50 ... 1 year $= £2\frac{1}{2}$;
..... £50 ... 3 years $= £2\frac{1}{2} \times 3$;
..... £50 ... $\frac{3}{4}$ year $= £2\frac{1}{2} \times \frac{3}{4}$;
..... £150 ... $\frac{3}{4}$ year $= £2\frac{1}{2} \times \frac{3}{4} \times 3$, etc.

NOTE.—When a rate per cent. is mentioned, the words *per annum* must be understood.

338. It should be **remembered** that, at 5 per cent., the interest upon £1 for 1 year is **one shilling** (*i.e.* 1*d.* per month): upon 15*s.* it is 9*d.*; upon 10*s.* it is 6*d.*; upon 5*s.*, 3*d.*; and so on. Thus, it can be seen at a glance that the interest at 5 per cent. upon £562. 17*s.* 6*d.* for 1 year is $562\frac{7}{8}$*s.*, *i.e.* £28. 2*s.* $10\frac{1}{2}$*d.*

At $2\frac{1}{2}$ per cent., the rate allowed on deposits in the Post Office Savings Bank and on Consols, the interest amounts to $\frac{1}{2}$*d.* per £1 per month.

339. The sum of money upon which interest is reckoned is called the **Principal**. When we add the interest to the principal we get the **Amount**, *i.e.* the sum to which the principal will amount at the end of a given time.

340. CASE I. **To find the Interest, or the Amount**, when the Principal, the Rate per cent. and the Time are given.

Example. *Find the simple interest on* £3265. 12*s.* 6*d.* *for* $5\frac{1}{2}$ *years at* 4 *per cent. per annum.*

Interest on £100 for $5\frac{1}{2}$ years $= £5\frac{1}{2} \times 4$,

$\therefore$ ..... £$3265\frac{5}{8}$ for $5\frac{1}{2}$ years $= £3265\frac{5}{8} \times \frac{1}{100} \times 5\frac{1}{2} \times 4$

$= £\frac{26125}{8} \times \frac{1}{25} \times \frac{11}{2}$

$= £\frac{11 \times 1045}{16} = £\frac{11495}{16}$

$= £718.\ 8s.\ 9d.$

Hence it will be seen that, **to find the simple interest, we multiply the principal by the rate per cent. and then by the number of years, and we divide the product by 100.**

341. The operations can easily be performed when the money has been expressed as the decimal of £1, and, since the product of the rate per cent. and the number of years is generally less than 100, it will, in most cases, be sufficient to decimalize the money correctly to 3 places only. The process will then give an answer correct to the nearest farthing. For practical purposes the nearest penny is generally sufficient.

Example i. *Find the simple interest on* £3265. 12*s.* 6*d. for* $5\frac{1}{2}$ *years at* 4 %.

Since $\frac{4 \times 5\frac{1}{2}}{100} = \frac{22}{100}$, we have to multiply the principal by 22 and to divide the product by 100.

£3265·625
22
65312·50
6531·25
100 | 71843·75
Interest = £718·438
= £718. 8*s.* 9*d.*

Example ii. *Find the simple interest on* £532. 16*s.* $9\frac{1}{2}$*d. for* $3\frac{1}{2}$ *years at* $5\frac{1}{4}$ %.

Since $\frac{5\frac{1}{4} \times 3\frac{1}{2}}{100} = \frac{147}{800}$, we have to multiply the principal by 147 and to divide the product by 800.

£532·840
147
53284·0
21313·60
3729·880
800 | 78327·48
Interest = £97·909
= £97. 18*s.* $2\frac{1}{2}$*d.*

Example iii. *Find the simple interest on* 13756 *f.* 75 *c. for* $3\frac{1}{4}$ *yrs. at* $2\frac{1}{2}$ %.

We have $\frac{3\frac{1}{4} \times 2\frac{1}{2}}{100} = \frac{65}{800}$.

13756·75 f.
65
825405·0
68783·75
800 | 894188·75
Interest = 1117·735... f.
= 1117 f. 74 c.

Example iv. *Find the simple interest on* 584 *dollars for* 42 *days at* 5 %.

We have $\frac{\frac{42}{365} \times 5}{100} = \frac{42}{7300}$.

584 dol.
42
2336
1168
7300) 24528 (3·36
2628
438
Interest = 3 dol. 36 c.

342. If the Interest is **Simple**, and we represent the *principal* by the letter P, the *rate* of interest by r, the *number* of years by t,

the amount of *interest* by I, and the sum of *principal and interest* by A, we may express their relations to one another in the form

$$A = P + I, \text{ and } I = P \times \frac{r}{100} \times t;$$

whence $$A = P + P \times \frac{r}{100} \times t = P \times \left(1 + \frac{r}{100} \times t\right).$$

When any 3 of the 4 quantities P, A, r and t, or P, I, r and t, are known, these relations enable us to determine the other.

EXAMPLES. 152. *Oral.*

Find the simple interest for one year on

1. £200 at 3 per cent.
2. £300 at 4 per cent.
3. £400 at $2\frac{1}{2}$ per cent.
4. £500 at 6 per cent.
5. £600 at 2 per cent.
6. £800 at 7 per cent.
7. £900 at 4 per cent.
8. £1000 at 8 per cent.
9. 2500 f. at 6 per cent.
10. 9500 f. at 3 per cent.
11. 350 f. at 4 per cent.
12. 650 f. at 7 per cent.
13. £850 at $2\frac{1}{2}$ per cent.
14. £450 at 3 per cent.
15. 575 dol. at 8 per cent.
16. 432 dol. at $2\frac{1}{2}$ per cent.
17. 617 dol. at 5 per cent.
18. 843 dol. at 5 per cent.
19. £937 at 5 per cent.
20. £741 at 5 per cent.
21. £586 at 5 per cent.
22. £672. 10*s.* at 5 per cent.
23. £578. 10*s.* at 5 per cent.
24. £1673. 10*s.* at 5 per cent.
25. £941. 15*s.* at 5 per cent.
26. £637. 15*s.* at 5 per cent.
27. £4514. 5*s.* at 5 per cent.
28. £6208. 5*s.* at 5 per cent.
29. £6287. 2*s.* 6*d.* at 5 p. c.
30. £2814. 7*s.* 6*d.* at 5 p. c.
31. £5623. 17*s.* 6*d.* at 5 p. c.
32. £5861. 12*s.* 6*d.* at 5 p. c.

EXAMPLES. 153.

Find the simple interest on

1. £650 for 1 yr. at 12 p. c.
2. £845 for 1 yr. at 15 p. c.
3. £832 for 1 yr. at 8 p. c.
4. £735 for 1 yr. at $2\frac{1}{2}$ p. c.
5. 942 dol. for 1 yr. at $3\frac{1}{2}$ p. c.
6. 572 m. for 1 yr. at $4\frac{1}{2}$ p. c.
7. 3265 dol. for 1 yr. at $2\frac{3}{4}$ p. c.
8. 2055 f. for 1 yr. at $5\frac{3}{4}$ p. c.
9. 215 f. for 1 yr. at $6\frac{1}{4}$ p. c.
10. 245 m. for 2 yrs. at 3 p. c.
11. £356 for 3 yrs. at $2\frac{1}{2}$ p. c.
12. £525 for 5 yrs. at $3\frac{1}{2}$ p. c.

Find the simple interest on

13. £650 for 8 yrs. at $4\frac{1}{4}\%$.
14. £960 for 12 yrs. at $2\frac{1}{4}\%$.
15. £10560 for 7 yrs. at $4\frac{1}{4}\%$.
16. £1545 for $5\frac{1}{2}$ yrs. at 4%.
17. £2763 for $4\frac{1}{2}$ yrs. at 5%.
18. 6208 dol. for $4\frac{1}{2}$ yrs. at 3%.
19. 5321 f. for $3\frac{3}{4}$ yrs. at $4\frac{1}{2}\%$.
20. £4762 for $6\frac{1}{3}$ yrs. at $5\frac{1}{2}\%$.
21. £325. 10*s.* for $2\frac{1}{2}$ yrs. at 4%.
22. £333. 10*s.* for 20 yrs. at $3\frac{3}{4}\%$.
23. £348. 10*s.* for 7 years at $4\frac{1}{2}$ per cent.
24. £774. 11*s.* 3*d.* for 4 years at 5 per cent.
25. £1715. 1*s.* 8*d.* for 5 years at 3 per cent.
26. 2333 f. 75 c. for 3 years at $1\frac{1}{2}$ per cent.
27. 1012 f. 75 c. for 20 years at $2\frac{1}{2}$ per cent.
28. £5982. 7*s.* 6*d.* for 4 years at $3\frac{3}{4}$ per cent.
29. £383. 5*s.* 8*d.* for $2\frac{1}{2}$ years at $3\frac{3}{4}$ per cent.

Find the amount of

30. £477. 3*s.* 4*d.* for $4\frac{1}{2}$ years at 4 per cent.
31. £635. 18*s.* $4\frac{1}{2}$*d.* for $3\frac{1}{2}$ years at 3 per cent.
32. 1396 m. 25 pf. for $3\frac{1}{2}$ years at $5\frac{1}{2}$ per cent.
33. 422 dol. 50 c. for $13\frac{1}{2}$ years at 4 per cent.
34. £3872. 13*s.* for 1 year at $2\frac{1}{3}$ per cent.
35. £753. 16*s.* 8*d.* for 8 years at $3\frac{1}{5}$ per cent.
36. £652. 7*s.* 6*d.* for 8 years at $4\frac{1}{2}$ per cent.
37. £7455 for 3 months at $2\frac{1}{2}$ per cent.
38. 8245 dol. for 4 months at 3 per cent.
39. £41. 13*s.* 4*d.* for 8 months at $4\frac{1}{2}$ per cent.
40. 670 m. 75 pf. for 8 months at 5 per cent.
41. £3460 for 8 months at $3\frac{1}{2}$ per cent.
42. £850 for 3 years 3 months at $4\frac{1}{2}$ per cent.
43. 8240 m. for 14 months at 3 per cent.
44. £1238. 14*s.* $4\frac{1}{2}$*d.* for 6 years 8 months at $1\frac{1}{2}$ per cent.
45. £1369. 17*s.* 4*d.* for 2 years 10 months at $4\frac{1}{8}$ per cent.
46. £365. 10*s.* for 8 months at $4\frac{1}{2}$ per cent.
47. £6870 for 73 days at $3\frac{1}{2}$ per cent.
48. £2160. 12*s.* 6*d.* for 1 year 73 days at 5 per cent.
49. £533. 6*s.* 8*d.* for 146 days at 3 per cent.
50. £183. 6*s.* 8*d.* for 1 year 146 days at $4\frac{1}{4}$ per cent.

XXXVIII. (b). TRUE DISCOUNT AND TRUE PRESENT WORTH.

343. It may happen that a person A is entitled to receive a sum of money at a future specified time, but finds it inconvenient to wait until the interval has elapsed. Being in want of ready-money, A might therefore try to find another person B who would give him money at once on condition that, when the interval had elapsed, B (instead of A) should have the right to receive the money which is then due.

The next step would be to determine *how much* money B should give A in exchange for this right. It must be considered that B would lose the *use* of his ready-money until the money came to him (instead of to A) at the end of the interval. B would thus lose the interest which his ready-money would have obtained for him, if he had lent it at interest elsewhere, or employed it in buying and selling at a profit. Theoretically, therefore, the money, which B (instead of A) will receive at the specified time, should repay to B not only the money which he had advanced to A, but, in addition, what this money would have earned at a fair rate of interest. Considered from this point of view, the amount which B should give A is called the **True Present Worth** of the sum with which B will be repaid at the specified time; and the difference between the two amounts is called the **True Discount.**

Hence we see that **the True Present Worth of £x, due at a specified time, is that sum which, with interest on itself for the interval, would amount to £x.** Thus

True Present Worth of £x + Int. on True Present Worth of £x = £x,
and **True Discount = Interest on True Present Worth.**

344. The new terms which we have introduced are but new names for things which we have already considered in the preceding section. The questions in the next set of examples are therefore of the same kind as those in the preceding set. The terms Principal and Interest are replaced by *True Present Worth* and *True Discount.*

345. **To find the Amount** of a sum of money due at some future time, when the True Present Worth, the Rate of interest and the Time are given.

Example. *If the true present worth of a sum of money, due 6 months hence, is £500, and the rate of interest is 5 per cent., find the sum then due.*

True present worth = £500;
∴ interest on true present worth for 6 mos. = £5 × 5 × ½ = £12. 10*s.*;
∴ the sum due = £512. 10*s.*

EXAMPLES. 154.

What is the amount of a sum of money

1. due 5 years hence, if its true present worth be £100, and the rate of interest be $3\frac{1}{2}$ per cent.?

2. due $2\frac{1}{2}$ years hence, if its true present worth be £350, and the rate of interest be 4 per cent.?

3. due $4\frac{1}{4}$ years hence, if its true present worth be £1375, and the rate of interest be $3\frac{3}{4}$ per cent.?

4. due $2\frac{3}{4}$ years hence, if its true present worth be £2050, and the rate of interest be $4\frac{7}{11}$ per cent.?

5. due 3 months hence, if its true present worth be £8721. 16*s*. 8*d*., and the rate of interest be $2\frac{1}{2}$ per cent.?

6. due 9 months hence, if its true present worth be £4575, and the rate of interest be $5\frac{1}{2}$ per cent.?

7. due 8 months hence, if its true present worth be 3375 f., and the rate of interest be $2\frac{1}{4}$ per cent.?

8. due 15 months hence, if its true present worth be 230 dol. 80 c., and the rate of interest be 4 per cent.?

XXXVIII. (c). TO FIND THE PRINCIPAL, OR THE TRUE PRESENT WORTH.

346. We have seen that

(i) Principal + Interest = Amount;

(ii) True Present Value + True Discount = Sum due at stated time;

and if any two of the quantities mentioned in (i) or (ii) be given, the third quantity can be readily found.

The other quantities which occur in questions on Interest or Discount are the *Time* and the *Rate per cent.* We have already considered the case in which the Principal, the Time and the Rate per cent. were given, and we had to determine the Interest or the Amount; and also the case in which the Present Worth, the Time and the Rate per cent. were given, and we had to determine the Discount or the Amount due at the stated time. We will now consider the other cases in turn.

347. Case II. **To find the Principal**, when the Amount (or the Interest), the Time and the Rate per cent. are given; or, which is the same problem, **to find the True Present Worth**, when the Amount due at the specified time (or the True Discount), the Time and the Rate per cent. are given.

Example i. *What sum will amount to* £300. 16*s.* *in* 4 *yrs. at* 5 %?

Interest on £100 in 4 years at 5 % = £5 × 4 = £20,

∴ amount of £100 in 4 years = £120,

∴ principal which in 4 yrs. will amount to £120 = £100,

∴ ..... £1 = £$\frac{100}{120}$ = £$\frac{5}{6}$,

∴ ..... £300$\frac{4}{5}$ = $\frac{5}{6}$ × £300. 16*s.*

= $\frac{1}{6}$ × £1504

= £250. 13*s.* 4*d.*

Example ii. *What sum will produce* £113. 8*s.* *interest in* 4$\frac{1}{2}$ *yrs. at* 4$\frac{1}{2}$ %?

Principal which produces £4$\frac{1}{2}$ in 1 yr. = £100,

∴ ..... £113$\frac{2}{5}$ in 4$\frac{1}{2}$ yrs. = £100 × $\frac{113\frac{2}{5}}{4\frac{1}{2}}$ × $\frac{1}{4\frac{1}{2}}$

= £100 × $\frac{567}{5}$ × $\frac{2}{9}$ × $\frac{2}{9}$

= £7 × 80

= £560.

Example iii. *Find the true present worth of* £808. 1*s.* 4*d.* *due* 3 *years and* 9 *months hence, reckoning interest at* 4 *per cent.*

Interest on £100 in 3$\frac{3}{4}$ yrs. at 4 per cent. = £4 × 3$\frac{3}{4}$ = £15,

∴ amount of £100 ..... = £115,

∴ true P.W. of £115 due ..... = £100,

∴ ..... £808$\frac{1}{15}$ ..... = £$\frac{100}{115}$ × 808$\frac{1}{15}$

= £$\frac{20}{23}$ × $\frac{12121}{15}$

= £$\frac{4}{3}$ × 527 = £$\frac{2108}{3}$

= £702. 13*s.* 4*d.*;

∴ true discount on £808. 1*s.* 4*d.* = £105. 8*s.* 0*d.*

EXAMPLES. 155.

1. How much must be lent at 4 % to produce £30 a year?

2. What sum will in 5 years amount to £411. 5*s.* at 3$\frac{1}{2}$ %?

3. Find the true present value of £424. 2*s.* 3*d.* due in 3$\frac{1}{4}$ years at 4$\frac{1}{2}$ per cent.

4. What sum, put out at interest for 18 months at 5 per cent., will amount to £188. 2*s.* 6*d.*?

5. Find the true present value of £5431. 15*s.* 11$\frac{1}{4}$*d.* due in 6 years at 4$\frac{1}{4}$ per cent.

6. What sum, put out at interest for 7$\frac{1}{2}$ years at 5 per cent., will amount to £1246. 13*s.* 4*d.*?

7. What sum, lent at 2$\frac{1}{2}$ per cent. for 5$\frac{1}{4}$ years, will at the end of that time amount to £814. 10*s.*?

8. What sum, put out at $3\frac{1}{2}$ per cent., will amount to £323. 11*s*. $5\frac{1}{2}$*d*. in $7\frac{1}{7}$ years?

9. Find the principal on which the interest for $3\frac{1}{2}$ years at $2\frac{1}{2}$ per cent. will be £551. 18*s*. $6\frac{3}{4}$*d*.

10. What sum, placed at simple interest at $4\frac{1}{4}$ per cent., will in 16 months amount to £39. 12*s*. 6*d*.?

11. Find the true present value of £350. 19*s*. $4\frac{1}{2}$*d*. due in $6\frac{1}{2}$ years at $4\frac{1}{4}$ per cent.

12. What sum will amount to £616. 13*s*. 4*d*. after being lent for $2\frac{1}{2}$ years at $4\frac{2}{5}$ per cent.?

13. What sum will amount to £44. 3*s*. $0\frac{1}{4}$*d*. in $2\frac{1}{2}$ yrs. at $3\frac{1}{4}$ %?

14. What amount will yield £273 interest in $4\frac{1}{2}$ years at $3\frac{1}{2}$ %?

15. What sum lent at $2\frac{1}{2}$ % will amount to £7680 in 3 years?

16. What principal will amount to £10. 1*s*. $10\frac{1}{2}$*d*. in 3 years 73 days at $6\frac{1}{4}$ per cent.?

Find the true ready-money value and the true discount in the case of the following sums of money:

17. £377. 12*s*. 6*d*., due in 18 months; interest at 4 %.

18. £578. 1*s*. 4*d*., due in $3\frac{1}{3}$ years; interest at $4\frac{1}{2}$ %.

19. £9393. 18*s*. $4\frac{1}{2}$*d*., due in $5\frac{1}{2}$ years; interest at $3\frac{1}{2}$ %.

20. £201. 4*s*. $7\frac{1}{2}$*d*., due in 25 days; interest at $3\frac{1}{2}$ %.

XXXVIII. (d). TO FIND THE TIME AND RATE PER CENT.

348. Case III. **To find the Time**, when the Principal, Rate per cent. and Interest (or Amount) are given; or, which is the same problem, **to find when the money will become due**, if the Amount, the True Discount (or the True Present Worth) and the Rate per cent. are given.

If we find the interest on the Principal for 1 year, and determine how many times this interest is contained in the given interest, the quotient will be the number of years required.

Example 1. *In what time will the simple interest on* £556. 13*s*. 4*d*. *amount to* £167 *at* 5 *per cent.?*

Interest on £556. 13*s*. 4*d*. for 1 year = £27. 16*s*. 8*d*.,

and ..... req^{d}. time = £167;

$$\therefore \text{ required time} = \frac{167}{27\frac{5}{6}} \text{ years} = \underline{6 \text{ years}}.$$

Example ii. *In what time will* £2833. 6*s.* 8*d.* *amount to* £3215. 16*s.* 8*d.* *at* 3 *per cent.?*

Amount of £2833. 6*s.* 8*d.* in req^{d}. time = £3215. 16*s.* 8*d.*,
$\therefore$ interest on ..... = £382. 10*s.* 0*d.*;
but interest on £2833. 6*s.* 8*d.* for 1 year = £85;

$$\therefore \text{ required time} = \frac{382\frac{1}{2}}{85} \text{ yrs.} = \frac{765}{170} \text{ yrs.} = \underline{4\tfrac{1}{2} \text{ yrs.}}$$

Example iii. *If the true discount on a sum of* £2674. 6*s.* *is* £324. 6*s.*, *interest being reckoned at* $4\frac{3}{5}$ *per cent., when is the sum due?*

Amount = £2674. 6*s.*,
and true discount = £324. 6*s.*;
$\therefore$ true present worth = £2350;
int. on true P.W. for 1 yr. at $4\frac{3}{5}$ per cent. $= £2350 \times 4\frac{3}{5} \times \frac{1}{100} = £\frac{47}{2} \times \frac{23}{5}$;
and int. on true P.W. for interval $= £324\frac{3}{10}$;

$$\therefore \text{ time required} = \frac{324\frac{3}{10}}{\frac{47}{2} \times \frac{23}{5}} \text{ yrs.} = \frac{3243}{1081} \text{yrs.} = \underline{3 \text{ years.}}$$

EXAMPLES. 156.

In what time will

1. £350 amount to £700 at 4 per cent.?
2. £225 amount to £256. 10*s.* at 4 per cent.?
3. £670 amount to £892. 15*s.* 6*d.* at $4\frac{3}{4}$ per cent.?
4. £540 amount to £712. 16*s.* at 4 per cent.?
5. £2700 amount to £3219. 15*s.* at $2\frac{3}{4}$ per cent.?
6. £1650 amount to £1891. 6*s.* 3*d.* at $4\frac{1}{2}$ per cent.?
7. £2175 amount to £2338. 2*s.* 6*d.* at $2\frac{1}{2}$ per cent.?
8. £1300 amount to £1493. 7*s.* 6*d.* at $3\frac{1}{2}$ per cent.?
9. the interest on £1260 at $3\frac{3}{4}$ per cent. be £236. 5*s.*?
10. the interest on £1637. 16*s.* 8*d.* at $3\frac{3}{4}$ per cent. be £491. 7*s.*?
11. the interest on £2151. 6*s.* 8*d.* at $2\frac{1}{3}$ per cent. be £215. 2*s.* 8*d.*?
12. the interest on £256. 6*s.* 8*d.* at $7\frac{1}{2}$ per cent. be £96. 2*s.* 6*d.*?
13. £3125 amount to £3681. 12*s.* $9\frac{3}{4}$*d.* at $3\frac{3}{4}$ per cent.?
14. £2977. 18*s.* 4*d.* amount to £3446. 18*s.* $9\frac{1}{4}$*d.* at $3\frac{1}{2}$ per cent.?
15. £445 amount to £486. 14*s.* $4\frac{1}{2}$*d.* at $2\frac{1}{2}$ per cent.?
16. the simple interest on any sum of money at $3\frac{3}{4}$ per cent be ·09375 of the principal?

When is the sum due, if the

17. true discount on £748 at $2\frac{1}{2}$ per cent. be £68?
18. true P.W. of £4972. 10*s.* at 10 per cent. be £3825?
19. true discount on £662. 15*s.* 8*d.* at 5 per cent. be £86 9*s.*?
20. true discount on £708. 12*s.* at $3\frac{3}{4}$ per cent. be £118. 2*s.*?
21. true discount on £1616. 2*s.* 8*d.* at 4 per cent. be £210. 16*s.*?
22. true discount on £481. 8*s.* $2\frac{1}{4}d.$ at $3\frac{1}{4}$ per cent. be £5. 3*s.* $2\frac{1}{4}d.$?
23. true P.W. of £238. 18*s.* 4*d.* at 5 per cent. be £203. 6*s.* 8*d.*?

349. Case IV. **To find the Rate per Cent.**, when the Principal, Time and Interest (or Amount) are given, or, which is the same problem, **to find the Rate of interest** when the Amount of the money, the True Discount (or the True Present Worth) and the Time are given.

If we find the interest on the Principal at the rate of 1 per cent. for the given time, and determine how often this is contained in the given interest, the quotient will be the rate per cent. required.

Example i. *At what rate per cent. will* £250. 13*s.* 4*d.* *amount to* £300. 16*s.* *in* 4 *years?*

Interest on £250. 13*s.* 4*d.* for 4 yrs. at 1 per cent. = £10. 0*s.* 6·4*d.*,

and ..... req^{d}. rate = £50. 2*s.* 8*d.*;

$$\therefore \text{ required rate per cent.} = \frac{\text{£50. 2s. 8d.}}{\text{£10. 0s. 6·4d.}} = \underline{5.}$$

Example ii. *A sum of* £1436. 10*s.* *is due in* 3 *years and* 7 *months, and the true discount is* £236. 10*s.*; *find the rate per cent.*

Amount = £1436$\frac{1}{2}$,

and true discount = £236$\frac{1}{2}$;

$\therefore$ true present worth = £1200;

$\therefore$ interest on £1200 for $3\frac{7}{12}$ yrs. = £236$\frac{1}{2}$,

$\therefore$ ..... £100 for 1 year $= £\frac{4}{2}\frac{7}{3} \times \frac{1}{1}\frac{}{2} \times \frac{1}{4}\frac{2}{3} = £\frac{1}{2}\frac{1}{}$;

$\therefore$ rate per cent. = $\underline{5\frac{1}{2}}$.

EXAMPLES. 157.

At what rate per cent. will

1. £500 amount to £567. 10*s.* in a year and a half?
2. £500 amount to £625 in 3 years?
3. £325 amount to £379. 3*s.* 4*d.* in 5 years?
4. £900 amount to £1120. 10*s.* in 7 years?

5. £250 amount to £302. 10*s.* in 2 years?

6. £1650 amount to £1784. 1*s.* 3*d.* in $2\frac{1}{2}$ years?

7. £1250 amount to £1962. 10*s.* in 6 years?

8. £3825 amount to £4972. 10*s.* in 3 years?

9. £3745 amount to £3932. 5*s.* in 2 years?

10. the interest on £1315 amount to £230. 2*s.* 6*d.* in $3\frac{1}{2}$ years?

11. the interest on £461. 10*s.* be £149. 19*s.* 9*d.* in 13 years?

12. the interest on £2416 amount to £634. 4*s.* in 5 years?

13. £1 amount to £1. 1*s.* in 5 years?

14. £39. 3*s.* 4*d.* amount to £43. 1*s.* 8*d.* in 3 years?

15. the interest on £250 amount to £18. 15*s.* in 73 days?

16. £400 amount to £409. 12*s.* in 146 days?

17. the true present worth of £1836, due 4 yrs. hence, be £1530?

18. the true P.W. of £151. 17*s.* 6*d.*, due in 4 years, be £125?

19. the true discount on £1356. 13*s.* 4*d.*, due 3 months hence, be £23. 6*s.* 8*d.*?

20. the true discount on £388. 17*s.* 9*d.*, due 18 months hence, be £22. 0*s.* 3*d.*?

21. the true P.W. of £808. 1*s.* 4*d.*, due in $3\frac{3}{4}$ years, be £702. 13*s.* 4*d.*?

22. the true discount on £1336. 11*s.* 3*d.*, due at the end of $3\frac{1}{2}$ years, amount to £199. 1*s.* 3*d.*?

Miscellaneous Examples on True Discount.

350. The following examples are not so direct as those which we have been considering, but they depend upon the same property, namely, that the **True Discount is the Interest on the True Present Worth.** In working these examples it will be useful to remember that

Amount due at specified time – True P.W. = True Discount;

$\therefore$ Int. on amount due at specified time – Int. on True P.W. = Int. on True Discount;

i.e. **Int. on amount due at specified time – True Discount = Int. on True Discount.**

Example i. *If the interest on £8825 at 4 per cent. be equal to the true discount on £11119. 10s., when is the latter sum due?*

Interest on £8825 = true discount on £11119. 10*s.*,

$\therefore$ £8825 = true P.W. of £11119. 10*s.*;

$\therefore$ true discount on £11119. 10*s.* = £11119. 10*s.* − £8825 = £2294. 10*s.*;

$\therefore$ int. on £8825 for req^{d}. time = £2294. 10*s.*;

but .... 1 year $= £8825 \times \frac{1}{25} = £353$;

$\therefore$ the required time $= \frac{2294\frac{1}{2}}{353}$ yrs. $= \frac{4589}{2 \times 353}$ yrs. $= \frac{13}{2}$ yrs.

$= 6\frac{1}{2}$ years.

Example ii. *If the true discount on a sum of money, due 8 months hence, at $2\frac{1}{2}$ per cent. be £88. 7s. 4d., what is the sum of money?*

Interest on true P.W. for 8 mos. $= £88\frac{11}{30}$;

$\therefore$ ..... 1 year $= £\frac{3}{2} \times \frac{2651}{30}$.

But ..... £100 ... 1 year $= £2\frac{1}{2}$;

$\therefore$ true P.W. of sum of money $= £\frac{\frac{3}{2} \times \frac{2651}{30}}{2\frac{1}{2}} \times 100$

$= £\frac{3}{2} \times \frac{2651}{30} \times \frac{2}{5} \times 100$

$= £5302$;

$\therefore$ required sum of money = £5390. 7*s.* 4*d.*

Example iii. *If the difference between the interest and the true discount on a sum of money for 4 months at $2\frac{2}{3}$ per cent. is £4. 10s., what is the sum?*

Interest on the true discount $= £4\frac{1}{2}$;

and ..... £100 $= £2\frac{2}{3} \times \frac{1}{3} = £\frac{8}{9}$;

$\therefore$ True Discount $= £\frac{4\frac{1}{2} \times 100}{\frac{8}{9}} = £\frac{2025}{4} = £506.\ 5s.$

Again, interest on true P.W. $= £506\frac{1}{4}$,

and ..... £100 $= £\frac{8}{9}$;

$\therefore$ True Present Worth $= £\frac{2025}{4} \times \frac{9}{8} \times 100 = £\frac{455625}{8}$

$= £56953.\ 2s.\ 6d.$;

$\therefore$ the sum req^{d}. = P.W. + Discount = £57459. 7*s.* 6*d.*

EXAMPLES. 158.

1. If the interest on £253. 2*s.* 6*d.* at 5 per cent. be equal to the true discount on £257. 6*s.* $10\frac{1}{2}d.$, for the same time at the same rate, when is the latter sum due?

2. If the interest on £4550 at 3 % be equal to the true discount on £5573. 15*s.* at 3 %, when is the latter due?

3. If the interest on £533. 6*s.* 8*d.* for 2 years be equal to the true discount at the same rate on £560 due 2 years hence, what is the rate of discount?

4. If the interest on £584. 6*s.* 8*d.* at $4\frac{1}{2}$ % be equal to the true discount on £682. 18*s.* $9\frac{1}{2}d.$ at $4\frac{1}{2}$ %, when is the latter due?

5. At what rate per cent. will the interest on £3729. 7*s.* 6*d.*, due 4 years hence, be equal to the true discount on £4661. 14*s.* $4\frac{1}{2}d.$ for the same time?

6. The interest on a certain sum is £28, and the true discount is £21. 17*s.* 6*d.* for the same time; what is the sum?

7. The interest on a certain sum is £10. 13*s.* $2\frac{2}{3}d.$, and the true discount is £10. 5*s.* for the same time; what is the sum?

8. The interest on a certain sum is £147. 19*s.* 3·3*d.*, and the true discount is £134. 10*s.* 3*d.* for the same time; find the sum.

9. If the true discount on a certain sum is £5. 9*s.* $4\frac{1}{2}d.$, and the interest is £1. 10*s.* $7\frac{1}{2}d.$ more, what is the sum?

10. Find a sum of money such that the difference between the interest on it and the true discount, for 6 months at 4 per cent., is 2*s.* 1·44*d.*

11. A certain sum is due 2 years hence, the true discount on it is £63. 17*s.*, and the interest on it for 2 years is £71. 16*s.* $7\frac{1}{2}d.$; find the sum and also the rate of interest.

XXXVIII. (e). PRACTICAL DISCOUNT.

351. It is a common custom for a merchant to pay for goods which he has bought, not in ready-money, but by means of what are called **Drafts** or **Bills of Exchange.** A bill of exchange is simply an order to pay the amount stated thereon on a specified day, or after a specified interval.

352. In the manner that has been explained in Art. 343, we can determine the true present value and the true discount on a bill of exchange so that buyer and seller may stand on equal terms. That method however is theoretical only, and is not adopted in practice. The buying and selling of *bills of exchange* form a large part of the business of a banker. There are, too, many private firms and persons to be found in all large business centres, who make what is called **bill-broking** their special business. A merchant holding bills may therefore take them to a banker, or to a firm of bill-brokers; but the discount which will be retained by them is the **interest on the bill itself,** instead

of the interest on *the true present worth* of the bill. When a bill is exchanged, or *sold* for ready-money, it is said to be **discounted.**

353. There is also another important custom to be noticed. The bill is nominally due on a certain day, but it is not customary to require payment until the third day after, the three extra days being called **Days of Grace.** For example, a bill which is dated March 23rd, and contains an order to pay a certain amount 3 months after that date, is not legally due until June 26th.

If therefore a bill of £100, due nominally on June 23rd, be taken to a banker on May 1st, the banker will reckon that between that day and June **26th,** when the bill will be paid, there are $30+23+3$, or 56, days; and he will give for the bill what remains of the £100 after the *interest* for 56 days has been deducted from it.

When specified dates are given in the examples which follow, we add on the Days of Grace; but when only the *period* before the bill falls due is given, as in Examples **159,** we shall suppose the Days of Grace already included in that period.

In calculating practical discount it is usual to reckon a fraction of a penny as a whole penny. This custom must be observed here.

Example. *Find the practical discount on* £397. 10*s.*, *due* 3 *months hence, at* 4 *per cent.*

Practical disc. at 4 per cent. on £100 due in 3 mos. = £1.

$\therefore$ ..... £397½ ..... 3 ..... $= £\frac{397\frac{1}{2}}{100} = £3{\cdot}975$

$= £3.\ 19s.\ 6d.$

The relation between Practical Discount and True Discount may be stated thus:

True Present Worth + True Discount = Amount of Bill;

$\therefore$ Int. on True P.W. + Int. on True Disc. = Int. on Amt. of Bill;

i.e. **True Discount + Int. on True Discount = Practical Discount.**

The Practical Discount is therefore *greater* than the True Discount by the *Interest on the True Discount.*

EXAMPLES. 159.

1. Find the banker's discount on £463. 10*s.*, due in $2\frac{1}{2}$ years, at $2\frac{3}{4}$ per cent.

2. A bill of £6400, due 1 year and 10 mos. hence, is discounted by a banker at $4\frac{3}{4}\%$; what will the holder of the bill receive?

3. If the mercantile discount on a bill due in 5 months at 4 per cent. be £1, for what sum was the bill drawn?

4. Find the difference between the practical and the true discount on £129. 3*s.* 7*d.*, due in 5 months, at $7\frac{1}{5}$ per cent.

5. Find the difference between the practical and the true discount on £724. 18*s.* 9*d.*, due in 9 months, at $4\frac{3}{4}$ per cent.

6. Find the difference between the customary and the true discount on £1462. 1*s.*, due 15 months hence, at $5\frac{1}{2}$ per cent.

7. Find the difference between the banker's and the true discount on a bill of £600. 5*s.*, due in 4 months, at $6\frac{1}{4}$ per cent.

8. Find the difference between the practical and the true discount on £150. 1*s.* 3*d.*, due 4 months hence, at $6\frac{1}{4}$ per cent.

9. How much less than the true present value will a banker give me for a bill of £950. 5*s.* 3*d.*, due 7 months hence, at $6\frac{1}{4}$ %?

10. If £900 be due 8 months hence, what is the difference between the true and the mercantile discount on it at 5 %?

11. How much less than the true present value will a banker give for a bill of £950. 8*s.*, which has $7\frac{1}{2}$ months to run, at 5 %?

12. Find the banker's discount on a bill of £731. 0*s.* 6*d.*, due $2\frac{1}{2}$ years hence, at $5\frac{1}{2}$ per cent. What will the banker gain?

Third-Tenth-and-Tenth Rule.

354. It will be seen that the calculation of Practical Discount frequently involves the calculation of Interest for a number of days. If the number of days be n, for example, the time may be expressed as $\frac{n}{365}$ of a year, a fraction which cannot be simplified unless 5 or 73 is a factor of n. We thus have to multiply the amount of the bill by $\frac{n}{365} \times \frac{r}{100}$, *i.e.* by $\frac{2nr}{73000}$. Consequently, to find the interest for a number of days we *multiply the principal by the rate per cent., and by* **twice** *the number of days, and we divide the product by* 73000. It will however be found that

$$\frac{1}{73000} = \frac{1}{100000}\left(1 + \tfrac{1}{3} + \tfrac{1}{30} + \tfrac{1}{300}\right), \text{ very nearly;}$$

so that **division by 73000** may be accomplished by adding to the dividend **one third** of itself, **one tenth** of *one third* of it, and **one tenth** of *one tenth of one third* of it, and, in the result, moving the decimal point 5 places to the left. The interest obtained by this method is not quite exact, but it may be corrected by deducting $\frac{1}{4}d.$ for every £10 in it.

355. In the following example we will suppose that the days of grace are to be allowed.

Example. *Find the practical discount on a bill of* £328. 10*s. which is dated January* 1*st*, 1903, *is payable in* 3 *months, and is discounted on February* 12*th; the rate of discount being* $2\frac{1}{2}$ *per cent.*

It will be noticed that 1903 is not a leap year, and that there are therefore only 28 days in February; also that the bill becomes payable, *nominally* on April 1st, *legally* on April 4th.

The number of days between Feb. 12th and April 4th = 16 + 31 + 4 = 51.

The rate per cent. × 2 × no. of days = $2\frac{1}{2} \times 2 \times 51 = 255$.

£328·5	£.
25 5	N = 83768
6570	$\frac{1}{3}$ N = 27923
16425	$\frac{1}{30}$ N = 2792
1642·5	$\frac{1}{300}$ N = 279
£83768	10^5 | 114762

Interest = £1·148, correct to 3 places,
= £1. 3*s.* 0*d.* (to next penny).

NOTE. Before dividing by 10^5, we need not in the working retain any figures to the right of the decimal point.

The interest can also be calculated by the method of Art. 341, without using the Third-Tenth-and-Tenth Rule.

EXAMPLES. 160.

Find the Simple Interest on

1. £2294. 18*s.* 9*d.* for 115 days at 5 per cent.
2. £535 for 117 days at 4·75 per cent.
3. £232. 7*s.* 8*d.* for 214 days at $3\frac{1}{8}$ per cent.
4. £300. 8*s.* for 321 days at £4. 12*s* 6*d.* per cent.
5. £568. 6*s.* 3*d.* for 143 days at $5\frac{3}{8}$ per cent.
6. £1472. 15*s.* 6*d.* for 281 days at £3. 2*s.* 6*d.* per cent.
7. £1271. 4*s.* 5*d.* for 327 days at $7\frac{3}{8}$ per cent.
8. £450 was borrowed on January 1st, 1902, at 5 per cent.; what sum would repay the debt on the 20th of October, 1902?
9. What will a banker retain on discounting a bill of £1275, drawn on the 4th of March at 10 months, and discounted on the 14th of August at 5 per cent.?
10. A bill of £347. 19*s.* 4*d.*, nominally due on the 1st of January, is discounted on the 29th of May; what will the banker retain, reckoning interest at $3\frac{1}{8}$ per cent.?
11. A bill of £307. 19*s.* 4*d.*, drawn on the 1st of April at

6 months, is discounted on the 31st of May; what is the banker's discount at $6\frac{1}{2}$ per cent.?

12. What will a banker retain on discounting a bill of £666. 13*s*. 4*d*., drawn on the 2nd of March at 8 months, and discounted on the 8th of June at $3\frac{1}{8}$ per cent.?

13. A bill of £170. 12*s*. 6*d*. is drawn on the 22nd of Dec., 1898, at 8 months, and is discounted on the 29th of April at $3\frac{3}{4}$ per cent.; how much will the banker retain?

14. A bill of £463. 10*s*. is drawn on the 3rd of May at 6 months, and is discounted at $2\frac{3}{4}$ per cent. on the 18th of May; what is the discount?

15. What is the banker's discount on a bill of £696. 10*s*. 10*d*., drawn on the 12th of June at 7 months, and discounted on the 26th of September at $6\frac{1}{4}$ per cent.?

16. What will a banker give on the 1st of July for a bill of £1931. 9*s*. 2*d*., drawn on the 10th of May at 6 months, at 5 %?

17. On the 27th of December, 1884, a bill of £423. 7*s*. 6*d*. was discounted at $4\frac{3}{4}$ per cent.; the bill was drawn on the 31st of July at 12 months; what was the discount?

18. A bill of £1231. 17*s*. 6*d*., drawn on April 6th at 9 mos., is discounted on June 4th; find the banker's discount at $3\frac{1}{4}$%.

19. Find the banker's discount on a bill of £342. 17*s*. 10*d*., drawn on May 1st at 4 mos., and discounted at $1\frac{1}{2}$% on June 15th.

20. Find the banker's discount on a bill of £362. 13*s*. 4*d*., drawn and discounted on March 13th, 1903, and nominally due on the 26th of March, 1904, interest being reckoned at 5 per cent.

XXXIX. COMPOUND INTEREST.

356. Money is said to be lent at **Simple Interest** when it is agreed between the lender and the borrower that the interest shall be paid over periodically as soon as it becomes due. It may however sometimes be agreed that the interest shall not be paid over, but shall be retained by the borrower, and shall be considered as an additional loan upon which he must subsequently pay additional interest. The loan thus becomes increased at regular periods, and with it is increased the sum upon which interest must be paid.

We will suppose, for example, that £100 is lent in this way, the rate of interest being 5 per cent. and the interest being reckoned yearly. At the end of one year the interest is £5. This is retained by the borrower, and the loan becomes £105.

For the second year the interest is £5. 5*s*. This again is retained, and the loan becomes £110. 5*s*. The interest for the third year is £5. 10*s*. 3*d*., and the loan becomes £115. 15*s*. 3*d*. Money lent upon these conditions is said to be lent at **Compound Interest.**

It will be seen that the amounts of interest for the successive years continually increase, owing to the continual increase in the principal; a sum of money will therefore increase more rapidly when lent at Compound Interest than when lent at Simple Interest; but the lender may not actually lose anything in the latter case, because, the successive amounts of interest being paid over to him, he may lend them elsewhere or use them in some other profitable manner.

357. The sum lent we will call the **Principal**; the sum to which it amounts with interest at the end of a stated time, we will call the **Amount**; the difference by which the Amount exceeds the Principal we will call the **Interest** or **Increase.**

358. Case I. **To find the Amount,** and hence the **Increase,** when the Principal, the Time and the Rate per Cent. are given.

The Principal should be decimalized to 5 places, and 5 places should be retained throughout the working, in order that the result may be correct to 3 places.

To find the Interest for one year we multiply the Principal by $\frac{r}{100}$, where r is the rate per cent. This fraction will often reduce to one which has 1 for its numerator and an easy divisor for its denominator. For example, when r is 5, $2\frac{1}{2}$, $1\frac{1}{4}$, ..., the fraction becomes $\frac{1}{20}$, $\frac{1}{40}$, $\frac{1}{80}$, ... In such cases the Interest for one year is obtained by dividing the Principal by an easy divisor such as 20, 40, 80, etc.

When the rate per cent. is a small integer, such as 3, the fraction is of the form $\frac{3}{100}$. In this case too the operation is simple; we multiply the Principal by 3 and then divide by 100.

We may sometimes express the fraction $\frac{r}{100}$ as the sum of *two or more fractions*, each of which has 1 for its numerator.

Thus, $$\frac{2\frac{3}{4}}{100}=\frac{2\frac{1}{2}}{100}+\frac{\frac{1}{4}}{100}=\frac{1}{40}+\frac{1}{400}.$$

These fractions, $\frac{1}{40}$ and $\frac{1}{400}$, of the Principal can easily be obtained, and their sum gives the Interest for the year. The following examples will illustrate the method of arranging the work.

Example i. *Find the amount, and the increase, of* £634. 9*s.* 4*d.* *in* 3 *years at* 5 % *compound interest.*

£634·46667 = Principal for 1st year.
5 % = $\frac{1}{20}$; ∴ 31·72333 = Interest for 1st year.
666·19000 = Principal for 2nd year.
5 % = $\frac{1}{20}$; ∴ 33·30950 = Interest for 2nd year.
699·49950 = Principal for 3rd year.
5 % = $\frac{1}{20}$; ∴ 34·97497 = Interest for 3rd year.

Amount at end of 3 years = £734·474
= £734. 9*s.* 5$\frac{3}{4}$*d.* ;
∴ Increase = £100. 0*s.* 1$\frac{3}{4}$*d.*

Example ii. *Find the amount, and the increase, of* £345. 13*s.* 8*d.* *in* 2$\frac{3}{4}$ *years at* 3 % *compound interest, to the nearest penny.*

For the last period, $\frac{3}{4}$ of a year, we proceed as if the rate of interest were $\frac{3}{4}$ of 3 %, *i.e.* 2$\frac{1}{4}$ %.

£345·68333 = Principal for 1st year.
3 % = $\frac{3}{100}$; ∴ 10·37050 = Interest for 1st year.
356·05383 = Principal for 2nd year.
3 % = $\frac{3}{100}$; ∴ 10·68161 = Interest for 2nd year.
366·73544 = Principal for 3rd year.
2 % = $\frac{1}{50}$; and $\frac{1}{4}$ % = $\frac{1}{400}$; ∴ 7·33471 + ·91684 = Interest for $\frac{3}{4}$ of a year.

Amount at end of 2$\frac{3}{4}$ years = £374·987
= £374. 19*s.* 9*d.* ;
∴ Increase = £29. 6*s.* 1*d.*

EXAMPLES. 161.

Find the amount, and the increase, of the following sums at Compound Interest, *to the nearest penny.*

1. £126 at 5 p. c. in 2 yrs.
2. £1234 at 5 p. c. in 3 yrs.
3. £625. 10*s.* at 5 per cent. in 2 years.
4. £716. 5*s.* at 2$\frac{1}{2}$ per cent. in 3 years.
5. £561. 2*s.* 6*d.* at 5 per cent. in 3 years.
6. £923. 17*s.* 6*d.* at 2$\frac{1}{2}$ per cent. in 3 years.
7. £258 at 10 per cent. in 3 years.
8. £2794 at 12$\frac{1}{2}$ per cent. in 3 years.
9. £2618. 5*s.* at 12$\frac{1}{2}$ per cent. in 3 years.
10. £374. 16*s.* 7*d.* at 8$\frac{1}{3}$ per cent. in 3 years.

Find the amount, and the increase, of the following sums at Compound Interest, *neglecting fractions of a penny.*

11. £3600 in 2 yrs. at 3 p. c.
12. £3752 in 2 yrs. at 4 p. c
13. £1256. 10*s.* in 2 years at 3 per cent.

Find the amount, and the increase, of the following sums at Compound Interest, *neglecting fractions of a penny.*

14. £7205. 5*s.* in 2 years at $4\frac{1}{2}$ per cent.
15. £506. 12*s.* 6*d.* in 3 years at 3 per cent.
16. £378. 5*s.* 8*d.* in 3 years at 3 per cent.
17. £3264. 11*s.* 7*d.* in 3 years at $2\frac{3}{4}$ per cent.
18. £3257. 13*s.* 9*d.* in 3 years at 4 per cent.
19. £5264. 7*s.* 10*d.* in 3 years at $3\frac{1}{4}$ per cent.
20. £2969. 16*s.* 3*d.* in 3 years at $2\frac{1}{4}$ per cent.
21. £1586. 13*s.* 6*d.* in 3 years at $4\frac{1}{2}$ per cent.
22. £1896. 15*s.* 2*d.* in $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent.
23. £1678. 9*s.* 11*d.* in $2\frac{1}{4}$ years at $4\frac{1}{2}$ per cent.
24. £3689. 8*s.* 3*d.* in $2\frac{1}{3}$ years at $4\frac{1}{2}$ per cent.
25. £2314. 17*s.* 2*d.* in $2\frac{2}{3}$ years at $3\frac{3}{4}$ per cent.

26. The population of a town in 1871 was 35740. If the rate of increase was 20 per cent. every 10 years, what was the population in 1901?

27. The population of a town increases yearly at the rate of 25 per 1000. It is now 378530. What will it be 3 years hence?

359. The intervals at which interest is to be reckoned and added may be any periods agreed upon, and moreover the rate of interest may vary from period to period.

Example i. *Find the amount, and the increase, of* £5423. 7*s.* 8*d.* *in* 2 *years at* 3 % *compound interest, the interest being added half-yearly.*

If the interest for one year is £3, the interest for half a year is £$1\frac{1}{2}$. We may therefore reckon the time to consist of 4 half-yearly periods and the rate per cent. for each period to be $1\frac{1}{2}$.

£5423·38333 = Principal for 1st half-year.
1 % = $\frac{1}{100}$; 54·23383 } = Interest for 1st half-year.
$\frac{1}{2}$ % = $\frac{1}{2}$ of 1 %; 27·11691 }
5504·73407 = Principal for 2nd half-year.
1 % = $\frac{1}{100}$; 55·04734 } = Interest for 2nd half-year.
$\frac{1}{2}$ % = $\frac{1}{2}$ of 1 %; 27·52367 }
5587·30508 = Principal for 3rd half-year.
1 % = $\frac{1}{100}$; 55·87305 } = Interest for 3rd half-year.
$\frac{1}{2}$ % = $\frac{1}{2}$ of 1 %; 27·93652 }
5671·11465 = Principal for 4th half-year.
1 % = $\frac{1}{100}$; 56·71115 } = Interest for 4th half-year.
$\frac{1}{2}$ % = $\frac{1}{2}$ of 1 %; 28·35557 }

Amount at end of 2 years = £5756·181
= £5756. 3*s.* 7$\frac{1}{2}$*d.*;

∴ Increase = £332. 15*s.* 11$\frac{1}{2}$*d.*

Example. ii. *Find how much* £243. 3*s.* 7*d. will amount to if lent at compound interest for* $\frac{3}{4}$ *of a year, interest being reckoned and added every three months: at the rate of* 5 *per cent. per annum for the first three months,* $3\frac{1}{2}$ *per cent. for the second, and* 3 *per cent. for the third.*

The interest on £100 for the successive periods of three months is £$1\frac{1}{4}$, £$\frac{7}{8}$ and £$\frac{3}{4}$ respectively, *i.e.* £$1\frac{1}{4}$, £$(1-\frac{1}{8})$ and £$(\frac{1}{2}+\frac{1}{4})$. The fraction $\frac{r}{100}$ therefore becomes $\frac{1\frac{1}{4}}{100}$, $\frac{1-\frac{1}{8}}{100}$ and $\frac{\frac{1}{2}+\frac{1}{4}}{100}$, or $\frac{1}{80}$, $\frac{1}{100}-\frac{1}{800}$ and $\frac{1}{300}+\frac{1}{400}$, for the three periods respectively.

	£243·17917	= Principal for 1st period.
$1\frac{1}{4}\% = \frac{1}{80}$;	3·03974	= Interest for 1st period.
	246·21891	= Principal for 2nd period.
$\frac{1}{8}\% = \frac{1}{100} - \frac{1}{800}$;	2·46219	
	·30777	
	2·15442	= Interest for 2nd period.
	246·21891	
	248·37333	= Principal for 3rd period.
$\frac{3}{4}\% = \frac{1}{200} + \frac{1}{400}$;	1·24187	= Interest for 3rd period.
	·62093	

Amount at end of 3rd period = £250·236 = £250. 4*s.* 8$\frac{3}{4}$*d.*

EXAMPLES. 162.

Find the amount of the following sums lent at Compound Interest, interest being added at the periods stated:

1. £325. 3*s.* 6*d.* for one year at 4 per cent. per annum; interest being added quarterly.

2. £513. 12*s.* 8*d.* for 4 months at 4 per cent. per annum; interest being added monthly.

3. £672. 8*s.* 10*d.* for $1\frac{1}{2}$ years at 5 per cent. for the first year and $3\frac{1}{2}$ per cent. per annum for the final period; interest being added half yearly.

4. £1236. 17*s.* 9*d.* for 2 years at 3 % for the first year and 2 % for the second; interest being added half-yearly.

5. £2693. 5*s.* 7*d.* for 2 years at $2\frac{1}{2}$ per cent. for the first six months, and $1\frac{1}{2}$ per cent. for the remainder of the time; interest being added half-yearly.

360. We will suppose r to denote the rate per cent. (which remains the same throughout the period), and £R to denote what £1 will amount to in 1 year; then

$$R = 1 + \frac{r}{100} \times 1 = 1 + \frac{r}{100} = \frac{100+r}{100}.$$

If £P be the original Principal, it follows that the sum to which it amounts at the end of the *first* year is £P × R. This is the Principal for the *second* year; therefore the amount at the *end* of the second year is £P × R × R, and so on. Thus, if £P be the original Principal, the amounts at the end of

1, 2, 3, 4, ... t years

will be £P . R, £P . R^2, £P . R^3, £P . R^4, ... £P . R^t respectively.

Hence if £A be the sum to which £P will amount after being lent for t years at Compound Interest, we see that

$$A = PR^t,$$

t being an integer. By means of this relation we can find the amount of any sum of money lent at Compound Interest at a given rate per cent. and for a given (integral) number of years.

If the last portion of the period is not a complete year, the last multiplier is not R, *i.e.* $1+\frac{r}{100}$, but it is what $1+\frac{r}{100}$ becomes when for r we put that number which is the same fraction of r as the final portion of the period is of a whole year.

We will now show how the formula $A = PR^t$ may be used.

361. Case II. **To find the Principal**, when the Amount, the Time and the Rate per Cent. are given.

Example. *Find the sum of money which, lent at Compound Interest at* $2\frac{1}{4}$%, *would amount to* £262. 13*s.* 5*d.* *in* $2\frac{1}{3}$ *years.*

For each of the 1st two years, $R = \frac{100+2\frac{1}{4}}{100} = 1{\cdot}0225$,

and for the last $\frac{1}{3}$ of a year, the multiplier $= \frac{100+\frac{3}{4}}{100} = 1{\cdot}0075$.

We will suppose that £P is the required Principal; then

$$P \times (1{\cdot}0225)^2 \times (1{\cdot}0075) = 262{\cdot}671;$$

$$\therefore\ P = \frac{262{\cdot}671}{(1{\cdot}0225)^2 \times (1{\cdot}0075)} = \frac{262{\cdot}671}{1{\cdot}0533475} = 249{\cdot}368;$$

$\therefore$ the required sum = £249. 7*s.* 4*d.* to the nearest penny.

362. The meanings of the terms **true discount** and **true present worth** when the interest is *compound* are the same as when the interest is *simple*. In the example in the preceding article we have found the present, or ready-money, value of a sum of £262. 13*s.* 5*d.*, which will be paid in $2\frac{1}{3}$ years. The difference between £262. 13*s.* 5*d.* and £249. 7*s.* 4*d.* is the *true discount.*

Example. *Find the ready-money equivalent of* £1034. 17*s.* 6*d.*, *to be paid in* $2\frac{1}{2}$ *years, reckoning Compound Interest at* 3 %.

For each of the first 2 years, $R = \frac{100+3}{100} = 1{\cdot}03$,

and for the remaining $\frac{1}{2}$ of a year, the multiplier $= \frac{100+1\frac{1}{2}}{100} = 1{\cdot}015$;

$$\therefore P \times (1{\cdot}03)^2 \times (1{\cdot}015) = 1034{\cdot}875;$$

$$\therefore P = \frac{1034{\cdot}875}{1{\cdot}0768135} = 961{\cdot}053;$$

$\therefore$ Ready-money value = £961. 1*s.* 1*d.* to the nearest penny.

EXAMPLES. 163.

Find how much, lent at Compound Interest, will amount to the following sums in the given times and at the given rates per cent. respectively; interest being added yearly.

1. £100 in 2 yrs. at 5 p. c.
2. £375 in 2 yrs. at 5 p. c.
3. £436. 10*s.* in 3 years at 3 per cent.
4. £518. 5*s.* in 3 years at 4 per cent.
5. £723. 14*s.* 8*d.* in $2\frac{1}{2}$ years at 4 per cent.
6. £391. 8*s.* 5*d.* in $2\frac{3}{4}$ years at 4 per cent.

Find the true present worth of the following sums of money, and also the true discount, reckoning compound interest at the given rates and neglecting fractions of a penny.

7. £500, due in 3 years, at 5 per cent.
8. £1200, due in $2\frac{1}{4}$ years, at 4 per cent.
9. £875, due in 3 years, at 3 per cent.
10. £2050, due in $2\frac{1}{2}$ years, at 3 per cent.
11. £327. 10*s.*, due in 3 years, at $2\frac{1}{2}$ per cent.
12. £861. 12*s.* 6*d.*, due in 3 years, at $4\frac{1}{2}$ per cent.
13. £462. 8*s.* 6*d.*, due in $2\frac{3}{4}$ years, at $5\frac{1}{2}$ per cent.
14. £1276. 17*s.* 2*d.*, due in $2\frac{1}{3}$ years, at $2\frac{1}{2}$ per cent. for the first year and 3 per cent. afterwards.
15. £2085. 3*s.* 9*d.*, due in $2\frac{2}{3}$ years, at 2 per cent. for the first 2 years and $2\frac{1}{4}$ per cent. afterwards.
16. £7621. 12*s.* 4*d.*, due in $2\frac{2}{3}$ years, at $4\frac{1}{2}$ per cent. for the first year and 3 per cent. afterwards.
17. £5013. 11*s.* 3*d.*, due in $2\frac{1}{2}$ years, at $2\frac{3}{4}$ per cent. for the first year and $2\frac{1}{2}$ per cent. afterwards.

363. CASE III. **To find the Principal,** when the Increase, time and rate per cent. are given.

Example. *On what sum of money will the increase at* $3\frac{1}{2}$ *per cent. for* $2\frac{1}{2}$ *years be* £169. 17*s.* 3*d.*?

Let £P be the sum of money required.

Amount of £P in $2\frac{1}{2}$ years at $3\frac{1}{2}$ p. c. compound interest

$$= £P \times 1{\cdot}035 \times 1{\cdot}035 \times 1{\cdot}0175$$

$$= £P \times 1{\cdot}0899714;$$

$$\therefore \text{ Increase} = £P \times {\cdot}0899714;$$

$$\therefore \ £P = £\frac{169{\cdot}8625}{{\cdot}0899714}$$

$$= £1887{\cdot}961$$

$$= £1887.\ 19s.\ 3d.$$

We have reckoned the fraction of a penny as a whole penny.

EXAMPLES. 164.

On what sum, to the next penny, will the increase at Compound Interest be

1. £18. 6*s.* 7*d.* in 2 years at 4 per cent.?
2. £59. 12*s.* 5*d.* in $2\frac{1}{2}$ years at 4 per cent.?
3. £130. 5*s.* 3*d.* in $2\frac{1}{2}$ years at 3 per cent.?
4. £254. 13*s.* 8*d.* in $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent.?
5. £362. 7*s.* 5*d.* in $2\frac{1}{4}$ years at 3 per cent.?
6. £534. 13*s.* 6*d.* in $2\frac{3}{4}$ years at $2\frac{1}{2}$ per cent.?
7. £226. 17*s.* 2*d.* in $1\frac{1}{2}$ years at $2\frac{1}{2}$ per cent. per annum, interest being added half-yearly?
8. £371. 3*s.* 9*d.* in 9 months at 4 per cent. per annum, interest being added quarterly?
9. £437. 10*s.* 6*d.* in $2\frac{1}{2}$ years, interest being at the rate of 5 % for the first year, and 4 % for the rest of the time?
10. £278. 9*s.* 8*d.* in 3 years, at 3 per cent. for the first year, $3\frac{1}{2}$ per cent. for the second, and 4 per cent. for the third?

364. CASE IV. **To find the rate per cent.,** when the Principal, Amount and Time are given.

The rate per cent. cannot be determined by the common arithmetical methods except in special cases.

Example. *Find the rate per cent. at which* £182. 5*s.* *would amount to* £250 *in 3 years at compound interest.*

We have $£182\frac{1}{4} \times \left(1+\frac{r}{100}\right)^3 = £250$;

$$\therefore \left(1+\frac{r}{100}\right)^3 = \frac{250}{182\frac{1}{4}} = \frac{1000}{729};$$

$$\therefore 1+\frac{r}{100} = \frac{10}{9};$$

$$\therefore \frac{r}{100} = \frac{1}{9};$$

$$\therefore \text{rate per cent.} = \frac{100}{9} = \underline{11\frac{1}{9}}.$$

EXAMPLES. 165.

(Questions 4, 5, 6 and 8 involve the finding of a cube root.)

Find approximately the rate per cent. Comp. Interest at which

1. £253. 14*s.* 3*d.* will amount to £259. 15*s.* 6*d.* in 2 years.
2. £147. 12*s.* 1*d.* will amount to £161. 4*s.* 9*d.* in 2 years.
3. £314. 3*s.* 10*d.* will amount to £342. 18*s.* 2*d.* in 2 years.
4. £1243. 9*s.* 4*d.* will amount to £1390. 5*s.* 8*d.* in 3 years.
5. £2086. 10*s.* 5*d.* will amount to £2373. 17*s.* 9*d.* in 3 years.
6. £1943. 7*s.* 8*d.* will amount to £2628. 14*s.* 3*d.* in $1\frac{1}{2}$ years, interest being added half-yearly.
7. £945. 6*s.* 7*d.* will amount to £1060. 3*s.* 2*d.* in 2 years, interest being added half-yearly.
8. £620. 4*s.* 11*d.* will amount to £669. 18*s.* 5*d.* in $1\frac{1}{2}$ years, interest being added quarterly.

MISCELLANEOUS EXAMPLES. 166.

(The Interest is to be considered *Simple*, unless *Compound* is specified.)

1. Find, to the nearest centime, the interest on 750 f. for 4 years 6 months at $3\frac{3}{4}$ per cent.
2. What is the interest, to the nearest centime, on 243 f. for 2 years 5 months at $2\frac{1}{2}$ per cent.?
3. Find the rate per cent., if the interest on 375 dol. amounts in 4 years to 56 dol. 25 c.
4. Find the interest on £6845. 15*s.* for $4\frac{1}{2}$ years at 5 per cent.
5. Find the rate per cent. if the interest on £451. 17*s.* 6*d.* for 4 years amounts to £36. 3*s.*
6. Find the interest on 1000 guineas for $12\frac{1}{2}$ years at $4\frac{1}{2}$ %.
7. Find the time for which £4951. 6*s.* 8*d.* is invested at $3\frac{1}{2}$ per cent., if the interest amounts to £433. 4*s.* 10*d.*

8. Find the true present worth of £252. 9*s*. $3\frac{1}{2}d$., due in $2\frac{1}{2}$ years, reckoning interest at $4\frac{3}{4}$ per cent.

9. Find the number of years in which £3713. 10*s*. at 7 per cent. will amount to £4038. 8*s*. $7\frac{1}{2}d$.

10. Find the sum which will amount to £1083. 10*s*. 11*d*. in $4\frac{1}{6}$ years at $5\frac{1}{3}$ per cent.

11. Find the amount of £2970. 16*s*. 8*d*. in 3 years 73 days at £3. 2*s*. 6*d*. per cent.

12. Find the sum of money on which the true discount for 6 years at $1\frac{2}{3}$ % is £16. 17*s*. 6*d*.

13. Find the number of years in which the interest on £1684. 3*s*. 4*d*. at $4\frac{1}{4}$ per cent. will amount to £178. 18*s*. $10\frac{1}{4}d$.

14. Find the true discount on 3762 f. for 6 years at $6\frac{1}{4}$ %.

15. Find the sum which will amount to £454. 2*s*. $1\frac{1}{2}d$. in $11\frac{1}{2}$ years at $2\frac{3}{4}$ per cent.

16. Find the rate per cent. if the interest on £869. 7*s*. 6*d*. for $3\frac{1}{2}$ years amounts to £121. 14*s*. 3*d*.

17. Find the number of years in which 28920 f. will amount to 32354 f. 25 c. at $4\frac{3}{4}$ per cent.

18. At what rate per cent. will 170 dol. 30 c. amount to 187 dol. 33 c. in 3 years 4 months?

19. Find the sum which will amount to £1152. 13*s*. 3*d*. in 7 years at $3\frac{1}{2}$ per cent.

20. In what time will £7466. 13*s*. 4*d*. amount to £8306. 13*s*. 4*d*. at $3\frac{3}{4}$ per cent.?

21. Find the rate per cent. if the true discount on £357. 17*s*. 8*d*. for 2 years is £29. 11*s*.

22. Find the true discount on 386 dol. 25 c. for 9 mos. at 4 %.

23. At what rate per cent. will £3235 yield £420. 11*s*. interest in 4 years?

24. If the simple interest on £3138. 3*s*. 6*d*. exceeds that on £2762. 13*s*. 6*d*. by £7. 0*s*. $9\frac{3}{4}d$. in 3 mos., find the rate per cent.

25. If the true discount on £775 for 5 years were £25, what would be the true discount on £765 for 3 years at the same rate?

26. If a sum of money be 19 times the true discount on it for a given time, and if the interest on the same sum for the same time at the same rate be £2. 3*s*. $4\frac{1}{2}d$., find the sum.

27. Find the interest on £5609. 10*s*. $9\frac{1}{2}d$. for 1 year 301 days at 4 per cent.

28. Find the Practical discount on a bill of £364. 3*s.* 9*d.* due nominally on July 17th and discounted at 4 % on March 5th.

29. Find the interest on £2105. 12*s.* 7*d.* from January 1st, 1891, to August 14th, 1893, at $3\frac{1}{2}$ per cent.

30. Find the Banker's discount on a bill of £329. 13*s.* 3*d.* due nominally on July 11th, 1893, and discounted at $2\frac{1}{4}$ % on Feb. 2nd, 1892.

31. Find the interest on £1325. 7*s.* 10*d.* from May 12th, 1893, to January 26th, 1895, at $3\frac{3}{4}$ per cent.

32. Find the interest on £2503. 18*s.* 5*d.* from January 20th, 1892, to March 30th, 1895, at 4 per cent.

33. Find the sum to which £586. 15*s.* will amount in 2 years at $2\frac{1}{2}$ per cent. compound interest.

34. Find the sum to which £264. 12*s.* $9\frac{1}{4}$*d.* will amount in $2\frac{1}{2}$ years at 3 per cent. compound interest.

35. Find the amount of £126. 17*s.* $3\frac{3}{4}$*d.* in $1\frac{1}{2}$ years at 4 per cent. compound interest, interest being reckoned half-yearly.

36. Find the sum which will amount to £1750 in 2 years at 4 % compound interest, interest being reckoned half-yearly.

37. Find the difference between the Simple and the Compound Interest on £453. 4*s.* 6*d.* for 3 years at $3\frac{1}{2}$ %.

38. Obtain the increase of £5723. 14*s.* 7*d.* when put out (i) at Simple Interest, (ii) at Compound Interest, at 3 per cent. per annum for $1\frac{3}{4}$ years, interest being reckoned half-yearly.

39. What sum of money will amount to £233. 4*s.* $4\frac{4}{5}$*d.* in 2 years, reckoning Compound Interest; interest being at the rate of $3\frac{1}{2}$ per cent. for the first year, and 4 per cent. for the second?

40. Find the sum of money at Compound Interest, on which the interest for the first year is £15. 12*s.* 6*d.*, and for the second year £16. 1*s.* $10\frac{1}{2}$*d.*

41. The difference between the simple and the compound interest on a certain sum for 3 yrs. at 5 % is £6. 2*s.* Find the sum.

42. The true present worth of a sum of money at compound interest is £150 if it is due in 3 years, and £160. 3*s.* 4*d.* if it is due in 2 years. Find the rate per cent.

43. A man takes a house for three years, the rent of which is £200 and is due at the end of each year. He pays down a certain sum at once in order to reduce his annual payment to £120. Reckoning compound interest at 4 %, find, to the nearest pound, the sum he pays down at once.

44. A person borrows £500: at the end of each year he pays £125 to reduce the principal, and also to pay interest at 4% on the sum which has been standing against him during the year. Find, to the nearest penny, how much he will owe after three payments.

XL. STOCKS AND SHARES.

365. We will now consider certain matters connected with the investment of money in the purchase of Government Annuities and of shares in Joint Stock Companies.

If a person wishes to open a shop of any kind, it is generally necessary for him to be provided beforehand with a certain amount of money wherewith to pay for the requisite furniture and for the goods which he proposes to sell. If two or more persons arrange to carry on business together, such an association is called a **Partnership**, each member of it is called a **Partner**, and each partner may be supposed to furnish a portion of the money required. At the end of certain periods, the **gross** profits earned will be calculated; from these will be deducted the expenses incurred; and the remainder or **net** profits will be divided among the partners, each of them receiving, we will suppose, the same fraction of the net profits as the money furnished by him is of the total amount subscribed by them all. The money used in starting and carrying on a business is called **Capital.**

366. When the capital and the business done are comparatively small, the whole affair may be in the hands of a very small number of persons, or even of *one* person only: and in such a case all the persons concerned may take part in conducting the business, that is, they may all be **active** partners. But when the capital is very large, as in the case of a great railway, it becomes necessary to increase the number of partners very considerably; and when this happens it is no longer possible for each one to take an *active* part. It is customary therefore for the general body of partners to elect a few of those most fit among themselves to manage and direct the affairs of the **Company.** The persons so elected are called the **Directors.**

367. Before starting the company, we may suppose that a few persons interested in the particular business, which it is proposed to carry on, meet together and choose a certain number of **Directors.** It will be decided that the capital necessary is, say, £3,000,000, and the directors will issue circulars to the public inviting them to subscribe towards it.

This capital may be divided into 300,000 equal **parts of £10** each, or into 30,000 equal parts of £100 each, or in other ways. In the former case each part is called a £10 **share.** In the latter case also each part may be a *share,* but it is more commonly described as **£100 stock,** three parts as £300 stock, and so on. The distinction between *share* and *stock* will be explained in Art. 371.

Any number of shares, or any amount of stock, may be subscribed for by the same person, and the subscriber's name will be entered in the books of the company as the proprietor of that portion of the capital. The proprietors of shares or stock in a company are called **Shareholders,** and the association is called a **Joint Stock Company.** At regular intervals, usually half a year, the accounts are made up, the directors are paid for their work, and the *net* profits are divided among the shareholders at so much per share or so much for each £100 stock. What each receives is called his **Dividend.**

368. If a shareholder wishes to withdraw from the company he may do so, not by demanding his money back from the directors, but by selling his stock or shares to some one else. For this purpose a special market called the **Stock-market** has been established, the business of which is carried on by **Stock-brokers.** It is the business of a stock-broker to find some one to buy the stock or shares which another wishes to sell; and to find some one willing to sell stock or shares that another wishes to buy. For doing this the broker will charge a certain sum for each £100 stock, or for each share, bought or sold. This charge is called the broker's **commission,** or the **brokerage.**

369. When the company is fairly started, if it does a good business and its *net* profits increase, the dividend on each share, or on each £100 stock, will become greater; and in consequence of this increase in the dividend, the market value of each share, or of each £100 stock, will also increase.

If, for example, the dividend paid were at the rate of 10 per cent. per annum and the business done was a steady one and comparatively free from risk of loss, the market value of a share of the *nominal* value of £10 might really be £25 or more.

If, on the other hand, the company were to pay only a small dividend, or no dividend at all, for several years, and if there should seem to be but a poor prospect of success, it might be possible to buy a £10 share for £2 or even less.

We must be careful, therefore, to remember the distinction between the **nominal** value of a share, which is only the amount of *capital* which it represents, and its **market** value, which is the money one can get for it in the stock-market. When the market value of a share is *greater* than the nominal value, the share is said to be **at a premium**; when the market value is *less* than the nominal value, the share is said to be **at a discount**; when the nominal value and the market value are the same, the share is said to be **at par**.

Thus, when the market value of £100 stock is £125, the stock is said to be at £25 *premium*; and when the market value is £85, the stock is said to be at £15 *discount*.

The market value is called the **price** of the share or stock.

370. To illustrate what we have said about the price depending upon the prosperity of the company, we quote the prices of £100 ordinary stock in a few railway companies on Nov. 17, 1885, June 30, 1896, and March 12, 1904.

The relative prices at any one time represent fairly the reputation of the companies at that time, but a general rise in prices and a general fall in prices depend mainly upon the political and monetary situation.

Company.	*Price*, 1885.	*Price*, 1896.	*Price*, 1904.
Caledonian	$100\frac{1}{2}$	163	100
Brighton	117	194	122
Chatham and Dover	$18\frac{1}{4}$	19	$13\frac{1}{4}$
Metrop. District	$41\frac{1}{2}$	$28\frac{3}{4}$	32
Midland	130	166	
North Western	166	200	144
South Western	125	215	$147\frac{1}{2}$

It will be seen that in 1885 the prices of equal amounts of similar stock ranged between £18. 5*s*. and £166. In 1896 all of the prices, except two, were very much higher. In 1904 all the prices, with one exception, were again much lower than in 1896; and one of the stocks had disappeared from the list, because of a change in its form.

371. The distinction between shares and stock is that when the capital of a company is in the form of *stock*, any odd amount of stock may generally be obtained; but when it is divided into *shares*, only *whole* shares can usually be obtained, parts of a share are not generally either procurable or saleable. There are however some exceptional cases.

372. There is another kind of *stock*, which might have claimed prior consideration. In the latter part of the 17th century the cost of the long wars was greater than could be borne by the ordinary revenue of the country. And, moreover, as posterity would reap much of the benefit expected to be derived from these wars, it was considered fair that posterity should bear part of the cost. It was suggested then that the public should be invited to *lend* money to the state, that a special tax should be imposed, and that, with the proceeds of the tax, interest should be paid upon the loan at a fixed rate, and the loan itself be gradually paid off. Several loans were raised in this manner at different times, and in each case a special tax was levied and the proceeds set apart. The money thus set apart was called the Funds; but in course of time the special tax was abolished, and the money was borrowed without any special arrangement being made for repayment. The name was then transferred to the loans themselves; and by the term **Funds** we now mean the permanent debt owed by the state.

373. On March 31, 1904, our national debt amounted to £794,498,000, comprising **Funded Debt** (or the **Funds**), **Unfunded Debt**, the capitalized value of **Terminable Annuities**, and other smaller liabilities; but there were assets valued at £31,868,000. If divided among the population the debt would amount to nearly £18 per head. In 1816 the total amount, reckoned in the same manner, was £900,046,000, or £45 per head. The Unfunded Debt consists of Exchequer Bills, Exchequer Bonds and Treasury Bills, all of which are made repayable at fixed dates. The Terminable Annuities consist partly of Life Annuities, and partly of Annuities granted for stated periods. A portion of the revenue, called the Sinking Fund, is set apart for the payment of interest on the debt; any balance and any revenue surplus is employed in buying stock in the open market, the stock bought being at once cancelled, and the debt thereby diminished.

Many states still borrow money or *raise loans*, promising to pay them off or **redeem** them at par at a stated time, and, in some cases too, the proceeds of special taxes are still set apart.

374. The Funded Debt will never be repaid in the way in which a *loan*, properly so called, would be; and it cannot therefore strictly be called a loan. In fact, anyone who now buys £100 of British stock, really buys a right to an annual payment, or **Annuity**, of £2$\frac{3}{4}$, £2$\frac{1}{2}$, or whatever the rate may be, and he may hand down this right to his descendants, or he may sell it, or give it away. The annuity will be paid so long as the country is

able and willing to pay it, and consequently the market price of a Government stock will depend mainly upon the country's reputation for honesty and solvency. The price may be affected temporarily in many ways, particularly by an outbreak of war. The price of the old British 3 per cent. stock, for example, was as high as 107 in 1739, but fell to $47\frac{1}{4}$ in 1798. In 1896, when the rate of interest was $2\frac{3}{4}$ per cent., the price rose to $113\frac{7}{8}$; in 1903, when the rate became $2\frac{1}{2}$ per cent., the price did not exceed $93\frac{5}{8}$, and in 1904 it has fallen below 86.

375. The British Funded debt has been subject to several changes from time to time. The rate of interest was reduced from 3 per cent. to $2\frac{3}{4}$ per cent. in 1888, and from $2\frac{3}{4}$ per cent. to $2\frac{1}{2}$ per cent. in 1903. It now consists of,

(i) the *Consolidated Annuities*, called briefly **Consols**, and formed by the amalgamation of several loans into one, on which interest is paid at the rate of $2\frac{1}{2}$ per cent. per annum. The Government has the right to pay off the whole amount *at par* after 1923, but no further change of any sort may be made before that date. The practical effect of such a right can only be that the *rate of interest* may be again *reduced*. The interest is paid quarterly on Jan. 5th, April 5th, July 5th, and Oct. 5th.;

(ii) some small amounts of $2\frac{1}{2}\%$ and $2\frac{3}{4}\%$ annuities, and debts to the Bank of England and to the Bank of Ireland.

376. In the reports published in the newspapers it will be found that to each stock are appended two prices, $112\frac{1}{8}$—$112\frac{3}{8}$, for example. This means that the stock-jobbers offer £112. 2*s.* 6*d.* for each £100 stock to those who wish to sell, and ask £112. 7*s.* 6*d.* for each £100 stock from those who wish to buy. The difference represents their profit.

In reckoning the cost of stock or shares, it is necessary to take account also of the stamp duties and the broker's commission. In the following examples we will generally suppose that these expenses are already included in the price quoted. It should however be noticed that any expenses *increase* the outlay of a buyer, and *diminish* the receipts of a seller.

377. **Given the amount of stock, to find the income.**

Example. *What annual income will be derived from* £2875 *of* 4% *stock?*

By 4 *per cent. stock* is meant a stock on every £100 of which is paid a regular dividend of £4 per annum.

Income from £100 stock = £4,

$\therefore$ ..... £2875 ..... $= £\frac{2875}{100} \times 4 = \underline{£115}.$

EXAMPLES. 167.

What annual income will be derived from

1. £3300 of 5 p. c. stock?
2. £4275 of 4 p. c. stock?
3. £800 of $3\frac{1}{2}$ p. c. stock?
4. 9015 f. of 5 p. c. stock?
5. £5638 of 5 p. c. stock?
6. 10775 dol. of 3 p. c. stock?
7. 72891 f. of 3 p. c. stock?
8. 8225 dol. of $2\frac{1}{2}$ p. c. stock?
9. £7650 of $4\frac{1}{2}$ p. c. stock?
10. £812. 10*s.* of 4 p. c. stock?
11. £4326. 10*s.* of $2\frac{1}{2}$ per cent. stock?
12. £2784. 15*s.* of $4\frac{1}{2}$ per cent. stock?
13. £6820. 12*s.* 6*d.* of $3\frac{1}{2}$ per cent. stock?
14. £3241. 17*s.* 6*d.* of 6 per cent. stock?
15. 3072 shares bearing a $\frac{1}{2}$ yearly dividend of 2*s.* 6*d.* each?
16. 1875 shares bearing a $\frac{1}{2}$ yearly dividend of 15*s.* each?
17. 250 shares bearing a $\frac{1}{2}$ yearly dividend of 12*s.* each?

378. Given the amount of money to be invested, to find how much stock can be bought.

Example. *How much stock at* $92\frac{3}{8}$ *(brokerage included) can be bought for* £1434. 17*s.* 10*d.*?

Amount of stock obtained for $£92\frac{3}{8}$ $= £100$,

$\therefore$ ..... $£1434\frac{107}{120} = £\frac{1434\frac{107}{120}}{92\frac{3}{8}} \times 100$

$= £\frac{172187}{120} \times \frac{8}{739} \times 100$

$= £\frac{20 \times 233}{3}$

$= £1553\frac{1}{3}$;

$\therefore$ required amount of stock $= £1553.\ 6s.\ 8d.$

EXAMPLES. 168.

How much stock at

1. 98 can be bought for £4900?
2. 105 can be bought for £2394?
3. $85\frac{1}{2}$ can be bought for 3591 f.?
4. 90 can be bought for £3249?
5. $90\frac{1}{4}$ can be bought for £6498?
6. $97\frac{3}{4}$ can be bought for £14076?
7. 90 can be bought for 12960 dol.?
8. 84 can be bought for £1245?

How much stock at

9. 88 can be bought for £10000?
10. 97 can be obtained for 7485 f.?
11. $93\frac{1}{2}$ can be bought for £525?

379. Given the amount of stock bought, to find the cost.

Example. *What must be paid for* £1553. 6*s.* 8*d. of* 3 *per cent. stock at* $92\frac{1}{4}$, *the brokerage being* $\frac{1}{8}$ *per cent.?*

Cost of £100 stock $= £(92\frac{1}{4}+\frac{1}{8}) = £92\frac{3}{8}$;

$\therefore$ ..... £1 ..... $= £\frac{739}{800}$;

$\therefore$ cost of £$1553\frac{1}{3}$ stock $= £\frac{4660}{3}\times\frac{739}{800} = £\frac{233\times739}{120} = £\frac{172187}{120}$;

$\therefore$ amount to be paid $=$ **£1434. 17*s.* 10*d.***

EXAMPLES. 169.

Find the cost of

1. 1500 f. French Rentes at 90.
2. 4550 f. French Rentes at 93.
3. £4500 stock at $112\frac{1}{2}$.
4. £2250 stock at 93.
5. £2137. 10*s.* stock at 80.
6. 3340 dol. stock at 98.
7. £8500 Consols at $97\frac{3}{4}$.
8. 1245 dol. stock at 84.
9. £7440 stock at $92\frac{7}{8}$.
10. £7580 stock at $90\frac{7}{8}$; brokerage, $\frac{1}{8}$ per cent.
11. £4450 stock at $51\frac{7}{8}$; brokerage, $\frac{1}{8}$ per cent.
12. £1234 stock at $88\frac{3}{4}$; brokerage, 2*s.* 6*d.* per cent.

What would the sale of the following stocks produce:

13. £1500 stock at $88\frac{3}{4}$; brokerage, 2*s.* 6*d.* per cent.?
14. £3725 stock at $113\frac{1}{8}$; brokerage, 2*s.* 6*d.* per cent.?
15. £12875 stock at $108\frac{1}{2}$; brokerage, $\frac{1}{8}$ per cent.?
16. £84350 stock at $8\frac{5}{8}$, allowing brokerage at 2*s.* 6*d.* per cent.?

380. Given the amount invested, to find the income.

Example. *What income will be derived from the investment of* £4537. 10*s.* *in a* 3 *per cent. stock at* $90\frac{5}{8}$, *brokerage being* $\frac{1}{8}$ *per cent.?*

Income from £100 stock $= £3$;

but price of £100 stock + brokerage $= £(90\frac{5}{8}+\frac{1}{8}) = £90\frac{3}{4}$;

$\therefore$ income from £$90\frac{3}{4}$ money $= £3$;

$\therefore$ ..... £1 ..... $= £\frac{3}{90\frac{3}{4}} = £\frac{12}{3623} = £\frac{4}{1211}$;

$\therefore$ ..... £$4537\frac{1}{2}$ ..... $= £\frac{4}{1211}\times\frac{9075}{2} =$ **£150.**

EXAMPLES. 170.

What income will be derived from investing

1. £2852 in 5 per cent. stock at 115?
2. £1274 in $3\frac{1}{2}$ per cent. stock at 91?
3. 25935 dol. in 3 per cent. stock at 90?
4. 4560 f. in $3\frac{1}{2}$ per cent. stock at 95?
5. 3220 dol. in $3\frac{1}{2}$ per cent. stock at $80\frac{1}{2}$?
6. £3519 in $3\frac{1}{2}$ per cent. stk. at $97\frac{5}{8}$; brokerage, $\frac{1}{8}$ per cent.?
7. £7560 in 3 per cent. stk. at $94\frac{3}{8}$; brokerage, $\frac{1}{8}$ per cent.?
8. If A invest £1210 in a $3\frac{1}{4}$ per cent. stock at $97\frac{1}{2}$, and a like sum in 5 per cent. stock at $90\frac{3}{4}$, what will be his income?
9. A invests £1695. 16*s.* 8*d.* in a $3\frac{1}{2}$ per cent. stock at $101\frac{3}{4}$, and B invests £1767. 3*s.* 9*d.* in a 3 per cent. stock at $94\frac{1}{4}$; what is the difference between the incomes obtained?
10. What income is obtained by investing £154. 14*s.* in a $3\frac{1}{2}$ per cent. stock at 91, and £183. 12*s.* in a $2\frac{1}{2}$ per cent. stock at 85?
11. Will the income in the preceding example be greater or less than that which would be derived from investing the whole amount in a 3 per cent. stock at 90? Find the difference.

381. Given amount invested and income, to find the price.

Example. *Find the price of a 3 per cent. stock when the investment of* £434. 12*s.* 6*d. produces an income of* £14. 5*s.*, *neglecting brokerage.*

Money reqd. to produce £$14\frac{1}{4}$ income = £$434\frac{5}{8}$;

∴ .. .. £1 ..... = £$434\frac{5}{8} \div 14\frac{1}{4}$;

∴ ..... £3 ..... = £$\frac{3\,4\,7}{1\,1\,4} \times 3 = £\frac{1\,8\,3}{2} = £91\frac{1}{2}$;

∴ the price of the 3 % Stock = $\underline{91\frac{1}{2}}$.

EXAMPLES. 171.

1. An income of £126 is obtained by investing £3591 in a 3 per cent. stock; what is the price of the stock?
2. An income of 552 f. is obtained by investing 17135 f. in a 3 per cent. stock. Find the price of the stock. (Brokerage $\frac{1}{8}$ %.)
3. What is the price of a 4 per cent. stock when an investment of £17430 produces an income of £830?
4. £7038 is invested in a $3\frac{1}{2}$ per cent. stock, and yields an income of £252; what is the price of the stock? (Brokerage $\frac{1}{8}$ %.)
5. Find the price of a 4 per cent. stock, in order that an investment of £4680 may produce an income of £180.

6. A person invests £4095 in a 4 per cent. stock, and receives an income of £173. 6*s.* 8*d.*; what is the price of the stock?

7. What is the price of a 3 per cent. stock, when an investment of 12967 f. 50 c. produces an income of 432 f. 25 c.?

8. Find the price of a 3 per cent. stock, if an investment of £616. 5*s.* produces an income of £20. 8*s.*? (Brokerage $\frac{1}{8}$ %.)

9. A person wishes to obtain an income of £100 per annum by investing £2666. 13*s.* 4*d.* in a 3 per cent. stock; find at what price it must be quoted when he buys in.

10. Find the price of $4\frac{1}{2}$ per cent. stock when an investment of 869 dol. 25 c. produces an income of 42 dol. 75 c.

Miscellaneous Questions on Stocks.

382. The following questions are more varied in character.

Example i. *Which produces the greater income, a* $5\frac{1}{2}$ *per cent. stock at* 105, *or a* $4\frac{1}{2}$ *per cent. stock at* $87\frac{1}{2}$?

In former case, income from £105 money = £$5\frac{1}{2}$,

$\therefore$ ..... £1 ..... = £$\frac{11}{210}$;

in latter case, ... £$87\frac{1}{2}$ ..... = £$4\frac{1}{2}$,

$\therefore$ ..... £1 ..... = £$\frac{9}{175}$.

It will be found in the usual way that $\frac{11}{210}$ is greater than $\frac{9}{175}$; therefore the *former* investment produces the greater income.

Example ii. *Find the alteration in income caused by selling* £6400 *of a* 3 *per cent. stock at* $86\frac{3}{8}$, *and with the money obtained buying* 4 *per cent. stock at* $114\frac{7}{8}$; *the brokerage being* 2*s.* 6*d.* *per cent. in each case.*

Income derived from 3 per cent. stock $= £6400 \times \frac{3}{100} = £192$;

money obtained by selling 3 per cent. stock $= £6400 \times \frac{86\frac{1}{4}}{100}$;

$\therefore$ income derived from 4 per cent. stock $= £64 \times \frac{345}{4} \times \frac{4}{115}$

$= £192$;

$\therefore$ there is *no change* in the income.

Example iii. *What rate of interest is obtained by investing in a* 4 *per cent. stock at* $92\frac{1}{2}$?

The rate per cent. required is to be reckoned on the *money* invested.

Income from £$92\frac{1}{2}$ money = £4,

$\therefore$ ..... £100 ..... = £$\frac{100}{92\frac{1}{2}} \times 4 = £\frac{800}{185} = £\frac{160}{37} = £4\frac{12}{37}$;

$\therefore$ rate of interest = $4\frac{12}{37}$ per cent.

Example iv. *If a 4 per cent. stock pays interest at the rate of $4\frac{12}{37}$ per cent. on the money invested, what is the price of the stock?*

$£4\frac{12}{37}$ = interest on £100 money;

$\therefore$ £4 = ... $£\frac{4}{4\frac{12}{37}} \times 100$ .....

= ..... $£\frac{148}{160} \times 100$ .....

= ..... $£\frac{185}{2}$ .....

= ..... $£92\frac{1}{2}$ .....;

but £4 = ... £100 stock;

$\therefore$ the price of the stock = $\underline{92\frac{1}{2}}$.

EXAMPLES. 172.

1. How much $3\frac{1}{2}$ per cent. stock at $97\frac{3}{4}$ can be purchased for £3519, and what income would be thence derived?

2. Find the yearly income arising from the investment of £4788 in a $3\frac{1}{2}$ per cent. stock at 105.

3. If the cost of £100 of 3 per cent. stock be $93\frac{1}{4}$, what income would be obtained by investing £2000 in that stock?

4. What annual income will be obtained from £13000 invested in a $3\frac{1}{2}$ per cent. stock at 91? and from the same sum invested in a 4 per cent. stock at 96?

5. What income will be derived from £5000, if £2000 be invested in a 3 per cent. stock at 98, and the remainder in a $3\frac{1}{2}$ per cent. stock at par?

6. By investing £3600 in a 3 per cent. stock, a yearly income was obtained of $4\frac{1}{6}$ per cent. on the investment; what was the price of the stock?

7. If a 3 per cent. stock is at $91\frac{1}{8}$, what interest does it give on £100 invested?

8. If £100 of 3 per cent. stock is worth £85 money; what rate of interest is this equivalent to?

9. What ought to be the average rate of interest when a 3 per cent. stock is at 84?

10. At what rate will a person receive interest, if he invests his capital in a 4 per cent. stock at 104?

11. Which produces the greater income, bank stock paying 10 per cent. at $234\frac{1}{2}$ or a 3 per cent. stock at $92\frac{1}{2}$?

12. What must I invest in a $3\frac{1}{2}$ per cent. stock at 96 in order to obtain an income of £711. 4*s.* $8\frac{3}{4}$*d.*? How much per cent. do I get by the investment?

13. The sum of £1245 is invested in a 4 per cent. stock at 84. How much money must be invested in a $3\frac{1}{2}$ per cent. stock at 98 in order to obtain the same income?

14. What half-yearly dividend is derived from an investment of £3000 in a $3\frac{1}{2}$ per cent. stock at $98\frac{3}{4}$, after deducting income-tax at the rate of 3*d.* in the pound?

15. What half-yearly dividend is due upon an investment of £2500 in a 3 per cent. stock at $87\frac{3}{8}$, after deducting 7*d.* in the pound for income-tax?

16. If railway stock bought at 28 per cent. premium pay $7\frac{1}{2}$ per cent. on the money invested, how much per cent. would it pay if it were bought at 10 per cent. discount?

17. If a person sells £1000 of 3 % stock at 91, and invests the proceeds at 5 %, by how much is his income increased?

18. What change of income is made by transferring £4275 stock from a 4 per cent. stock at 80 to a $5\frac{1}{2}$ per cent. stock at 99?

19. A person sells out £2250 of 3 per cent. stock at 93, and invests the proceeds in 6 per cent. stock at 108. Find the change in his income.

20. If a 3 per cent. stock be at 92 and a 4 per cent. stock at 115, how much will one's income be increased by selling £10000 of the former, and investing the proceeds in the latter?

21. £2500 is invested in a 3 per cent. stock at 88. The stock is afterwards sold at 92, and the money is put out at interest at 4 per cent. Find the change in the income.

22. A person sells £1000 of a 4 % stock at 90 and invests the proceeds in a 3 % stock at 72. How much of the latter stock will he obtain, and what will be the difference in his income?

23. If a 3 per cent. stock be at 91, and a 5 per cent. stock at 117, how much will the income of a man be affected by his selling £2250 of the former, and investing the proceeds in the latter?

24. How should I affect my income by selling out of a $1\frac{3}{4}$ per cent. stock at $47\frac{1}{4}$, and investing in a $3\frac{3}{5}$ per cent. stock at 99?

25. Find the alteration in income occasioned by selling £3200 of 3 % stock at $86\frac{3}{8}$, and with the proceeds buying 4 % stock at $114\frac{7}{8}$, the brokerage being $\frac{1}{8}$ % on each transaction.

26. If I sell £43400 of a three per cent. stock at 96, and buy 4 per cent. stock at 105, find the difference in my income.

27. If I hold £10000 of a 3 per cent. stock, and sell out at $94\frac{1}{2}$, investing the money obtained in a 4 per cent. stock at 105, what change would I make in my income?

28. A person invests £7560 in a 3 per cent. stock at $94\frac{1}{2}$, and when the price falls to 90 he sells out $\frac{1}{4}$ of his stock: afterwards when the price rises to $94\frac{3}{4}$, he sells the remainder. Find the alteration in his capital.

29. How much money must be invested in a 3 per cent. stock at $92\frac{1}{2}$ in order to produce the same income as £1710 invested in a $3\frac{1}{2}$ per cent. stock at 95?

30. A person invests half his capital in a 3 per cent. stock at 90, and half in a 4 per cent. stock at par. Which investment produces the larger income? If his total income be £550, what was his capital?

31. If I buy 10 shares of £20 each at $27\frac{1}{2}$, and sell out at $37\frac{1}{2}$ after receiving a dividend of 15 %, how much shall I gain in all?

32. A person investing in a 3 per cent. stock receives $3\frac{1}{3}$ per cent. interest for his money. What is the price of the stock?

33. What is gained by investing £1950 in stock at $97\frac{3}{8}$ and selling out at 104; brokerage being $\frac{1}{8}$ per cent. on each transaction?

34. Which produces the greater income, French 3 % Rentes at $64\frac{1}{4}$, or United States 5 % stock at $102\frac{3}{4}$? Find the difference between the incomes derived from investments of £1000 in each.

35. A person invests £1365 in a 3 per cent. stock at 91; he sells out £1000 stock when the price has risen to $93\frac{1}{2}$, and the remainder when it has fallen to 85: how much does he gain or lose by the transaction?

36. Determine the price of a 3 per cent. stock when £5230 stock can be bought for £4975. 0*s.* 9*d.*; and find the income thereby obtained. What sum of money would produce the same income if invested in 4 per cent. railway stock at $104\frac{3}{4}$?

37. What income will be obtained by investing £1232. 10*s.* in a 3 per cent. stock at $90\frac{5}{8}$? If the stock is sold out at $88\frac{3}{4}$, and the proceeds invested in railway stock at $106\frac{1}{2}$, paying a dividend of 6 per cent., what will be the increase in income?

38. A person invests £4700 in shares which are at 98 and pay $3\frac{1}{2}$ per cent., and the same sum in a 3 per cent. stock which is at 94. What difference would it have made in his annual income if he had invested the whole amount in the shares?

39. A person sells £5000 of a 3 per cent. stock, and buys a $3\frac{1}{2}$ per cent. stock at $87\frac{1}{2}$. If the increase in his income be £5, what is the price of the three per cent. stock?

40. A man has £9000 of 3 per cent. stock; he sells out at 91 and invests in a 4 per cent. stock, thereby increasing his income by £76. 13*s.* 4*d.* Find the price of the 4 per cent. stock.

41. A person sells £10000 of a $3\frac{1}{2}$ per cent. stock, realizing £9350. What must be the price of a 4 per cent. stock, if the above sum, when invested in it, produces £10 more income?

42. A person buys £800 of a 3 per cent. stock at 85, and £500 more when it is at 97; how much per cent. will he get for his money, after deducting an income-tax of 7*d.* in the £?

43. By selling out £4500 of a 5 per cent. stock at $112\frac{1}{2}$, and investing the proceeds in a 7 % stock, a person finds his income increased by £168. 15*s.*; what is the price of the latter stock?

44. If a 3 per cent. stock is at $93\frac{1}{8}$, how much must be spent in the purchase of stock to obtain an income of £552? If the purchaser afterwards sells out at 95, and invests the proceeds in mortgages at 4 per cent., what will be the increase in his income?

45. A man has £6680 stock in a 4 per cent. stock at 98. When the price has fallen 2 per cent., he transfers his capital to a $3\frac{1}{2}$ per cent. stock at $83\frac{1}{2}$. Find the alteration in his income.

46. A person invests £25935 in a 3 per cent. stock at 90; what is his income? If the first year's dividend be invested in the same stock at 91, and the dividend for the second year at 95, what will be his income for the third year?

47. A man invests £6498, half in a 3 per cent. stock at 90, and the other half in a 4 per cent. stock at 95. Each stock rises 5 per cent.; he then sells out, and invests the proceeds of each stock in the other. The stocks fall to their original value, and he again sells out. Find his gain or loss.

48. A man buys £700 of a 3 per cent. stock at $94\frac{1}{2}$, and also invests £585 in Russian 5 per cent. stock at $97\frac{1}{2}$. How much stock has he altogether standing in his name? If he sells out of the 3 per cent. stock at 95 and out of the 5 per cent. stock at $96\frac{1}{4}$, how much does he gain or lose by the transaction?

49. A man invests £550 in a $3\frac{1}{2}$ per cent. stock at $88\frac{1}{4}$; afterwards when they are at 93 he sells out, and invests the money at $5\frac{1}{4}$ per cent. What difference does this make in his income?

50. The price of £100 bank stock which pays $6\frac{1}{2}$ per cent. is $185\frac{3}{4}$; £6250 of this stock is transferred to a 3 per cent. stock, when the price of the latter is $92\frac{7}{8}$: find the loss in annual income.

51. The difference between the incomes derived from investing a certain sum in 6 per cent. stock at 126 and in 9 per cent. stock at 210 is £22. 10*s.* Find the amount invested.

52. If a 3 per cent. stock be at $90\frac{5}{8}$, what sum must I invest in order to secure a yearly income of £470 after paying an income-tax of 5*d.* in the £, brokerage being $\frac{1}{8}$ per cent.?

53. A invests £3500 in buying equal amounts of a 3 per cent. stock at $78\frac{1}{8}$ and a 6 per cent. stock at $109\frac{3}{8}$. B invests the same sum, half in one stock and half in the other. Find (i) the difference in their incomes, (ii) the ratio of their rates of interest.

54. A person invested £1911 in a 3 per cent. stock at $79\frac{1}{2}$; he sold out, and realized a gain of £150 after paying $\frac{1}{8}$ per cent. commission on each transaction. At what price did he sell?

55. A certain 3 per cent. stock is at $91\frac{1}{2}$, and a 4 per cent. stock at 123; one person buys £5000 stock in each, and another person invests £5000 money in each: compare the respective rates of interest obtained by the two on their whole investments.

56. When a 3 per cent. stock is at $87\frac{1}{2}$, and £100 shares paying 5 per cent. are at $130\frac{1}{4}$, which produces the better income? What sum does a person invest if the difference between the incomes resulting from the two investments is £561?

57. If a 3 per cent. stock is at $89\frac{3}{8}$, what must be the price of 5 per cent. stock in order that it may be advantageous to transfer from the former into the latter, $\frac{1}{8}$ per cent. commission being charged both on the sale and on the purchase of stock?

58. A railway proprietor receives one year a dividend of 6% on his stock, and pays an income-tax of 4*d.* in the £. The next year he receives a dividend of $6\frac{1}{2}$%, and pays an income-tax of 3*d.* in the £. He finds that his income is £249 more in the latter year than in the former. How much stock does he hold?

59. If a 3 per cent. stock is at 91, and a $3\frac{1}{2}$ per cent. stock at 106, what is the difference between the rates per cent. obtained on money invested in them? What will this difference become if we allow for a commission of $\frac{1}{8}$ per cent. on the latter stock?

60. A person has a quantity of 3 per cent. stock which yields £240 a year. He sells out $\frac{1}{4}$ of the stock at $87\frac{1}{4}$, and invests the proceeds in stock at $174\frac{1}{2}$. What dividend ought the latter to pay in order that he may thereby increase his income by £40?

61. A person invests £14970 in the purchase of a 3 per cent. stock at 90 and a $3\frac{1}{4}$ per cent. stock at 97. His total income being £500, how much of each stock did he buy?

62. A man invested 500 guineas in a 3 per cent. stock at $93\frac{1}{2}$, and, after having received one quarter's dividend, sold out at $96\frac{3}{4}$. Find how much stock he was able to buy, and how much he gained by the whole transaction.

63. 144 shares in a company are worth £856. 16*s.* when the dividend is at the rate of 7 per cent.; how many shares ought to be worth £1193. 8*s.* when the dividend is at 9 per cent.?

XLI. PROPORTIONAL PARTS AND PARTNERSHIPS.

(a) Proportional Parts.

383. When we know the ratio between the parts of a given quantity, however many the parts may be, we can easily determine the parts themselves.

Suppose, for example, that there are two parts in the ratio of 3 : 4; then, if we divide the whole quantity into *sevenths*, the first part contains 3 sevenths, and the other 4 sevenths.

Again, if there are 5 parts in the ratio of 2 : 3 : 4 : 5 : 6, the relations of the parts to the whole are represented by the fractions $\frac{2}{20}$, $\frac{3}{20}$, $\frac{4}{20}$, $\frac{5}{20}$, $\frac{6}{20}$; for these fractions are proportional to 2, 3, 4, 5, 6, and their sum is a whole unit.

NOTE.—The ratios which quantities bear to each other are not altered when we multiply all the quantities, or divide all the quantities, by any, the same, number. Suppose, for example, that the parts into which a quantity is divided are proportional to the numbers $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{1}{6}$. We may multiply each of these numbers by 60, without altering the ratios, and we thus find that the parts are proportional to the whole numbers 30, 40, 45, 48, 10. Similarly, if all the quantities have a common factor, this factor may be removed without the ratios being altered.

Example i. *Divide* £550 *among* 4 *people, so that their shares may be proportional to* 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$ *respectively.*

The shares are proportional to 2, $2\frac{1}{2}$, 3 and $3\frac{1}{2}$, *i.e.* to 4, 5, 6 and 7, the sum of which is 22. We may therefore divide £550 into 22 equal parts, and give the people 4, 5, 6 and 7 of these parts respectively.

Therefore, 1st share $= \frac{4}{22}$ of £550 $=$ £100;

2nd share $= \frac{5}{22}$ of £550 $=$ £125;

3rd share $= \frac{6}{22}$ of £550 $=$ £150;

4th share $= \frac{7}{22}$ of £550 $=$ £175.

Example ii. *Divide* £333. 6*s.* 8*d.* *among* A, B *and* C, *so that* B *may have* $1\frac{2}{3}$ of A's *share, and* C *as much as* A *and* B *together.*

B's share $= 1\frac{2}{3}$ of A's share,

and C's share $=$ A's share $+ 1\frac{2}{3}$ of A's share $= 2\frac{2}{3}$ of A's share;

$\therefore$ the 3 shares are proportional to 1, $1\frac{2}{3}$ and $2\frac{2}{3}$, *i.e.* to 3, 5 and 8;

$\therefore$ A's share $= \frac{3}{16}$ of £333. 6*s.* 8*d.* $= \frac{1}{16}$ of £1000 $=$ £62. 10*s.*,

B's share $= \frac{5}{16}$ of £333. 6*s.* 8*d.* $= \frac{1}{16}$ of £1666. 13*s.* 4*d.* $=$ £104. 3*s.* 4*d.*,

and C's share $= \frac{8}{16}$ of £333. 6*s.* 8*d.* $= \frac{1}{2}$ of £333. 6*s.* 8*d.* $=$ £166. 13*s.* 4*d.*

NOTE.—The student should always check his results by verifying that the sum of the parts is equal to the whole.

EXAMPLES. 173.

1. Divide £45 among 3 persons so that their shares may be as the numbers 3, 5, 7.

2. Divide 702 into three parts proportional to 2, 3 and 4.

3. Divide 351 into three parts proportional to $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$.

4. A prize of £2770 was divided among three persons in the proportion of $\frac{3}{4}$, $\frac{1}{1}\frac{4}{5}$ and $\frac{5}{8}$; what were their respective shares?

5. A yacht and its fittings cost £2011. 4*s.* and the yacht itself cost 5 times as much as the fittings; what did the fittings cost?

6. A General levied a contribution of £870 on 4 villages containing 250, 300, 400 and 500 inhabitants respectively; what had each village to pay, if each paid in proportion to its population?

7. A force of police, 1921 in number, is to be distributed among 4 towns in proportion to the populations, which are respectively 4150, 12450, 24900 and 29050; how many should be allotted to each town?

8. A body of 50000 soldiers is to be raised in 4 provinces which contain respectively 175002, 82434, 22116 and 20448 men fit for service; find the number which each province must supply.

9. A sum of £25000 has to be raised by rate from 4 townships A, B, C and D. A is rated at £87501, B at £41217, C at £11058 and D at £10224. If each township has to pay a share proportional to the sum at which it is rated, find the share paid by each.

10. Divide £954. 9*s.* among A, B and C, so that A's share may be to B's as 3 is to 5, and B's share to C's as 10 is to 11.

11. At an examination three candidates obtain altogether 1460 marks. A's marks are to B's in the ratio of 4 to 3, and A's to C's as 7 is to 6; how many marks did each get?

12. Divide £35. 5*s.* $1\frac{1}{2}d.$ among A, B and C, so that B may get twice, and C three times, as much as A.

13. Divide £180 among A, B and C, so that A may receive 3 times as much as B, and B and C together $\frac{1}{2}$ as much as A.

14. Divide £1714. 11*s.* $4\frac{1}{2}d.$ among 4 persons A, B, C and D, so that A may have twice as much as B, B three times as much as C, and C four times as much as D.

15. Gunpowder is composed of 33 parts of nitre, 7 of charcoal and 5 of sulphur; what weight of each of these components is contained in a ton of gunpowder?

16. £2228. 16*s.* was left to a boy and two girls: the boy was to have 3 parts, and each girl 2 parts; what did each receive?

17. Three persons, A, B, C, agree to pay their hotel bill in the proportion of 4 : 5 : 6. A pays the first day's bill, which amounts to £2. 10*s.* 10*d.*; B the second, £3. 12*s.* 2*d.*; and C the third, £3. 17*s.*; how must they settle accounts?

18. Divide £1690 among 4 persons, so that the first may receive $\frac{1}{11}$ as much again as the second, the second $2\frac{2}{3}$ times as much as the third, and the fourth as much as the second and third.

19. A debt of £18. 15*s.* 10*d.* was paid in crowns, shillings and pennies. How many coins of each kind were there, if the numbers were proportional respectively to 3, 2, 1?

20. A sum of £630 is divided among 10 men, 32 women and 48 children. Each man receives twice as much as a woman, and the 32 women receive as much as the 48 children; what will be the share of each person?

21. In coining shillings at the Mint, a composition is used consisting of silver and copper in the ratio of 37 to 3. The price of silver being 2*s.* 2*d.* per ounce, and the weight of a shilling $\frac{2}{11}$ of an ounce, what is the value of the pure silver in a shilling?

22. Six coins of equal weight, made of gold and silver mixed, are melted together and recast. In one of them the gold and silver are in the ratio of 2 : 1, in two others in the ratio of 3 : 5, and in the rest in the ratio of 7 : 5; in what ratio will the gold and silver be mixed in the new coins?

23. The total cost of two articles is 26 dol. 25 c. They are both sold at the same price, the one at a gain of 12 %, and the other at a loss of 2 %. Find the cost of each.

24. The incomes of two men would be equal if one were increased by 15 % and the other diminished by 8 %. The sum of their incomes is £671. 8*s.* What is the income of each?

25. Two sums of money are together equal to £609. 10*s.* The interest on one sum for 3 years at 4 % is equal to the true discount on the other for 3 years at 4 %. Find the two sums.

(b) Partnerships

384. As an example of proportional division we will consider the distribution of the profits, or the losses, of a partnership. We will suppose that they are distributed among the partners in proportion to each one's share of the capital.

Example i. *Two partners,* A *and* B, *begin business together with a capital of* £3500, *of which* A *contributes* £2000 *and* B *the rest. Their profits at the end of the year are* £2100. *What is the share of each?*

Total capital = £3500;

A's share of capital = £2000 = $\frac{2000}{3500}$ of whole = $\frac{4}{7}$ of whole,

and B's ..... = £1500 = $\frac{1500}{3500}$ ..... = $\frac{3}{7}$ ..... ;

∴ A's share of profits = $\frac{4}{7}$ of £2100 = £1200,

and B's ..... = $\frac{3}{7}$ of £2100 = £900.

Example ii. A *puts* £348. 16*s.* 6*d.*, B £804. 11*s.* 4*d.* and C £621. 12*s.* 2*d.* *into a speculation which yields* £795. 18*s.* 8*d.* *profit. Find their shares of this profit.*

Rough approximation.—The contributions are roughly £350, £800, and £600, *i.e.* in the ratio of 7 : 16 : 12. Hence the shares are roughly $\frac{7}{35}$ of £800, $\frac{16}{25}$ of £800, and $\frac{12}{35}$ of £800, *i.e.* £160, £366 and £274 roughly.

A's contribution = £348·825;

B's ..... = £804·56667;

C's ..... = £621·60833;

∴ Total capital = £1775;

∴ Profit on £1775 = £795·93333;

∴ ..... £1 = £·448413;

∴ A's share = £156·418

= £156. 8*s.* 4$\frac{1}{2}$*d.*;

B's ..... = £360·778

= £360. 15*s.* 6$\frac{3}{4}$*d.*;

C's ..... = £278. 14*s.* 9*d.*

```
                0·448413
17,75 ) 795·93333
        85 93
        14 933
            7333
             233
              55

 ·44841 | 30
     348 | ·825
 ------
 134·52390
  17·93652
   3·58730
    ·35873
       897
       224
 ------|
 156·418

 ·44841 | 30
     804 | ·56667
 ------
 358·73040
   1·79365
    ·22421
      2690
       269
        27
         3
 ------|
 360·778
```

EXAMPLES. 174.

1. Two persons gained £375 in trade, one having put into the business £500 and the other £850; what part of the profits ought each to receive?

2. A bankrupt owes £16000 and £8000 to two creditors; his assets are £7000; what does each creditor lose?

3. Three persons contribute £5000, £7500, and £2500, respectively, to a business. Their profits amount to £3000. What is the share of each?

4. A puts £480 into a business, B £720 and C £240. Divide a profit of £2880 among them.

5. Three partners invest £3000, £1200 and £2000 respectively in a business, and at the end of a year the profits are £744; what will be the share of each partner?

6. Three partners in a firm invest in it respectively £3000, £3500 and £4000. If the whole profits amount to 1678 guineas, what share does each receive?

7. A, B and C, respectively, invested £6413, £5247 and £4081 in a business of which the net profits in one year were £1123. 1*s.* 2¼*d.*; what was the share of each?

8. A, B and C trade in partnership with capital to the amount of £241, £365, £472 respectively. How will they share a profit of £416. 10*s.*?

9. A, B and C are partners, A receiving $\frac{2}{5}$ of the profits, and B and C sharing the remainder equally. A's income is increased by £220 when the profits rise from 8 to 10 per cent. Find the respective capitals invested.

10. A and B are partners in a business in which A has £4900 and B £1400: B, as working partner, receives 6 per cent. of the whole profit, and the rest is divided between them in proportion to their capital. What does each receive of £450 profit?

11. A is a working, B a sleeping, partner in a business. A owns £1200 of the capital, B £2000. A receives 10 per cent. of all profits for managing, and the rest is divided proportionally. Find their respective shares of £800 profit.

12. A has £5000 engaged in a concern, and B £7500. The gross profit for a year is £3200, $\frac{1}{8}$ of which is spent in salaries, and £30 in insurance of the premises. A then receives 8 per cent. on his capital, and B 4 per cent. on his; and the remainder of the profit is divided between them in proportion to their capital. Find the net receipts of A and B respectively.

(c) Temporary Partnerships.

385. If one or more of the partners join the others *after* they have started, the sharing of the profits will depend not only upon the capital contributed by each, but also upon the *time* during which each share of the capital is employed.

If, for example, A begins the year alone with £2000 capital, and is joined 4 months afterwards by B with £1500 capital, it will be seen that A has £2000 employed for 12 months, and B £1500 for 8 months. Now 12 months' use of £2000 may be con-

sidered equivalent to *one* month's use of £24000, and 8 months' use of £1500 as equivalent to *one* month's use of £12000. Hence, supposing the profits for the year to be £2100, we may state the case as follows:—"*If* A *and* B *trade for* one *month with a capital of* £36000, *of which* £24000 *belongs to* A *and* £12000 *to* B, *what will be their respective shares of a profit of* £2100? We see then that

A's share of the profit $= \frac{24000}{36000}$ of £2100 $= \frac{2}{3}$ of £2100 $=$ £1400;

and B's share of the profit $= \frac{12000}{36000}$ of £2100 $= \frac{1}{3}$ of £2100 $=$ £700.

Example i. A, B *and* C *rent a field jointly for* £25. 12*s.* 5*d.* A *puts in* 15 *sheep for* 6 *months,* B 45 *sheep for* 8 *months,* C 81 *sheep for* 4 *months. What should each pay in rent?*

A has 6 months' use of the field for 15 sheep,
which is equivalent to 1 month's use for 90 sheep;

B has 8 months' use of the field for 45 sheep,
which is equivalent to 1 month's use for 360 sheep;

C has 4 months' use of the field for 81 sheep,
which is equivalent to 1 month's use for 324 sheep;

∴ their shares of the use of the field are proportional to 90, 360, 324;

∴ A's share of rent $= \frac{90}{774}$ of £25. 12*s.* 5*d.* $= \frac{5}{43}$ of 6149*d.* $= 715d.$ $=$ £2. 19*s.* 7*d.*

Similarly, B's share of rent $= \frac{360}{774}$ of 6149*d.* $= \frac{20}{43} \times 43 \times 143d.$ $=$ £11. 18*s.* 4*d.*;

∴ C's share of rent $= \frac{324}{774}$ of 6149*d.* $= \frac{18}{43} \times 43 \times 143d.$ $=$ £10. 14*s.* 6*d.*

Example ii. A *has a business in which he employs a capital of* £70000. *He sets free part of this capital by selling a share in the business to* B *after* 2 *months, for* £22000, *and another share to* C *for* £8000 *after* 3 *months more. At the end of* 7 *months from the start the profit is* £19600. *What does each receive?*

A employs £70000 capital for the first 2 months,
£48000 capital for the next 3 months,
and £40000 capital for the last 2 months;

this is equivalent to £140000 capital for 1 month,
£144000 capital .....,
and £80000 capital .....;

i.e. altogether, £364000 capital for 1 month

B employs £22000 capital for 5 months, or £110000 capital for 1 month.
C employs £8000 capital for 2 months, or £16000 capital for 1 month.

Hence A's share of profit $= \frac{364000}{490000}$ of £19600

$= \frac{364}{490}$ of £19600

$= 52 \times 280$ $=$ £14560;

B's share of profit $= \frac{110}{490}$ of £19600 $=$ £4400;

C's share of profit $= \frac{16}{490}$ of £19600 $=$ £640.

EXAMPLES. 175.

1. Two persons, A and B, hold a field in common at a rental of £54. A puts in 23 horses for 27 days, and B 21 horses for 39 days; how much of the rent should be paid by each?

2. Three persons, A, B and C, join in business. A puts in £500 for 3 months, B £650 for 8 months, and C £300 for 11 months. They gain £420. Find the share of each.

3. A, B and C use the same field for pasture. A has 8 oxen in it for 10 months, B has 7 oxen for 12 months, and C has 20 oxen for 5 months. If the rent is £66, how much should each pay?

4. A and B undertake a contract; A provides 5 men for 20 days, and 6 men for the next 15 days; B provides 10 men for 12 days, and 9 men for the next 20 days. If A receives £332. 10*s.*, what should B receive?

5. A puts £600 into a concern, and B, three months after, £300; six months after the start they put in £300 each, and they gain £375 in 18 months; how must they share the profit?

6. Four merchants, A, B, C and D, trade together. A's stock of £300 was in use for 12 mos., B's stock of £330 for 10 mos., C's stock of £375 for 8 mos., and D's stock of £395 for 6 mos. The whole gain was £818. What should each receive?

7. A and B rent a field for 10 months at 30*s.* per month, and A puts in 10 cows for 9 months; how many can B put in for the remaining month, if he pays £3 less than A?

8. A, B and C rent a field of 40 acres. A puts in 20 cows for 3 months, B 40 for 2 months, C 60 for $1\frac{1}{2}$ months. A's share of the rent is £11. 10*s.*; what are B's and C's shares?

9. A and B begin business together with £300, and C joins them after 4 months. How much capital must he bring to the business if, at the end of the year, each receives $\frac{1}{3}$ of the profits?

10 A began business with £450, and was joined subsequently by B with £1350. When did B join, if the profits are divided equally at the end of the year?

11. A, B and C together purchase an estate, their shares being as $1\frac{1}{2} : 2\frac{1}{2} : 4$. C sells one-third of his share to A, and A sells 96 acres to B. If A and C have then equal portions, how many acres had each at first?

12. A, B and C entered into partnership, A contributing £175 capital and B £210. Their profits amounted to £422. 10*s.*, of which C's share was £172. 5*s.* Find what capital C contributed

XLII. AVERAGES.

386. If a person reckons his expenditure during an ordinary year, and divides the amount by 365, the quotient gives what is called his **average** daily expenditure.

If a man buys 3 horses for £43, £56 and £38 respectively, he buys 3 horses for £137, at an *average* price of £45. 13*s.* 4*d.* each.

If a train travels 200 miles in 5 hours, the speed may vary from time to time, but the *average* rate is 40 miles per hour.

If a school consists of 340 boys in 17 classes, it is unlikely that all the classes will be of exactly the same size, but we may say that the *average* number of boys in a class is $\frac{340}{17}$, or 20.

The **average** of any number of quantities of the same kind is the result of dividing the *sum* of the quantities by the *number*. The average is also called the **mean** of the quantities.

Example. *Find the average of the four numbers* $2\frac{3}{4}$, $5\frac{1}{3}$, $4\frac{1}{6}$, $8\frac{1}{2}$.

$$\begin{aligned}\text{The average} &= \tfrac{1}{4}\left[2\tfrac{3}{4}+5\tfrac{1}{3}+4\tfrac{1}{6}+8\tfrac{1}{2}\right]\\ &= \tfrac{1}{4}\left[19+\tfrac{9+4+2+6}{12}\right]\\ &= \tfrac{1}{4}\left[20+\tfrac{3}{4}\right]\\ &= \underline{5\tfrac{3}{16}}.\end{aligned}$$

387. Generally, the average of n quantities of the same kind, denoted by a, b, c, d..., is $\frac{1}{n}(a+b+c+d+\ldots)$; and conversely, if A be the average of n quantities the sum of the quantities will be nA.

388. If we are given the average value of two, or more, sets of quantities, the average value of the whole is *not* obtained by finding the average of the averages, unless there is the same number of quantities in every set. For example, if the average value of n quantities in one set be a, and the average value of m quantities of the same kind in another set be b, the sum of the $(m+n)$ quantities in the two sets is $an+bm$. Therefore the *average* value of the whole is $\frac{an+bm}{m+n}$, **not** $\frac{a+b}{2}$, unless $m=n$.

Example i. *A man's average daily expenditure is* 2*s.* 3*d. during May,* 4*s.* $0\frac{1}{2}$*d. during June, and* 2*s. during July. Find his average daily expenditure for the three months.*

$$\begin{aligned}\text{The total expenditure} &= 27d.\times 31+48\tfrac{1}{2}d.\times 30+24d.\times 31\\ &= 837d.+1455d.+744d.\\ &= 3036d.;\end{aligned}$$

$$\text{the number of days} = 31+30+31=92;$$

$$\therefore\ \text{the average daily expenditure} = 3036d.\div 92 = 33d. = \underline{2s.\ 9d.}$$

Example ii. *A ship sails out to a mark at the rate of* 10 *knots and sails back at the rate of* 15 *knots. What is its average rate of sailing?*

Let x = number of knots out to the mark.

Then $\frac{x}{10}$ = number of hours taken to go out;

and $\frac{x}{15}$ = ... to return;

$\therefore \frac{x}{10}+\frac{x}{15}$ = ... to go out and return;

$\therefore$ average rate of sailing $= 2x \div \left(\frac{x}{10}+\frac{x}{15}\right)$ knots $= 2x \div \frac{x}{6}$ knots $=$ 12 knots

EXAMPLES. 176.

1. Find the averages of the following sets of numbers:

(i) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

(ii) 7, 9, 12, 25, 17, 34, 50. (iii) ·25, 6·12, 89·4, ·0516.

(iv) $7\frac{1}{2}$, $6\frac{1}{4}$, $8\frac{3}{4}$, $9\frac{1}{3}$, $2\frac{1}{5}$. (v) $8\frac{2}{3}$, $5\frac{17}{60}$, $7\frac{11}{30}$, $25\frac{1}{2}$.

2. Find the average of $12\frac{6}{2\cdot5}$, 41, $6\frac{1}{2}$, ·0012, 0, $203\frac{1}{2}$, and $29\frac{7}{20}$; express the result as a decimal.

3. The populations of 3 towns are 16501, 18927, and 30543; what is the average population?

4. A cricketer, during a season, scores in successive innings 15, 3, 24, 0, 97, 215, 11, 47, 61, 0, 145, 19, 2, 169. What is his average per innings?

5. The population of England in 1841 was 14995132, and in 1851 it was 16854142. Find the average annual increase between the two dates.

6. What is the average daily expenditure of a man whose expenses for a common year amount to £1000?

7. On the six days respectively of one week a profit and loss account shews the following:—Profits of £27. 10*s.* 9*d.*, £35. 15*s.* 3*d.*, £56. 14*s.* 6*d.*, and £8. 2*s.* 1*d.*; and losses of £18. 16*s.* 3*d.* and £5. 6*s.* 4*d.* Find the average profit per day for the week.

8. Of 21 people, 13 lose £116. 7*s.* 8*d.* each, and 8 lose £93. 0*s.* 9*d.* each. What is the average loss per man?

9. A boat's crew weighed 10 st. 11 lb., 10 st. 13 lb., 11 st. 7 lb., and 10 st. 7 lb.; find the average per man.

10. The weights of a Cambridge crew are 10 st. $5\frac{1}{2}$ lb., 11 st. 2 lb., 12 st. $5\frac{1}{4}$ lb., 13 st. $6\frac{1}{4}$ lb., 14 st. 1 lb., 12 st. 3 lb., 11 st. $3\frac{1}{2}$ lb., and 10 st. 13 lb. Find the average weight per man.

11. If the keep of a prisoner cost £26. 19*s.* 10¾*d.* for a common year, find the average cost per day.

12. A farmer gains £90. 10*s.* by selling 120 oxen at the rate of £400 for 20; find the average cost per head.

13. In a school the numbers for the week were:—Mon. morn. 67; Tues. morn. 60; Wed. morn. 65; Thurs. morn. 68; Fri. morn. 62; Mon. aft. ·5 more than the average of Mon. and Tues. mornings; Tues. aft. 59; Wed. aft. ·5 less than the average of Tuesday; Thurs. aft. the average of Mon. morn. and Tues. aft.; Fri. aft. 60. Find the average attendance for the week.

14. The weights of Bow Bells are respectively: 2 tons 13 cwt. 25 lb., 1 ton 14 cwt. 2 qrs. 6 lb., 1 ton 6 cwt. 13 lb., 1 ton 1 cwt. 23 lb., 16 cwt. 4 lb., 13 cwt. 2 qrs. 22 lb., 12 cwt. 7 lb., 10 cwt., 9 cwt. 1 qr. 5 lb., 8 cwt. 3 qrs. 7 lb. Find the average weight.

15. If 40 articles are bought at 12*s.* 6*d.* each, and 60 more of the same kind at 15*s.* 6*d.* each, what is the whole cost and the average price of each article?

16. If the price of wheat in 1859 averaged £18 a load, in 1860, £19. 10*s.*, and in 1861, £20. 2*s.* 6*d.*, what is the average price per load for the three years?

17. The average weight of a crew of 8 men is 11 st. 6 lb. 6 oz. each. Three of them weigh 36 st. 4 lb.; what is the average weight of the others?

18. The average of twenty-five results is 18; that of the first twelve being 14, and that of the last twelve 17. Determine the thirteenth result.

19. The populations of five parishes are 618, 226, 182, 258, and 1715 respectively; what is the population of a sixth parish, if the average population of the six is 628⅓?

20. The average dividend paid by a railway company during 8 years is 3·8125 per cent. The dividends paid in the first 7 years were respectively 3½, 3¼, 3½, 4, 4½, 4¼, 4 per cent. What was the dividend in the 8th year?

21. On the occasion of the Thanksgiving for the Prince of Wales, in 1872, it was estimated that the route was 7 miles in length, that there were on an average 20 rows of people on each side, and that each person presented a front of 15 inches. On this estimate, what was the number of spectators?

22. The populations of 3 towns in 1861 were respectively 22350, 43425 and 10630. In 1871 the two former had increased by 6 per cent. and 8 per cent. respectively, and the latter had decreased by 10 per cent.; find the average population in 1871.

23. The goal averages of a certain football club are 3·4 for, and 3·6 against. If they win the next match by 5 goals to 2, their goal averages for and against will be equal. How many matches have they played?

24. A batsman in his 17th innings makes a score of 85, and thereby increases his average by 3. What is his average after the 17th innings? He had never been 'not out.'

25. A man buys 48 horses at an average price of 625 f. each, and sells 14 at 500 f. each, 10 at 562 f. 50 c. each, 9 at 800 f. each, and 15 at 1125 f. each. If the expenses of the sale average 25 f. per horse, what does he gain per cent. on his original outlay?

26. A man whose bowling average is 12·4 takes 5 wickets for 26 runs, and thereby decreases his average by ·4. How many wickets had he taken before the last match?

27. A man invests a quarter of his capital in $2\frac{1}{4}\%$ stock at 90, and the remainder in $3\frac{1}{2}\%$ stock at 105. Find the average rate per cent. which he receives on his capital.

28. A train travels from A to B at the rate of 20 miles an hour, and from B to A at the rate of 30 miles an hour. What is the average rate for the whole distance?

XLIII. MIXTURES.

(a) To find the Value of a Mixture.

389. We may mix two or more qualities of wine at different prices so as to produce wine at any price between the greatest and least of those that are mixed. If, for example, we mix one bottle of wine at 2*s.* 6*d.* with one at 3*s.* 6*d.*, we can make up two bottles worth 3*s.* each; and again if we mix one bottle at 2*s.* 6*d.* with 99 at 3*s.* 6*d.*, we can make up 100 bottles worth a trifle less than 3*s.* 6*d.* each.

We will first consider how to determine the value of a mixture when the quantity and value of each ingredient are known.

Example. *If 30 tons of coal at 20s. 6d. per ton be mixed with $22\frac{1}{2}$ tons at 24s. per ton, find the price of the mixture.*

Cost of 30 tons at 20*s.* 6*d.* $= £1\frac{1}{40} \times 30 = £30.\ 15s.$,

cost of $22\frac{1}{2}$ tons at 24*s.* $= £1\frac{1}{5} \times 22\frac{1}{2} = £27$;

$\therefore$ cost of $52\frac{1}{2}$ tons of mixture $= £57\frac{3}{4}$;

$\therefore$ cost of 1 ton of mixture $= £\dfrac{57\frac{3}{4}}{52\frac{1}{2}} = £\frac{2}{2}\frac{3}{1}\frac{1}{0} = £\frac{1}{1}\frac{1}{0} = £1.\ 2s.$;

$\therefore$ price required $= £1.\ 2s.$ per ton.

EXAMPLES. 177.

1. A man bought 2 qrs. of corn at £1. 6*s.* per qr., and 7 bush. at £1. 12*s.* per qr.; he mixed them and sold the mixture at 4*s.* 2*d.* per bush.; how much did he gain?

2. A wine merchant mixes 50 litres of wine at 3 f. 75 c. per litre with 25 litres at 4 f. 20 c. per litre; what is the value of the mixture per litre?

3. A mixture is made of 1 gallon of ale at 5*d.* per gallon, 3 at 9*d.*, 4 at 1*s.*, and 12 at 4*d.*; how much per gallon is the mixture worth?

4. If 18 Kg. of coffee at 2 f. 70 c. per Kg. be mixed with 144 Kg. at 4 f. 5 c., what is the value per Kg. of the mixture?

5. A mixture of two kinds of wine, in the proportion of 4 gallons of one to 11 gallons of the other, is worth £1. 8*s.* per gallon. The first kind being worth £1. 15*s.* 4*d.* per gallon, what is the value of the second kind?

6. A mixture of coffee and chicory is made so that the chicory is $\frac{1}{9}$th of the whole. If the mixture be worth 2*s.* per lb., and the chicory 4*d.* per lb., what is the value per lb. of pure coffee?

7. A mixture is made of 27 metric tons of coal at 17*s.* 9*d.* per metric ton, 12 met. tons at 22*s.* 6*d.* per met. ton, and 11 met. tons at 19*s.* 4*d.*; find the value per metric ton of the mixture.

8. A grocer mixes 1 cwt. 3 qrs. of sugar at $5\frac{1}{2}d.$ per lb. with 2 cwt. 1 qr. 14 lb. at $2\frac{3}{4}d.$ per lb. At what price per lb. must he sell the mixture so as not to lose by the sale?

9. A grocer mixes 2 cwt. 15 lb. of sugar at $2\frac{3}{4}d.$ per lb. with 4 cwt. 30 lb. at $1\frac{1}{4}d.$; at what price per lb. must he sell the mixture so as not to lose by the sale?

10. An innkeeper buys 10 gallons of spirits at 12*s.* per gallon, 15 at 4*s.* 6*d.* and 18 at 5*s.* 9*d.*; how must he sell the mixture per gallon that he may gain £2. 5*s.* 10*d.* on his outlay?

11. A grocer mixes 72 lb. of tea at 1*s.* $5\frac{1}{4}d.$ per lb. with 90 lb. of tea at 1*s.* 3*d.* per lb.; at what price per lb. must he sell the mixture so as to gain $2\frac{1}{2}d.$ per lb.?

12. A wine merchant buys two sorts of spirit at 16*s.* 6*d.* and 10*s.* 6*d.* per gall. He mixes them in the proportion of 5 parts of the cheaper to 7 of the dearer. At what rate per gallon must he sell the mixture in order to make 25 per cent. profit?

13. A man mixes 20 gallons of ale, which cost him 16*d.* a gallon, with 65 gallons of beer which cost him 9*d.* a gallon; at what price must he sell the mixture in order to gain 20 per cent.?

14. If 80 gallons of brandy at £1. 5*s.* per gallon be mixed with 55 gallons at £1. 7*s.* 6*d.* per gallon, at what price per gallon must the mixture be sold to gain 15 per cent. on the outlay?

15. A wine merchant buys 6 dozen of wine at 72 f. per dozen, and 20 dozen more at 54 f. per dozen. What is his gain per cent., if he sells the mixture at 63 f. per dozen?

16. A publican mixes 4 gallons of gin worth 15*s.* a gallon, with 4 gallons of water and with a gallon of base spirit worth 10*s.*; what will he gain per cent. on his outlay if he sells the mixture at 2*s.* 10*d.* per bottle, six bottles making one gallon?

17. A grocer buys coffee at £8. 10*s.* per cwt. and chicory at £2. 10*s.* per cwt., and he mixes them in the proportion of 5 parts chicory to 7 parts coffee; at what rate per lb. must he sell the mixture so as to gain $16\frac{2}{3}$ per cent. on his outlay?

18. I purchase $2\frac{1}{2}$ cwt. of tea at £6. 10*s.* 10*d.* per cwt. and $1\frac{1}{2}$ cwt. at £9. 1*s.* per cwt. I mix them, and sell the mixture at 1*s.* 9*d.* per lb. What is the profit per lb., the profit per cent. on the outlay and the profit per cent. on the selling price?

19. One liquid contains $22\frac{1}{2}\%$ of water; another contains 27 % of water. A glass is filled with 5 parts of the first liquid and 7 parts of the second. How much per cent. of it is water?

20. One alloy contains 75 % of gold; another contains $66\frac{2}{3}\%$ of gold. A new alloy is made of 2 parts of the first to 3 parts of the second. How much per cent. of the mixture is gold?

(b) To Form a Mixture of a Given Value.

390. We will next consider how to mix two or more ingredients together in order to produce a mixture of given value.

Example i. *Two wines, worth* 32*s. and* 24*s. per gallon respectively, are mixed together. If the mixture be worth* 27*s.* 6*d. per gallon, in what ratio are the wines mixed?*

The price of 1st is 9 sixpences per gall. above that of the mixture,

and .... 2nd is 7 ... . below ..... .

Hence we must take such quantities of each that the loss on the first wine is exactly balanced by the profit on the second. To do this we mix the wines in the proportion of 7 gallons of the first to 9 of the second.

We may also proceed thus:

Let x = the number of gallons of first wine,

and y = ... second wine.

Then $32x + 24y = 27\frac{1}{2}(x + y)$,

since the former expression is the number of shillings in the value of x gallons at 32*s.* and y gallons at 24*s.*, and the latter expression is the number of shillings in the value of $(x+y)$ gallons at 27*s.* 6*d.* Hence

$$32x+24y=27\tfrac{1}{2}x+27\tfrac{1}{2}y;$$
$$\therefore\ 4\tfrac{1}{2}x=3\tfrac{1}{2}y;$$
$$\therefore\ 9x=7y;$$
$$\therefore\ x:y=7:9.$$

Thus we mix 7 gallons of the first with 9 of the second.

Example ii. *Mix wine at* 25*s.*, 18*s.*, 16*s.* 6*d.* *and* 15*s.* *a gallon respectively, so that the mixture may be worth* 17*s.* 6*d.* *a gallon.*

The prices of the wines are respectively

15 sixpences and 1 sixpence *above*,
and 2 sixpences and 5 sixpences *below*

that of the mixture; and we have to mix the wines so that the excesses may make up for the deficiencies. One way to do this is to take

1 gallon of the 1st with 3 of the 4th,
and 1 ..... 3rd ..... 2 ..... 2nd;

for the excess in the value of 1 gallon of the 1st will balance the deficiency in that of 3 gallons of the 4th; and the excess in the value of 1 gallon of the 3rd will balance the deficiency in that of 2 gallons of the 2nd. We may therefore mix the wines in the ratio of 1, 2, 1 and 3.

We might also take 2 gallons of the 1st with 15 of the 3rd,
and 5 ..... 2nd ..... 1 ..... 4th,

in which case the ratio would be 2, 5, 15 and 1.

Note.—Examples of this kind generally have several different solutions.

EXAMPLES. 178.

1. How must a grocer mix black tea at 1*s.* 2*d.* a lb. with green tea at 1*s.* 10*d.* a lb., so as to make a mixture at 1*s.* 4*d.* a lb.?

2. Wine at 18*s.* a gall. is mixed with wine at 25*s.* a gall., and the mixture is sold for 22*s.* 6*d.* a gall.; how were they mixed?

3. A grocer buys teas at 4 f. 80 c. and 6 f. 60 c. per Kg. How must he mix them so as to gain 20% by selling at 7 f. 20 c. per Kg.?

4. A grocer mixes coffee at 2*s.* with chicory at 6*d.* per lb. How are they mixed, if by selling at 1*s.* 9*d.* per lb. he makes 5% profit?

5. How must a grocer mix teas at 1*s.* 6*d.*, 2*s.* 9*d.* and 3*s.* 9*d.* per lb. in order to sell the mixture at 3*s.* per lb.?

6. Mix ale at 2*s.* 8*d.*, 2*s.* and 1*s.* 10*d.* per gallon with water, so that the mixture may be sold at 1*s.* 6*d.* per gallon.

7. How must a dealer mix spirits at 4*s.* $7\frac{1}{2}$*d.*, 5*s.* $3\frac{1}{2}$*d.* and 3*s.* $4\frac{1}{2}$*d.* per quart, so as to sell the mixture at 4*s.* 10*d.* per quart?

8. Three kinds of tea at 4 f. 80 c. per Kg., 6 f. 40 c. per Kg. and 8 f. 40 c. per Kg. respectively are mixed together; if the price of the mixture is 7 f. 20 c. per Kg., how were they mixed?

9. Currants at 2*d.* per lb. are mixed with currants at 3*d.*, $4\frac{1}{2}d.$ and $5\frac{1}{2}d.$ per lb. respectively, and the mixture is sold at $3\frac{3}{4}d.$ per lb.; in what proportion were they mixed?

10. In what proportion must wines of 15*s.*, 20*s.*, 26*s.* and 30*s.* a gallon respectively be mixed, so that the mixture may be sold at 24*s.* a gallon at $14\frac{2}{7}$ per cent. profit?

11. How much water must be added to a cask containing 70 gallons of spirits, worth 13*s.* 4*d.* a gallon, in order to reduce the price to 11*s.* 8*d.* a gallon?

12. A dealer, buying spirit at 5 f. per litre, dilutes it with water so that, by selling it at 4 f. 50 c. per litre, he gains 20 per cent. How much water is there in a litre of the compound?

13. A dairyman buys milk at $2\frac{1}{2}d.$ per quart, dilutes it with water, and sells the mixture at 3*d.* per quart. His profits are 60 per cent. upon his outlay. How much water is there in each quart?

14. A person buys equal quantities of apples at 2 a penny and 3 a penny respectively, and mixes them. How many may he then sell for 5*s.*, so as neither to gain nor to lose?

15. A grant is made at the rate of 5*s.* per head for all the children in elementary schools. If the grant be distributed at the rate of 5*s.* 9*d.* per child in town, and 3*s.* 6*d.* per child in country schools, how many per cent. of the children are in town and country respectively?

16. One vessel contains 60% of wine and the rest water; another contains 85% of wine and the rest water. In what proportion must the contents be mixed, if the mixture contains 30% of water?

XLIV. PROBLEMS.

(a). Work, Pipes, etc.

391. If we know the time in which two or more persons can separately do a certain piece of work, we can easily find the time that they will take to do the same when working together, supposing that each person still works at the same rate as when alone. Problems which involve the rate at which water or any liquid flows along a pipe are of a similar character. We will work out an example of each.

Example i. A *working alone can do a piece of work in* $2\frac{1}{2}$ *days,* B *in* $3\frac{3}{4}$ *days, and* C *in* 5 *days; in what time could they do it if they all worked together?*

A can do the whole in $2\frac{1}{2}$ days, B in $3\frac{3}{4}$ days, and C in 5 days;

$\therefore$ A can do $\frac{2}{5}$ of the work in 1 day,
B ..... $\frac{4}{15}$ ..... 1 .....,
C ..... $\frac{1}{5}$ ..... 1 .....;

$\therefore$ A, B and C together can do $\frac{2}{5}+\frac{4}{15}+\frac{1}{5}$ of the work in 1 day,

$\therefore$ ..... $\frac{6+4+3}{15}$ ....,

$\therefore$ ..... $\frac{13}{15}$ .....,

$\therefore$ ..... the whole work in $\frac{15}{13}$ days;

$\therefore$ time required $= 1\frac{2}{13}$ days.

Example ii. *A vessel has* 3 *pipes connected with it,* 2 *to supply and* 1 *to draw off. The first alone can fill the vessel in* 4 *hours, and the second in* 3 *hours; the third can empty it in* $1\frac{1}{2}$ *hours. If all the pipes be opened when the cistern is full, how soon will the cistern be empty?*

1st pipe fills $\frac{1}{4}$ of cistern in 1 hour,
2nd ..... $\frac{1}{3}$ .....,
3rd pipe empties $\frac{2}{3}$ .....;

$\therefore$ when all the pipes are open

$\left(\frac{2}{3}-\frac{1}{4}-\frac{1}{3}\right)$ of cistern will be emptied in 1 hour,

i.e. $\frac{1}{12}$ ....;

$\therefore$ the whole will be emptied in 12 hours.

392. Generally, if x men could separately do a piece of work in a, b, c, d, ... days respectively, they would respectively do $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \frac{1}{d}, \ldots$ of the work in *one* day. Hence, *when working together*, they would do $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\ldots$ of the work in *one* day, and would need $1 \div \left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\ldots\right)$ days to do the whole.

EXAMPLES. 179.

1. If A can do in 20 days a piece of work which B can do in 25 days, in what time will they complete it together?

2. If A does in 8 days a piece of work which B can do in 10, in what time will they do it together?

3. If a cistern can be filled by one pipe in 3 hours, by a second in 5 hours, and by a third in 7 hours, in what time will it be filled by all three in action together?

4. A can do a piece of work in 6 days, B in 8 days, and C in 9 days; in how many days would all together finish it?

5. A can dig a trench in 3 days, B in 4, and C in 5 days. How long will it take them all together to dig the same length of trench, and what portion of the work will be done by each?

6. A acting alone can do a certain work in $2\frac{1}{2}$ hours, B in $3\frac{3}{4}$, C in 4, D in $7\frac{1}{2}$. In what time could it be done if they all worked together?

7. Two pipes together can fill a cistern in 8 min., and one of them alone in 24 min. How long would the other alone take?

8. A can do a piece of work in 20 days, B in 12 days; A and B work at it together for 6 days, and then C finishes it in 2 days; in how many days could C have done it alone?

9. A and B together can do a piece of work in 15 days. After they had worked together for 6 days, A finished it by himself in 30 days. In what time could each of them have done the whole?

10. A and B working together do a piece of work in 7 days and B alone can do it in 13 days. If B stops after 3 days, how long afterwards will A take to finish the work?

11. Pipes A and B can fill a cistern in 3 minutes and 5 minutes respectively, and C can empty it in $7\frac{1}{2}$ minutes. In what time will the cistern be filled when A, B and C are all turned on?

12. A cistern has 2 pipes, which can fill it in 2 hrs. and 3 hrs. respectively. A third pipe can empty it in 5 hours. If all be turned on when the cistern is empty, when will it be filled?

13. A cistern can be filled by 2 pipes, A and B, in 20 and 24 minutes respectively, and can be emptied by a pipe C in 30 minutes. What part of it will be filled in 15 minutes, if A, B and C are all turned on together?

14. A and B together can do a certain task in 30 days. After 11 days, however, B is called off, and A finished it by himself in 38 days more. In what time could each alone do the whole?

15. A can do a certain work in 15 hours; B takes 4 hours to do what A can do in 3; and B can do in 5 hours as much as C can do in 6. How long would the work have taken C alone?

16. Three taps, A, B and C, can fill a cistern, A by itself in 24 min., B in 10 min., and C in 27 min. They are all turned on at once, but after $4\frac{1}{2}$ min. B and C are turned off. How much longer will A by itself take then to fill the cistern?

17. A cistern has three pipes, A, B and C; A and B can fill it in 4 and 5 hours respectively, and C can empty it in 2 hours; if these pipes be opened in order at 1, 2 and 3 o'clock a.m., find when the cistern will be empty.

18. 2 men and 5 boys are employed on a piece of work and do $\frac{1}{2}$ of it in 6 days. After this, 1 more man and 1 more boy are put on, and $\frac{1}{3}$ more is done in 3 days; how many more men must then be put on that the whole may be finished in 1 day more?

19. For doing a certain work, A's ability is equal to the joint ability of B and C. If A and B together could do it in 9 hrs. 36 min., and C by himself in 48 hrs., in what time could B alone do it?

20. Two pumps discharging respectively 80 and 100 gallons per minute are employed to drain a well. It is observed that if the smaller pump alone is at work, the water sinks one inch per minute, and if both are employed from the first, the well is pumped dry in two hours. Find its depth, and find how long the larger pump alone would take.

(b). Races, Games of Skill, Trains.

393. The method of dealing with questions relating to races, or games of skill, will be understood if the following example is carefully studied.

Example. A *gives* B 10 *yards, and* C 15 *yards, in a race of* 100 *yards. How much should* B *give* C *in* 150 *yards?*

A can run 100 yards while B runs 90,
and A ..... 100 ..... C 85;
$\therefore$ B ..... 90 ..... C 85,
$\therefore$ B ..... 30 ..... C ..... $\frac{85}{3}$,
$\therefore$ B ..... 150 ..... C ..... $\frac{85}{3} \times 5$,
$\therefore$ B ..... 150 ..... C ..... $141\frac{2}{3}$;
$\therefore$ B can give C $8\frac{1}{3}$ yards in 150 yards.

394. In questions concerning trains the pupil must clearly distinguish between two cases: (i) that in which a train has to pass a fixed point, and (ii) that in which it has to pass an object of given length, such as a station or another train at rest. In order to pass a fixed point a train must move forward through a distance equal to its own length; in order to pass completely a fixed object of given length the train must travel a distance equal to the sum of the two lengths, its own length and that of the object.

Example. *A person, standing on a railway platform* 264 *yards long, noticed that a train, which passed him in* 8 *seconds, passed completely through the station in* 20 *seconds. Find the length of the train, and the rate of the train in miles per hour.*

Let x = the number of yds. in the length of the train. Then the train travels x yards in 8 seconds and $(264+x)$ yards in 20 seconds;

$$\therefore \frac{x}{8}=\frac{264+x}{20},$$

since each of these expressions is the number of yards which the train travels per second;

$$\therefore \quad 5x=528+2x;$$
$$\therefore \quad 3x=528;$$
$$\therefore \quad x=176;$$
$$\therefore \text{ the length of train} = \underline{176 \text{ yards}}.$$

Hence the train travels $(264+176)$ yards in 20 sec., *i.e.* $\frac{1}{4}$ mile in $\frac{1}{3}$ min., *i.e.* 45 miles per hour.

395. When trains are moving in opposite directions it is clear that their **rate of approach** before meeting, or their **rate of separation** afterwards, is equal to the *sum* of their rates of travelling; and when they are going in the same direction their **rate of passing**, or **of separation**, is equal to the *difference* of their rates of travelling. The following example will illustrate these principles.

Example. *A train travelling at the rate of* 60 *miles an hour overtakes another train,* 450 *ft. long, and completely passes it in* 42 *seconds. If the trains had been going in opposite directions they would have passed one another in* 6 *seconds. Find the length of the first train, and the rate of the second train in miles per hour.*

Rate of 1st train $= \frac{60\times 1760\times 3}{60\times 60}$ ft. per sec. $=88$ ft. per second.

Let x = no. of feet travelled in one second by the 2nd train,
and y = no. of feet in the length of the 1st train.

When going in opposite directions, they pass one another at the rate of $(88+x)$ feet per sec., and, when going in the same direction, at the rate of $(88-x)$ feet per sec. In order to pass one another completely one train must travel relatively to the other a distance equal to the sum of the two lengths, *i.e.* $(450+y)$ feet;

$$\left.\begin{aligned}\therefore\ 450+y&=(88+x)\times 6,\\ \text{and } 450+y&=(88-x)\times 42;\end{aligned}\right\}$$
$$\therefore\ 6(88+x)=42(88-x);$$
$$\therefore\ 88+x=7(88-x);$$
$$\therefore\ 88+x=616-7x;$$
$$\therefore\ 8x=528;$$
$$\therefore\ x=66; \quad \text{.....(i)}$$
$$\therefore\ 450+y=(88+66)\times 6=154\times 6=924;$$
$$\therefore\ y=474; \quad \text{.....(ii)}$$

$\therefore$ rate of 2nd train $= 66$ ft. per sec. $= \underline{45 \text{ miles per hour}}$,

and length of 1st train $= 474$ feet $= \underline{158 \text{ yards}}$.

EXAMPLES. 180.

1. If A can beat B by 20 yds., and B can beat C by 5 yds., in a race of 100 yds., by how much would A beat C in 200 yds.?

2. If A can beat B by 31 yds., and C by 18 yds., in a race of 200 yds., by how many yards could C beat B in 350 yds.?

3. At a game of skill A can give B 2 points out of 19, and A can give C 5 out of 47. Of B and C, which is the better player, and how many points can he give the other in 500?

4. At a game of rackets A can give B 3 points out of 15, and A can give C 7 points out of 15. How many points can B give C so as to make an even match?

5. At billiards A can give 15 points to B and 20 to C in a game of 50; how many can B give C in 70 in order to make an even match?

6. At a game of skill A can give B 10 points in 50, and B can give C 10 in 50. How many points in 50 can A give C to make an even match?

7. A, B and C run a mile. A wins, beating B by 11 yds. and C by 64 yds. By how much would B beat C in 2 miles?

8. A can beat B by 5 yds. in 100 yds., and B can beat C by 10 yds. in 200 yds. By how much can A beat C in 400 yds.?

9. A can give B 110 yds. in 2 miles, and A can give C 120 yds. in 5 miles. If B and C run 1 mile, which will win, and by how much?

10. In a game of skill A can give B 10 points in 100, and B can give C 15 in 75 and D 9 in 90; how many points could A and D give B and C in a game of 362 to make an even match?

11. A and B run a mile, and A wins by 80 yds. A and C run over the same course and A wins by 20 seconds. B and C run and B wins by 5 seconds. In what time can A run a mile?

12. A man, standing on the platform of a railway station, noticed that a train passed him in $4\frac{1}{2}$ sec. If the train was moving at the rate of 40 miles per hour, how long was the train?

13. Two trains start at the same time and move towards each other at the rate of 60 and 45 miles an hour respectively. If they meet in 12 min., how far apart were they when they started?

14. A train, travelling at the rate of 40 miles an hour, passed an observer in 5 sec. and passed through the station in 15 sec. Find the length of the train and the length of the station.

15. A train, whose length is 176 feet, is moving at the rate of 45 miles an hour and overtakes a second train going at the rate of 30 miles an hour. How long does the first train take to pass a passenger in the second train?

16. A man standing on the platform of a station, which is 132 yds. long, noticed that a train, which took 4 seconds to pass him, passed completely through the station in 9 seconds. What was the speed of the train in miles per hour?

17. Two regiments, which are respectively 120 yards and 150 yards in length, are marching in opposite directions at the respective rates of 4 miles and $3\frac{1}{2}$ miles an hour. How long after the meeting of their front ranks will their rear ranks be 500 yards apart?

18. Two trains, the faster of which travels at the rate of 45 miles an hour, pass each other in 12 sec. when moving in opposite directions, and in 1 min. when moving in the same direction. Find the rate of the second train in miles per hour. If one train be 270 yds. in length, what is the length of the other?

19. A train 88 yards long overtook a person walking at the rate of 4 miles an hour, and passed him completely in 10 seconds: it afterwards overtook another person and passed him completely in 9 seconds. At what rate was the second person walking?

20. A man rows 7 miles against the stream in 2 hours. If the stream be flowing at the rate of 3 miles an hour, how long would he take to row 13 miles in still water?

21. Rowing at a uniform rate a man can row 3 miles down stream in 18 minutes, and he can row back again in 24 minutes. Find the rate of the stream in miles per hour.

22. A boat moves down stream at the rate of a mile in 6 minutes and up stream at the rate of 6 miles in an hour. What is the rate of the stream in miles per hour?

23. A train reaches B 5 min. late, when it goes from A to B at the rate of $29\frac{1}{2}$ miles per hour: and it is 2 min. late when it travels at 30 miles an hour. What is the distance from A to B?

24. A train, approaching a station at the rate of 25 miles an hour, passes over two fog-signals placed half a mile apart. Find the interval between the times at which their reports are heard at the station, sound travelling at the rate of 1100 feet per second.

25. A takes a minute more to walk 13 miles than B does to walk 12 miles; but A takes a minute less than B to walk one mile. How long does each take to walk one mile?

26. Two trains start at the same time from A and B and approach each other at the rates of 30 and 50 miles an hour respectively. When they meet, one train has travelled $14\frac{1}{2}$ miles farther than the other. Find the distance between A and B.

27. A and B set out from the same place in the same direction and travel uniformly. After 9 days A is 72 miles in front of B: he then turns and travels back for a distance equal to that which B would travel in 9 days; he then turns again and overtakes B in $22\frac{1}{2}$ days from the start. What is the rate of each?

(c). Clocks and Circular Tracks.

396. The minute-hand of a clock travels all round the dial while the hour-hand travels $\frac{1}{12}$ of it. If therefore the minute-hand travel through any number of the 60 divisions of the dial, the hour-hand during the same time will travel through $\frac{1}{12}$ of that number; consequently the minute-hand will gain $\frac{11}{12}$ of that number upon the hour-hand. In 15 minutes, for example, the minute-hand travels through 15 divisions, and the hour-hand through $\frac{1}{12}$ of 15; hence, whatever may have been the original positions of the hands, the minute-hand during the $\frac{1}{4}$ of an hour gains $\frac{11}{12} \times 15$ divisions upon the other.

Suppose now that the time is 2 o'clock: the minute-hand is then 10 divisions behind the hour-hand. In $\frac{1}{4}$ of an hour the minute-hand will gain $\frac{11}{12} \times 15$, or $13\frac{3}{4}$ divisions, and will therefore at 2.15 be $3\frac{3}{4}$ divisions in front of the other.

The following examples should now present no difficulty.

Example i. *At what time between* 4 *o'clock and* 5 *o'clock will the hands of a watch be* (i) *at right angles*, (ii) *in the same straight line?*

At 4 o'clock the minute-hand is 20 divisions behind the other.

(i) When the hands are at *right angles* the minute-hand will be 15 divisions from the hour-hand; and this will be the case when the minute-hand has gained 5, and also when it has gained 35 divisions. But the minute-hand gains

55 divisions in 60 minutes,

and ∴ 5 ..... $\frac{60}{11}$ .....,

and 35 ..... $\frac{420}{11}$ .....

Hence the hands will be at right angles,

for the 1st time, after an interval of $\frac{60}{11}$, or $5\frac{5}{11}$, minutes,

and for the 2nd time, ... $\frac{420}{11}$, or $38\frac{2}{11}$, ... ,

i.e. at $5\frac{5}{11}$, and at $38\frac{2}{11}$, minutes past 4.

(ii) When in the same straight line as the hour-hand, the minute-hand may be exactly over the hour-hand, or exactly opposite to it. Hence the minute-hand has to gain either 20 or 50 divisions.

But it gains 55 divisions in 60 minutes,

$\therefore$ ..... 5 ..... $\frac{60}{11}$ ..... ,

$\therefore$ ..... 20 ..... $\frac{240}{11}$ ..... ,

and ..... 50 ..... $\frac{600}{11}$ ..... .

Hence the required times are $21\frac{9}{11}$ and $54\frac{6}{11}$ minutes past 4.

Example ii. *On looking at my watch between* 4 *and* 5 *o'clock and again between* 8 *and* 9 *o'clock, I found that the hands had exactly changed places. What was the time when I first looked at my watch?*

Let x minutes past 4 be the time when I first looked at my watch. The minute-hand was then x minute-divisions past 12. But the hour-hand, during those x minutes, will have travelled forward $\frac{x}{12}$ divisions from 4, and therefore, when I first looked at my watch, the hour-hand must have been $\left(\frac{x}{12}+20\right)$ divisions past 12.

Again, between 8 o'clock and 9 o'clock the minute-hand is where the hour-hand was between 4 o'clock and 5 o'clock, *i.e.* the minute-hand is $\left(\frac{x}{12}+20\right)$ divisions past 12; and during this time the hour-hand must have moved forward $\frac{1}{12}\left(\frac{x}{12}+20\right)$ divisions beyond 8. But the hour-hand is now where the minute-hand was before, *i.e.* x divisions past 12, or $(x-40)$ divisions past 8. Hence it follows that

$$x-40=\frac{1}{12}\left(\frac{x}{12}+20\right);$$

$$\therefore\ 144x-5760=x+240;$$

$$\therefore\ 143x=6000;$$

$$\therefore\ x=\frac{6000}{143}=41\frac{137}{143}.$$

Hence we see that the time when I first looked at my watch was $41\frac{137}{143}$ min. past 4, and when I looked again it was $23\frac{71}{143}$ min. past 8.

EXAMPLES. 181.

At what times between the hours stated below are the hands of a watch (i) coincident? (ii) opposite one another? (iii) at rt. angles? (iv) 10 divisions apart? (v) 25 divisions apart?

1. 1 o'clock and 2 o'clock. 2. 3 o'clock and 4 o'clock.

3. 4 ..... 5 ..... 4. 6 ..... 7 .....

5. 7 ..... 8 ..... 6. 10 ..... 11 .....

7. A watch, which is 10 minutes too fast at noon on Monday, gains 3 min. 12 sec. every 24 hours. What will be the time by it at 10.15 a.m. on the following Saturday morning?

8. A watch shows the true time at 6.0 a.m. on Sunday, and at noon on Tuesday has gained 54 minutes. Find what the true time will be when the watch shows 8 a.m. on Friday.

9. A watch, which gains one second in every minute, was set right at 6 a.m. What was the true time in the evening of the same day when the watch indicated 9.15 p.m.?

10. A clock, which gains uniformly, shows the time at noon to be 11.45 a.m. At 1.0 p.m. next day it is 5 min. fast. When did it indicate the true time?

11. A watch, which is set correctly at noon, indicates 9.10 p.m. on the same day when the true time is 10.0 p.m. What is the true time when the watch indicates 11.0 p.m. on the same day?

12. If a clock gains 2 min. 24 sec. in 24 hours of true time, and you wish to have it right at 6 p.m. on Thursday, how would you arrange the hands at 11 a.m. on the previous Tuesday?

13. Two clocks, one of which gains as much as the other loses,—viz. 75 sec. in 24 hours,—are set right at noon on April 1st. When will there be a difference of 48 minutes between them?

14. Two clocks, one of which gains 1 min. 15 sec. per day and the other loses 45 seconds per day, are put right at noon on Jan. 6th, 1904. When will they again indicate the same time, and what time will they then shew?

15. How many times in 24 hours are the hands of a clock at right angles?

16. On looking at my watch between 4 o'clock and 5 o'clock, and again between 7 o'clock and 8 o'clock, I found that the minute-hand and the hour-hand had changed places. What were the times when I looked at my watch?

17. Three watches are set right at noon on Tuesday, March 15th, 1904. One gains 10 minutes per week, another gains 15 minutes per week, and the third loses 5 minutes per week. If they all continue going at the same rate, find the date when they will all indicate again the same time.

18. At what times between 7 o'clock and 8 o'clock is the minute-hand midway between the hour-hand and XII?

19. A circular track is 440 yards in circumference, and A and B run round it in opposite directions. If their speeds are in the ratio of 6 to 5, and the race is for a mile, find the places where they pass each other. Where is B when A has finished?

20. Three men A, B, C, who can run 854, 714 and 504 yards respectively in the same time, start together to run round a circular track, 440 yards in circumference. They agree to stop when they are all together again for the first time. How far must each man run?

(d). Pasture with Growing Grass.

397. Examples of the following kind should be noticed, in which a pasture is used for feeding cattle, and the concurrent growth of the grass is taken into account.

Example i. *The grass on a certain pasture, growing uniformly, would be consumed by* 70 *cows in* 24 *days, and by* 30 *cows in* 60 *days. How many cows would consume it in* 96 *days?*

Original crop + 24 days' growth = 70 × 24 × daily consumption per cow,
and ..... + 60 ..... = 30 × 60 × ..... ,
and ..... + 96 ..... = 96 × reqd. no. × ..... ;

∴ by subtraction, 36 ..... = (1800 − 1680) × ..... ,
and 36 ..... = (96 × reqd. no. − 1800) × ..... ;

∴ 96 × reqd. no. − 1800 = 1800 − 1680 ;

∴ 96 × reqd. no. = 3600 − 1680 = 1920 ;

∴ reqd. no. of cows = $\frac{1\,9\,2\,0}{9\,6}$ = 20.

Example ii. *With the same data as in Example* i., *find for how long one day's growth of grass would feed a cow, and for how long the original crop would do so. Find also how long it would take* 20 *cows to exhaust the pasture.*

(i) Original crop + 60 days' growth = 30 × 60 × cow's daily consumption,
and ..... + 24 ..... , = 70 × 24 × ..... ;

∴ 36 ..... = (1800 − 1680) × ..... ;
∴ 36 ..... = 120 × ..... ;
∴ 1 ..... = $3\frac{1}{3}$ × ..... ;

∴ one day's growth would feed a cow for $3\frac{1}{3}$ days.

(ii) Again, 36 days' growth = 120 × cow's daily consumption ;
∴ 12 ..... = 40 × ..... ;
∴ 60 ..... = 200 × ..... ;
∴ original crop = (1800 − 200) × ..... ;
= 1600 × ..... ;

∴ the original crop alone would feed a cow for 1600 days.

(iii) Let x be the number of days.

Orig. crop + x × days' growth = 20 × x × cow's daily consumption ;
∴ $1600 + x \times 3\frac{1}{3} = 20 \times x$;
∴ $(20 - 3\frac{1}{3}) \times x = 1600$;
∴ $50 \times x = 4800$;
∴ $x = 96$;

∴ 20 cows would exhaust the pasture in 96 days.

EXAMPLES. 182.

1. In a pasture ground whereon grass grows uniformly, and which contains several days' growth of grass, 29 cows eat all the grass in 7 days. If 25 cows be kept on the same pasture ground, the grass would last for 9 days. How many cows may be allowed to graze so that all the grass may be eaten up in 6 days?

2. If the grass on a certain pasture grows uniformly, and would be consumed by 20 sheep in 96 days and by 30 sheep in 60 days, by how many sheep would it be consumed in 24 days?

3. If 29 sheep would eat up a field of uniformly growing grass in 7 weeks, and 25 sheep would do so in 9 weeks, how many sheep would exhaust it in 6 weeks?

4. The uniformly growing grass on a certain meadow would be consumed by 150 oxen in 15 days and by 120 oxen in 20 days. How many oxen are put in, if 30 are removed after 8 days and the whole of the grass is consumed in 4 days more?

5. If 17 horses would exhaust a field of uniformly growing grass in 30 days, and 19 horses would do so in 24 days, how many horses must be put in to eat up the whole in 8 days, if 4 of them are removed after 6 days?

6. If 19 sheep consume the uniformly growing grass on a field in 13 days, and 16 of them would consume the grass on the same field in 16 days; in how many days would 107 sheep exhaust the field?

7. If 40 horses would consume the grass of a field, which grows uniformly, in 12 days, and 25 horses would consume the grass of the same field in 20 days, find in how many days 30 horses would consume it.

8. If 48 horses would consume a field of uniformly growing grass in 8 days, and 64 horses in 5 days, in how many days would 40 horses consume it?

9. The grass on a meadow is of uniform quality and growth; 20 horses would exhaust it wholly in $25\frac{1}{2}$ days, and 21 horses would do so in 24 days; in how many days would 26 horses exhaust it?

10. In a certain pasture there is a crop of 525 stone of grass, which grows uniformly. Eleven oxen turned in would eat all the grass in 48 days, and 6 oxen in 98 days. What weight of grass would each ox eat per day?

11. The grass growing uniformly on a certain meadow would be consumed by 50 cows in 60 days and by 40 cows in 80 days.

How many cows must be turned in for 120 days, so that the quantity of grass remaining at the end of the time may be equal to $\frac{2}{3}$ of the quantity there would have been if the meadow had not been used at all?

12. A cistern, partly full, into which a steady stream of water is flowing, has a number of equal holes in the bottom, which can be opened and closed at will. If 10 holes were opened, the cistern would be emptied in 20 minutes; if 8 were opened, it would be emptied in 35 minutes. If 12 holes are opened, how soon will the cistern be emptied?

XLV. METRIC MEASURES.

398. The metre was originally intended to be $\frac{1}{10,000,000}$th of the distance from the equator to the pole, measured along a circle of longitude. In order to ascertain this length, the distance along the meridian from Dunkirk to Barcelona was measured by Delambre and Méchain in 1791-1798, and from their results the total distance from pole to equator was calculated.

A rod of platinum equal in length to $\frac{1}{10,000,000}$ of this total distance, as calculated by the mathematicians, was then constructed, and the length of the rod became the **standard metre**: but it has been discovered since that time that the calculated distance from equator to pole was not quite accurate, and that the length of the platinum rod is less than what it was intended to be by about $\frac{1}{5000}$th of itself. The length of the rod, however, has not been altered in consequence of this discovery, but the original definition of the metre has been abandoned. The metre, therefore, is not the natural unit which it was designed to be, and is simply an arbitrary length. It is now defined as the length of a certain platinum rod kept in the Palais des Archives at Paris

From this unit of length all the other measures of length, area, capacity and weight, employed in the Metric System, were derived. The **gram** was defined as **the weight of a cubic centimetre of pure water at 4° C.**; and the **litre** as **the volume of 1000 cubic centimetres, or 1 cubic decimetre**. These too are no longer natural units; the *gram* is simply the weight of a certain piece of metal kept at Paris, and the *litre* is the volume of a kilogram of pure water at 4° C.

399. The following equivalents are taken from an Order in Council, dated 19th day of May, 1898:

Measures of Length.

1 inch = 25·400 mm.	1 cm. = 0·3937 inch.
1 foot = 0·30480 metre.	1 metre = 1·0936143 yards.
1 yard = 0·914399 metre.	1 Km. = 0·62137 mile.
1 mile = 1·6093 Km.	1 metre = 39·370113 inches.

Approximately, **1 kilometre** = 5 furlongs; **1 metre** = $1\frac{1}{11}$ yards; and **20 metres** = 1 chain, or the length of a cricket pitch.

Measures of Area.

1 sq. inch = 6·4516 sq. cm.	1 sq. cm. = 0·15500 sq. inch.
1 sq. foot = 9·2903 sq. dm.	1 sq. m. = 10·7639 sq. feet.
1 sq. yard = 0·836126 sq. metre.	= 1·1960 sq. yards.
1 acre = 0·40468 hectare.	1 are = 119·60 sq. yards.
1 sq. mile = 259·00 hectares.	1 hectare = 2·4711 acres.

Approximately, **1 hectare** is a little less than $2\frac{1}{2}$ acres; an **are** is about $\frac{1}{40}$ of an acre, and a **centiare** is about $1\frac{1}{5}$ sq. yds.

Measures of Capacity.

1 cu. inch = 16·387 c.cm.	1 litre } = 61·024 cu. inches.
1 cu. foot = 0·028317 c.m.	1 c.dm. } = 1·75980 pints.
1 pint = 0·568 litre.	1 Dl. = 2·200 gallons.
1 gallon = 4·5459631 litres.	1 Hl. = 2·75 bushels.

Approximately, **1 litre** is a little more than $1\frac{3}{4}$ pints.

Measures of Weight.

1 gram = 15·432 grains.	1 grain = 0·0648 gram.
1 Kg. = 2·2046223 lb.	1 lb. = 0·45359243 Kg.
1 tonne = 0·9842 ton.	1 cwt. = 50·80 Kg.

Approximately, **1 gram** is about $15\frac{1}{2}$ grains, **1 kilogram** is about $2\frac{1}{5}$ lb., and **1 tonne** is rather less than 1 ton.

400. Both the yard and the metre being arbitrary and independent lengths, it follows that the one can be compared with the other in no other way than by actual measurement. The relations between the measures of length, which we have given above, are therefore subject to the limitations which necessarily exist as regards such measurements. It is found practically that results which depend upon measurement cannot be relied upon for more than 6 figures. For example, we know that one metre is equivalent to rather more than 39 inches. The excess over and above 39 inches can be expressed as the decimal of an inch; but, as there are 2 figures already in the integral part 39, we cannot rely upon the decimal to more that 4 places. If there had been

only 1 digit in the integral part we could have relied upon 5 places in the decimal; and if there had been 4 digits in the integral part, we could have relied upon only 2 places in the decimal.

401. When we convert a British measure to the equivalent metric measure, we shall usually have to multiply one number by another. In order to ascertain how many decimal places we must retain in the working it will therefore be necessary to ascertain beforehand, by a rough approximation, how many figures we may expect in the *integral* part of the result, or how many decimal places there will be before the *first significant figure*.

Example i. *If* 1 *inch* = 2·539954 *cm., find, correct to* 5 *significant figures, the number of metres in* 1 *mile.*

Taking $2\frac{1}{2}$ cm. as equivalent to 1 inch, we find, as a rough approximation, that

$$1 \text{ mile} = 1760 \times 36 \times 2\tfrac{1}{2} \text{ cm.} = 1760 \times 90 \text{ cm.} = 158400 \text{ cm.};$$

we see therefore that there will be 6 figures to the left of the decimal point. To obtain 5 figures correct, we retain 7 figures, in accordance with the usual rule; that is, we arrange the calculation so as to obtain 1 decimal place in the product.

The calculation is shewn in the margin; whence we see that

$$1 \text{ mile} = 63360 \times 2{\cdot}539954 \text{ cm.}$$
$$= 160930 \text{ cm.}$$
$$= \underline{1609{\cdot}3 \text{ metres.}}$$

2·5	39954 cm.
6336 0	
152397·2	
7619·9	
762·0	
152·4	
160930 cm.	

Example ii. *If* 1 *metre* = 39·3708 *inches, and* 1 *cubic foot of water weigh* 1000 *oz., find, to* 4 *significant figures, the number of pounds in* 1 *kilogram.*

Rough approx.

$$1 \text{ Kg.} = \frac{(3{\cdot}9\ldots)^3}{1728} \times \frac{1000}{16} \text{ lb.} = \frac{(3{\cdot}9\ldots)^3}{27{\cdot}648} \text{ lb.}$$
$$= \frac{(3{\cdot}9\ldots)^3}{30} \text{ lb.} = \frac{64}{30} \text{ lb.} = 2{\cdot}1\ldots \text{ lb.}$$

The integral part of the result will therefore consist of one figure. We shall consequently require **3** decimal places correct in the result, and we shall therefore retain **5** decimal places in the working. Moreover, $(3{\cdot}9\ldots)^3$ is divided by about 30, so that, if the multiplication be correct to 3 decimal places, the error in the final result will not be much more than $\frac{1}{30}$ of ·0005, *i.e.* the error will be in the *fifth* decimal place.

3·93708	
3	·93708
11·81124	
3·54337	
·11811	
2756	
31	
15·50059	
3	·93708
46·50177	
13·95053	
·46502	
·10850	
124	
61·027	

We have 1 kilogram = the weight of 1 litre of water,

and $1 \text{ litre} = 1 \text{ cu. dm.} = (3{\cdot}93708)^3 \text{ cu. in.};$

$$\therefore 1 \text{ Kg.} = \frac{(3{\cdot}93708)^3}{1728} \times \frac{1000}{16} \text{ lb.} = \frac{61027}{27648} \text{ lb.} = \underline{2{\cdot}207 \text{ lb.}}$$

	2·2073
27,6,4,8)	61027
	57310
	2014
	79

402. The ratio which the weight of any volume of a substance bears to the weight of an equal volume of water is called the **specific gravity** of the substance.

When the volume and the specific gravity of a body are known, its weight can at once be found. The weight of x cubic centimetres of water is x grams. If the specific gravity of a substance be s, the substance will be s times as heavy as water, and therefore the weight of x cubic centimetres of the substance will be $s \times x$ grams. Thus the number of grams in the weight of a body will be found by multiplying the specific gravity of the substance by the number of cubic centimetres in its volume.

EXAMPLES 183.

1. What is the weight of 27 hectolitres of water?

2. What is the weight of 20 cubic centimetres of mercury, whose specific gravity is 13·5?

3. How many ares of land are there in a strip measuring 1 kilometre by 7 metres?

4. How many square slabs of marble, each 150 sq. cm. in area, will be required to pave a court whose area is 50·7 sq. metres?

5. How many square metres will 240 sheets cover, if each sheet measures 303 millimetres by 195 millimetres?

6. A cistern is 4 metres long, 24 dm. wide and 80 cm. deep. Find its capacity in cubic metres and in litres.

7. Find the time required to travel 31 kilometres 150 metres at the rate of 1 min. 28 sec. per kilometre.

8. If a man walks 11 metres in 5 sec., how many kilometres can he walk in an hour at the same rate?

9. Find the weight in kilograms of 15 gallons of water.

10. Reduce 532 kilometres to miles, furlongs and chains.

11. How many revolutions will the wheels of a carriage make in travelling 41 kilometres, if the wheels are 125 cm. in diameter? [circumference = 3·1416 diam.] (Answer to nearest integer.)

12. If a square field contain 10·24 hectares, find the length of a side in metres.

13. A cubical cistern holds 1331 kilograms of water: what is the length of an inner edge?

14. If 3 hectares contain 35881 sq. yards, find, to 5 significant figures, the number of yards in a metre.

15. What is 1*s.* 6*d.* per gallon in francs per litre, assuming that 1 gallon of water weighs 10 lb., 1 Kg. = $2\frac{1}{5}$ lb. and £1 = 25 f.

16. Given that 1 metre = 39·3707 inches, prove that the difference between 5 miles and 8 kilometres is nearly 51 yards.

17. Assuming that a metre = 39·37 inches, find the number of sq. centimetres in a sq. foot, to the nearest sq. centimetre.

18. If a sq. metre = 1550·031 sq. inches, find, to the fifth decimal place, the number of sq. centimetres in a sq. inch.

19. A man pays 12000 francs for 5 ares of land, and sells it for 25 f. 20 c. per sq. metre. How much does he gain?

20. If a farmer lays 2 tons of lime on an acre of land, how many grams is it per sq. metre? [1 metre = $39\frac{3}{8}$ in., 1 Kg. = $2\frac{1}{5}$ lb.]

21. Determine, to three decimal places, the number of cubic metres in a box 2 yds. long, 1 yd. wide and $2\frac{1}{2}$ ft. deep; reckoning 1 cu. yd. = ·76453 cu. metres.

22. A box measures 3·75 metres by 3·50 metres by 50 cm. Find how many litres of olive oil it will hold, and the weight of the oil in kilograms; the oil being ·914 as heavy as water.

23. How many litres are there in a vat measuring 2 metres by 75 cm. by 50 cm.? Find also the weight in kilograms of the sulphuric acid, specific gravity 1·840, required to fill it.

24. Find the weight in kilograms of a bar of gold 10 cm. long, 30 mm. wide and 25 mm. thick; its specific gravity being 19·36

25. If the specific gravity of sea water be 1·026, and that of olive oil be ·915, find the weight of 1 Hl. of each in kilograms.

26. If the specific gravity of alcohol be ·8, how many kilograms of alcohol will be required to fill, one-third full, a tank measuring 1·50 metres by 3·20 metres by 80 centimetres?

27. If a cubic foot of oil weighs 55 lb., what will a cubic metre weigh in kilograms? [1 ft. = ·3048 metres, 1 Kg. = $2\frac{1}{5}$ lb.]

28. A rectangular sheet of tin of uniform thickness is 85 centimetres wide and 2·7 metres long, and weighs 536 grams. Find its thickness, if the specific gravity of tin be 7·3.

29. A plate of iron 137 cm. long, 643 mm. wide and 43·1 mm. thick weighs 277·54 Kg. Find its sp. gravity, to one decimal place.

30. If a person steps 80 cm. each time, how many steps will he take in walking round a square field containing 18 Ha. 625 ca.?

31. How many metric tons of air (sp. gravity = ·00129206) are there in a space, 70 metres by 50 metres by 35 metres?

32. If a metre = 39·3708 inches, express a hectare in acres, correct to three decimal places.

33. Supposing a linear yard to be $\frac{32}{35}$ of a metre, find, to two

places of decimals, the difference, in cubic inches, between the volume of a cubic yard and that of a stère.

34. How many miles will be travelled in 1 hr. 28 min. 21 sec. at the rate of 5 kilometres an hour? (Answer to 3 decimal places.)

35. Reduce 10 gal. 3 qts. 1 pt. 3 gills to litres: assuming 1 gal. = 277·27 cu. inches, and 1 metre = 39·37 inches.

36. Find the cost of paving a court 25·34 metres by 18·36 metres at 1 f. 8 c. per sq. yard. [1 sq. metre = 1·196 sq. yds.]

37. If a rectangular pathway, which measures 787·4 metres by 1·526 metres, is made at a cost of 1 f. 50 c. per sq. metre, find the length in English measure of a similar pathway 6 feet wide, which costs 1*s.* 1½*d.* per sq. yard, the total cost being the same. [Take 1 metre = 39·37 in. and £1 = 25 f., and neglect fractions of an inch.]

38. Frenchmen on the average consume yearly 1160 grams of coffee and 9 grams of tea, and Englishmen 473 grams of coffee and 1679 grams of tea; if tea be 1*s.* 6*d.* and coffee 1*s.* 8*d.* per lb., and a franc = 9½*d.*, find in francs the excess of the value of an Englishman's consumption over that of a Frenchman's. [1 kg. = 2⅕ lb.]

39. The specific gravity of a certain liquid is 1·63. Find, to the nearest integer, how many cubic centimetres of the liquid must be added to 75 c.cm. of water so that the specific gravity of the mixture may be 1·43.

40. Given that 1 inch = 2·54 cm. and that one gallon contains 277·274 cu. inches, find to 3 significant figures the equivalent of 1 pint in litres.

41. Given that 1 metre = 1·0936 yards, find to 4 significant figures the equivalent of 15 acres in hectares.

42. Given that 1 metre = 39·37079 inches, and that 1000 oz. is the weight of a cubic foot of water, find to 4 significant figures the equivalent of 7 Kg. in pounds.

43. If 1 Kg. = 2·2046 lb. and a cubic foot of water weighs 1000 oz., find to 6 significant figures the equivalent of 1 cubic foot in cubic centimetres.

XLVI. MONETARY SYSTEMS AND EXCHANGE.

403. In consequence of the intercourse and traffic which is carried on between one country and another, it is continually necessary to determine how much in the monetary system of one country is equivalent to a sum of money expressed in the monetary system of another.

For example, a person about to leave London for Paris may wish to change a certain sum in English money for its equivalent in francs and centimes; or a merchant in London with money in the form of pounds, shillings and pence may wish to pay for goods bought in Berlin, the value of which is stated in marks, etc.

In former times such a calculation was much more troublesome than it is now, owing to the large number of small states, with independent systems of coinage, into which Europe was then divided. But at the present time one uniform system is in use throughout what is now the German Empire, while another uniform system has been adopted in common by the States which together form the Latin Monetary Union.

404. Until 1872 some of the European States practised **Bi-metallism**, that is, they coined at their Mints *on demand* both gold and silver, the value of any quantity of gold being fixed at $15\frac{1}{2}$ times that of an equal weight of silver. The relative values of the two metals were consequently fixed within narrow limits, for no one would sell silver for less than could be obtained for it by sending it to the French Mint. Since 1872, however, no country has coined both gold and silver as *standard money*.

In England the standard is gold. Silver pieces are merely *tokens* of certain fractions of the standard gold coin, and are legal tender only up to 40*s*. In France also gold is the standard, but the legal tender of silver pieces is not limited as in England.

In India the standard is silver, but owing to the serious and continued decline in the gold price of silver, and the consequent decline in the *exchange* value of the Rupee, the Indian Government decided in June, 1893, that their Mints should be opened to the free coinage of gold at the rate of 15 rupees to £1, and that they should no longer be open for the free coinage of silver. Until 1893, anyone might take any weight of silver bullion to the Indian Mints, and demand in exchange for it an equivalent weight of current silver coin (less 2 per cent. for seigniorage, and 1 per mille for melting). By an act of Parliament of 1898, the British sovereign was made legal tender and equivalent to 15 rupees, a rupee being thus made equivalent to 1*s*. 4*d*.

405. For purposes of exchange we can ascertain the comparative values of a gold and a silver coin only by estimating the value of the pure metal in each at the market price of silver for the day. There is no fixed exchange value for silver, or for a silver coin, any more than for an article of merchandise.

In England a florin is worth much more than a rupee, although it actually contains less silver; for the value of a florin in

England is its token value, $\frac{1}{10}$ of £1, while a rupee has only its exchange value, which is now about 1*s.* 4*d.* In 1861 the value of the rupee was as high as 2*s.* 2*d.* It has since been less than 1*s.* 1*d*

406. For coinage purposes pure gold and pure silver are seldom, if ever, used. It is customary to mix with them small quantities of more durable metals. There are two ways of estimating the quantity of pure gold in such a mixture. In England it is expressed by the number of **carats**, or *twenty-fourth* parts. Thus **18-carat** gold denotes a mixture containing **18** parts of pure gold to 6 parts of alloy. **Standard gold**, which is the mixture used for coining sovereigns, is 22-carat gold, *i.e.* it contains 22 parts of pure gold to 2 parts of alloy. This is sometimes expressed by saying that standard gold is $\frac{11}{12}$ **fine.**

On the continent of Europe the quantity of gold in a compound is generally expressed in **millièmes**, or *thousandth* parts. Thus French Standard gold is described as 900 *millièmes fine, i.e.* it contains 900 parts of pure gold to 100 parts of alloy.

The quality of silver is expressed by a fraction which denotes the ratio of the quantity of pure silver to the whole. Thus silver which contains 32 parts of pure silver to 8 parts of alloy is described as $\frac{32}{40}$ *fine.* The quality of silver may also be expressed with reference to some fixed standard. In this country the standard is called **sterling silver** and is $\frac{37}{40}$ fine.

A troy ounce of standard gold contains 440 grains fine gold and 40 grains alloy. It sells for £3. 17*s.* 10½*d.* A troy ounce of sterling silver contains 444 grains fine silver and 36 grains alloy; it used to sell for 5*s.*, but now sells for about 2*s.* 1*d.* only.

407. The principal monetary systems, most of which are now decimal, are the following.

Gold-standard Countries.

Name	Standard coin	Legal weight in grains	Exchange value	Subdivisions
United Kingdom	Sovereign	123·27	£1	£1 = 20 shillings
Latin Union	10-Franc piece	49·78	7*s.* 11¼*d.*	1 f. = 100 centimes
German Empire	10-Mark piece	61·4	9*s.* 9½*d.*	1 m. = 100 pfennige
Austria-Hungary	10-Krone piece	52·28	8*s.* 4*d.*	1 kr. = 100 heller
Holland and Java	10-Florin piece	103·71	16*s.* 6¼*d.*	1 fl. = 100 cents
Norway, Sweden and Denmark	10-Krone piece	69·14	11*s.* 0¼*d.*	1 kr. = 100 öre
United States	10-Dollar (Eagle)	258·0	41*s.* 1¼*d.*	1 dol. = 100 cents
Turkey	Pound	111·36	18*s.* 0⅜*d.*	£T.1 = 100 piastres
Japan	20-Yen piece	257·21	40*s.* 11⅜*d.*	1 yen = 100 sen

Silver-standard Countries.

Name	Standard coin	Legal weight in grains	Exchange value	Subdivisions
India*	Rupee	165·0	Exchange value in gold depends upon market price of silver	16 annas
Russia	Rouble	277·7221		100 copeks
Mexico	Dollar (silver)	277·0586		100 cents
China	Tael (a weight)	516·4058		1000 cash

The states which form the Latin Monetary Union are France, Italy, Switzerland, Belgium and Greece, and their gold coins are alike in quality and weight. The same system has been adopted in part by Spain, Roumania, Servia, Bulgaria and Finland. The *francs* and *centimes* of France are called *lire* and *centesimi* in Italy, *dinars* and *paras* in Servia, *pesetas* and *centimos* in Spain, *drachmai* and *lepta* in Greece, *leys* and *banis* in Roumania, *leva* and *stotinkis* in Bulgaria, *markka* and *penni* in Finland.

408. In Art. 369 we explained that a share in a Joint Stock Company has two values :—(i) its *nominal* value, or the amount of capital it represents, (ii) its *market* value, or the amount of money it will sell for, which may vary from day to day.

A sum of money expressed in £. *s. d.* has, similarly, two values with reference to francs and centimes ; (i) its *nominal* or *intrinsic* value, which depends only upon the quantity of gold which it represents, (ii) its *market* value, which may differ slightly from the intrinsic value, and may vary from day to day. The market value depends upon the state of trade, as will be explained.

In determining in the monetary system of one country the equivalent of a given amount expressed according to that of another, we determine what is called the **rate of exchange**.

409. When English money is exchanged for French money, the *nominal* rate is 25 francs (or more accurately 25·22½ francs) for £1 ; and, in the case of American money, one dollar for 4*s.* 2*d.* These rates are called the **par of exchange**. The use of the term *par* to express equality between the nominal and the actual rate is similar to the use of the same word to express that the nominal value of a share, or of stock, is the same as its market value.

410. Business transactions between one country and another are commonly carried on with what are called **Foreign Bills of Exchange**, or briefly **Foreign Bills**, by means of which large accounts may be settled without sending from one country to another any large quantities of gold and silver.

Suppose, for example, that A in London sends goods to B in Paris, and that C in Paris sends goods of exactly the same value to D in London. We have here a case in which D in London

* By Act of Parliament, £1 exchanges for 15 rupees.

owes C in Paris exactly as much as B in Paris owes A in London, and the transaction might be settled by D paying A, and B paying C.

Practically, the matter will often be arranged thus:—C will **draw a bill** on D, that is, an order for the payment of a specified sum of money after a certain interval. D will then **accept** this bill, that is, he will write his name across it with the word *accepted*. This signifies his agreement to pay the specified amount at the time stated. B in Paris now buys the *bill* which C has drawn on D, who is in London, and sends it by post to A, who is also in London. A then gets the money from D at the stated time and the matter ends, without any gold or silver passing between the two places.

411. The case we have imagined is a simple one, but it is sufficient to illustrate the way in which such transactions are settled. A merchant avoids remitting gold and silver, for that is a costly process. Instead of doing so, he buys *bills of exchange* and sends them to his creditor by post. In many cases however there may be a balance which cannot be paid except in gold. The medium between a merchant who wants to buy a bill and a merchant who wants to sell one is generally a bill-broker, to whom the one sells bills and from whom the other buys them.

When Paris owes London more than London owes Paris, *bills on London* are sought after in Paris, and a merchant there is willing therefore to pay more for them than their *nominal* value. In Paris bills on London are then at a *premium*, and in London at the same time *bills on Paris* are at a *discount*. The premium is of course never more than the cost of transmitting gold; either as **specie**, in the form of coin, or as **bullion**, in the uncoined form.

When bills are neither at a premium nor at a discount they are said to be *at par*. In this case pounds and francs exchange at their intrinsic values in gold, *i.e.* £1 for $25{\cdot}22\frac{1}{2}$ francs In Paris, when bills on London are at a premium, more than $25{\cdot}22\frac{1}{2}$ f. are given as equivalent to £1; when at a discount, less. The rate of exchange on any day is called the **course of exchange**.

412. We have considered exchange with reference only to Paris and London, *i.e.* to France and England, but all that we have said applies also to any other pair of countries.

It must be noticed too that the *course* of exchange between any two countries depends upon the state of *direct* trade between them. It might therefore sometimes be advantageous to the merchant in London, instead of buying bills on Paris, to buy bills

on Antwerp, and to instruct his agent at Antwerp to buy bills there on Paris. Or again he might remit to Paris by way of New York, or by way of Antwerp and New York, or even by a more circuitous route. The determination of the course of exchange when one or more places intervene is called the **arbitration of exchange.**

413. We will now work out a few examples to illustrate what we have said. If any monetary systems occur which have not been described above, sufficient information about them will be found in the questions themselves.

Example i. *Exchange* 7528 *rupees for English money at* 1*s.* 4*d. per rupee.*

7528 rupees $= 16 \times 7528d. = 120448d. = 10037s.\ 4d. =$ £501. 17*s.* 4*d.*

Example ii. *A person in London wishes to remit* 27432 *pesetas to Madrid when the exchanges are* 27 *pesetas* = 15*s.*, £1 = 25·4 *francs*, 19 *francs* = 27 *pesetas. Is it more advantageous to remit directly to Madrid or circuitously through Paris?*

Direct: $27432 \text{ pesetas} = £27432 \times \frac{3}{4} \times \frac{1}{27}$
$= £254 \times 3$
$= £762.$

Through Paris: $27432 \text{ pesetas} = 27432 \times \frac{19}{27} \text{ francs}$
$= £27432 \times \frac{19}{27} \times \frac{1}{25\cdot4}$
$= £760;$

$\therefore$ by remitting through Paris there is a saving of £2.

Example iii. *Exchange* 600 *American dollars for English money, when English money is at a premium of* 20 *per cent.*

English money being at 20 per cent. premium means that it is worth 20 per cent. more American money than it would be if it were at par: thus

at par 4*s.* 2*d.* = 1 dollar,

$\therefore$ at 20 per cent. prem., 4*s.* 2*d.* = 1 dollar + $\frac{1}{5}$ dollar = $\frac{6}{5}$ dollar;

$\therefore$ 1 dollar $= \frac{5}{6} \times 50d.$;

$\therefore$ 600 dollars $= 600 \times \frac{5}{6} \times 50d. = 25000d. = 2083s.\ 4d. =$ £104. 3*s.* 4*d.*

EXAMPLES. 184.

1. A German 20-mark piece is worth £·979; find, to the nearest farthing, the value in English money of 3725·39 marks.

2. If 25·4 francs be equivalent to £1 sterling, find the value in English money of 511·175 francs.

3. Convert £246. 13*s.* 10*d.* to francs, the course of exchange being 25 f. 18 c. per £.

4. Exchange 12687 f. 50 c. into English money, the rate of exchange being 25 f. $32\frac{1}{2}$ c. per £ sterling.

5. Find, to the nearest franc, the value of 10,000 rupees when 1 rupee = 1*s.* 4*d.* and £1 = 25·24 f.

6. If 24 rupees be worth 32*s.*, and 25 francs be worth 20*s.*, change 1000 francs into rupees.

7. What would £1 be worth at Paris if a bill for 12 fl. 15 c. sells for £1 at Amsterdam, and 9 fl. 50 c. are worth 20 f. at Paris?

8. Find, to the nearest centime, the difference in francs between 57·576 Napoleons and £47. 19*s.* $7\frac{1}{4}$*d.* [£1 = 25·2 f. and 1 Napoleon = 20 f.]

9. What is the rate of exchange between London and Vienna, through Paris, when £1 = 25·23 f. and 50 kronen = 47·45 f. (Answer in kronen per £1 to two decimal places.)

10. What is the rate of exchange between London and Berlin, through Vienna, when £1 = 24 kronen, and 118 kronen = 105 m.?

11. If £1 in France exchanges for 25·2 francs, and if 20 francs in England exchange for 15*s.* 9*d.*, how much do you lose in £100 by the two exchanges?

12. A tourist just before leaving London has £23. 12*s.* 6*d.*, and on his arrival at Paris he exchanges what remains for 23 Nap. 12 f. 50 c.; find what he has already spent, the rate of exchange being 25 f. 20 c. per £ sterling.

13. In Paris £1 = 25·24 f., and in London 20 f. = 15*s.* 8*d.* A person changes £75 in London into French money, spends 1000 f., and changes what he has left into English money in Paris. How much English money does he get?

14. A person on leaving England exchanged £12 for French money at the rate of 25 francs for a sovereign, and on arriving at Munich received 235 m. 55 pf. for the French money; what was his loss (i) in English money, (ii) in French money, supposing a mark to be worth $11\frac{3}{4}$*d.*?

15. If 5 silver roubles be worth 15*s.* $7\frac{1}{2}$*d.*, and £4 be worth 5 Napoleons, and 1 Napoleon be worth 4 dollars 25 cents, how many dollars are worth 45·056 silver roubles?

16. If £1 is worth 25·15 f., and 322 francs are worth 266 marks, and 400 piastres are worth 73·8 marks, find to the nearest penny the value in English money of 756 piastres.

17. A person in London owes another in St. Petersburg 460 roubles, which must be remitted through Paris. He pays the requisite sum to his broker when the exchange between London and Paris is 23 francs for £1, and between Paris and St. Peters-

burg 2 francs for one rouble. The remittance is delayed until the rates of exchange are 24 francs for £1, and 3 francs for 2 roubles. What does the broker gain or lose by the delay?

18. A person in London owes another at St. Petersburg 920 roubles, which must be sent him through Paris. He pays the needful sum to his broker at a time when the exchange between London and Paris is 25 f. 15 c. for £1, and between Paris and St. Petersburg 1·2 f. for one rouble. The broker delays remitting until the rates are 25 f. 35 c. for £1, and 1·15 f. for one rouble. What does the broker gain or lose by the delay?

19. The Guernsey pound contains 18 ounces avoirdupois, and the Guernsey shilling contains 13 English pence. If a Guernsey pound of butter costs 1*s.* 6*d.*, Guernsey money, what will be the price in English money of $2\frac{1}{2}$ lb. avoirdupois?

20. If £1 = 25 f. $22\frac{1}{2}$ c. at par, how much are 570 f. worth in English money, when the latter is at a premium of 5 per cent.?

21. If French money is at a discount of 10 per cent., find the value of £67. 15*s.* 6*d* in francs. [£1 = 25 f. $22\frac{1}{2}$ c. at par.]

22. If United States money is at a premium of 20 per cent, how much are 2000 marks worth in dollars, if 1 dollar = 4·2 m.?

23. Exchange 5220 dollars for English money when it is at a premium of $7\frac{1}{2}$ per cent.

24. Exchange 12000 francs for English money when it is at a discount of 5 per cent. [At par, £1 = 25·$22\frac{1}{2}$ f.]

25. If 25·50 francs are obtained in Paris for £1 sterling, is English money at a premium or at a discount in Paris, and how much per cent.? [At par, £1 = 25·$22\frac{1}{2}$ f.]

26. A man's income is £1723. 6*s.* 8*d.* He exchanges it in Paris from English money to French. When French money is at par, 1 f. = 9·4*d.*, but at the time of his exchange it is at a discount, being worth 2·2 per cent. less than when it is at par. How much did he get?

27. A merchant in New York wishes to remit 5110 dollars to London, a dollar being equal to 4*s.* 2*d.* English; for what sum in English money must he draw his bill when bills *on* London are at a premium of $9\frac{1}{2}$ per cent.?

28. French gold is 900 millièmes fine and 10 Napoleons weigh $64\frac{1}{2}$ grams. English gold is $\frac{11}{12}$ fine and 480 oz. are coined into 1869 sovereigns. Find the par of exchange in francs per £.

29. If 6 oz. of 18-carat gold be mixed with 5 oz. of 9-carat gold, what is the fineness of the mixture, to the nearest millième?

30. What profit per cent. would be made by the Mint in buying 12 oz. of silver at 2*s.* 5*d.* per oz., and coining it into 66 shillings, neglecting the cost of making the coins?

31 The United States gold coin of 10 dollars is $\frac{9}{10}$ fine, and weighs 258 grains. If 480 oz. of 22-carat gold be coined into 1869 sovereigns, find the value of a sovereign in dollars and cents.

32. What is the value of a bar of gold weighing 875·56 ounces, if the gold is $5\frac{1}{4}$ carats worse than standard gold, and standard gold is worth £3. 17*s.* $10\frac{1}{2}$*d.* per ounce?

XLVII. EQUATION OF PAYMENTS.

414. When a number of bills for various amounts fall due at various times, it is convenient to be able to determine a date on which the aggregate amount of the bills may be settled in one payment, with equal regard to the interests of payor and payee.

The date on which this may be done is called the **Equated Time**, and the period between the initial date and the equated time is called the **Equated Period**. The problem itself is called the **Equation of Payments**, and involves a principle similar to that already explained in the section on **Partnership**.

Suppose, for example, that on one and the same day a bill is drawn on a merchant for £350 payable in 3 months, another for £600 payable in 4 months, and a third for £230 payable in 6 months. At what time may the aggregate amount of £1180 be paid in one sum? It will be seen that the periods of the bills allow the use of

(i) £350 for 3 mos., which is equivalent to that of £1050 for 1 mo.,
(ii) £600 for 4 mos., „ „ £2400 for 1 mo.,
(iii) £230 for 6 mos., „ „ £1380 for 1 mo.
Altogether, this is equivalent to the use of £4830 for 1 mo.,
i.e. „ £1180 for $\frac{4830}{1180}$ mo.,
i.e. „ £1180 for $4\frac{11}{118}$ mo.

Hence the *equated period* is $4\frac{11}{118}$ months, and the *equated time* is $4\frac{11}{118}$ months after the date on which the bills are drawn.

Example. *Find the equated period of four bills drawn on the same day, viz., £250 due in 2 months, £123 due in 3 months, £432 due in 4 months, and £56 due in 6 months.*

Number of months in equated period $= \frac{250\times2+123\times3+432\times4+56\times6}{250+123+432+56}$
$= \frac{2953}{861}$
$= 3\frac{370}{123}$.

The *equated time* can be ascertained by reducing $\frac{370}{123}$ of a month to days, and reckoning onwards from the date on which the bills were drawn.

415. The principle is the same if the bills are drawn on different dates, and we will now consider a practical case in which a merchant supplies goods to a customer from time to time, and from time to time also receives cash to be placed to his customer's credit in part payment of the account. The matter to be determined is the date on which the balance should be paid on closing the account.

Example. *A merchant supplies goods, and receives cash, as stated below. Determine the date on which the balance should be paid to close the account.*

1904	Dr.	£	*s.*	*d.*	1904	Cr.	£	*s.*	*d.*
15 Mar.	To goods, at 2 mos.	253	4	2	22 Mar.	By Cash	110	10	0
18 Mar.	,, 3 ,,	326	15	0	16 May	,,	450	0	0
19 June	,, 4 ,,	572	18	5	5 Sept.	,,	394	17	6

We might reckon dates from 15th March, the date of the first bill, or from the 1st day of that month. We will adopt the latter date.

The balance of the account is £197. 10*s.* 1*d.* To determine when this should be paid, we multiply each amount by the number of days between 1st March and the day on which the bill is due or the cash is paid, and we find the sum of these products on each side of the account. We then subtract the smaller sum from the larger, and divide the remainder by the balance £197. 10*s.* 1*d.* The quotient will be the number of days between 1st March and the date on which this balance should be paid. We will express each amount as a decimal of a £ to 3 places.

The number of days from 1 March, 1904, to 15 May, 18 June, 19 October, 22 March, 16 May, 5 September are respectively 75, 109, 232, 21, 76, 188.

Dr.		Cr.	
£253·208 × 75	= £ 18990·600	£110·500 × 21	= £ 2320·500
£326·750 × 109	= £ 35615·750	£450 000 × 76	= £34200·000
£572·921 × 232	= £132917·672	£394·875 × 188	= £74236·500
£1152·879	£187524·022	£955·375	£110757·000
£ 955·375	£110757·000		
197·504)	76767·022 (388·6		
	17516		
	1716		
	135		

The equated period is therefore 388·6 days, and the balance, £197. 10*s.* 1*d.*, would be due at this interval after 1st March, 1904. It would therefore be due on 25th March, 1905. Discount would be allowed if the amount were paid before this date, and interest would be added if it were not paid till after.

NOTE.—The balance of the above account is in favour of the merchant. If it had been in favour of the customer, *i.e.* if the customer's payments had exceeded the amounts due, the equated period would be reckoned *backwards* from 1st March; and the merchant would have to repay him the amount of the balance, together with interest on it for the interval between the equated time and the date on which the repayment is made.

EXAMPLES. 185.

1. £300 is due in 2 months, £250 in 3 months, and £200 in 4 months; find the equated time of payment.

2. Find the equated time of payment of £700 due in 4 months, £650 due in 5 months, and £150 due in 8 months.

3. Find the equated time of payment of £5000, of which one half is due in 3 months, one quarter in 4 months, and the remainder in 5 months.

4. A man owes £259. 10*s.* 6*d.*, whereof £100 is to be paid in 1 month and the balance in 4 mos.; find the equated time.

5. A man owes £730 to be paid in 8 months; but he pays £300 at the end of 4 months, and £260 at the end of 6 months. When will the remainder be due?

6. Find the equated time of payment of £647. 12*s.* 6*d.* due in 3 mos., £325. 9*s.* $4\frac{1}{2}d.$ due in 4 mos. and £500 due in 6 mos.

Determine the dates on which the cash balances of the following accounts should be paid.

7.

1904	Dr.	£	*s.*	*d.*	1904	Cr.	£	*s.*	*d.*
5 June	To goods, 3 mos.	237	0	0	20 June	By Cash	150	0	0
18 July	,, 2 ,,	1365	0	0	25 July	,,	1000	0	0
10 Aug.	,, 5 ,,	128	0	0	5 Nov.	,,	200	0	0

8.

1903	Dr.	£	*s.*	*d.*	1903	Cr.	£	*s.*	*d.*
15 Feb.	To goods, 4 mos.	1105	10	4	21 Mar.	By Cash	100	0	0
23 May	,, 6 ,,	2318	15	5	16 June	,,	1007	15	6
14 Aug.	,, 3 ,,	527	18	2	7 July	,,	323	13	4

9.

1901	Dr.	£	*s*	*d.*	1901	Cr.	£	*s.*	*d.*
18 Jan.	To goods, 3 mos.	375	2	5	25 Jan.	By Cash	350	0	0
10 Feb.	,, 4 ,,	503	16	9	19 Mar.	,,	259	10	0
7 Apr.	,, 6 ,,	264	0	0	4 May	,,	320	0	0
14 May	,, 3 ,,	437	12	8	21 Aug.	,,	180	0	0

XLVIII. APPROXIMATION.

(a). Degree of Accuracy. (i). Addition.

416. When a quantity is given correct to 3 decimal places, the true value of the quantity may be greater or less than the given value by anything not greater than ·0005; *i.e.* there is a possible error, but this error is not greater than ·0005. If now a number of such quantities be added together, it will be clear that the errors also are added, and it is therefore necessary to examine to what extent the aggregate error may affect the result of the whole operation.

417. We will suppose that 6 quantities, each of which is correct to 3 decimal places, have to be added together. Each of the quantities may consequently be too large or too small by an amount not greater than ·0005. Some of them may be too small, and others may be too large, or they may *all* be too small, or *all* too large. If we take the extreme case, in which the 6 quantities are all too large, or all too small, by the greatest possible amount, the error in the sum will clearly be not greater than $\cdot0005 \times 6$, or ·003; *i.e.* the sum may be too large, or too small, by a quantity not greater than ·003. The figure in the 3rd decimal place may therefore be wrong, and that in the 2nd place, if not correct, will be wrong by only 1. Thus we can generally find the sum correct to *one* decimal place.

It will be seen therefore that, if n quantities be added together, and if the error in each be not greater than ·0005, the error in the sum will be not greater than $\cdot0005 \times n$; and generally, if x be the possible error in each of n quantities, the **limits of error** in the sum are $\pm nx$.

(ii). Subtraction.

418. If one quantity be subtracted from another, each being correct to 3 decimal places, the case which would produce the greatest error is that in which one quantity is too *large* by nearly ·0005, and the other too *small* by the same amount. In that case the possible error in the difference is not greater than $\cdot0005 + \cdot0005$, or ·001. The figure in the 3rd decimal place may therefore be wrong by 1, but the figure in the 2nd place will generally be correct, and the difference can therefore be found correct to *one* decimal place.

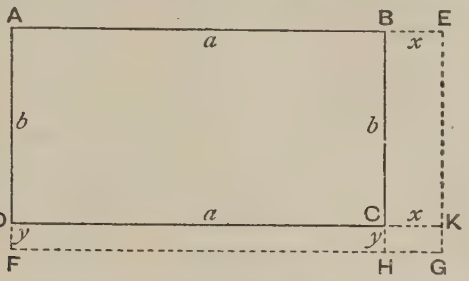
Generally, if the possible error in each of two quantities be not greater than x, the **limits of error** in their difference are $\pm 2x$.

(iii). Multiplication.

419. We will first consider the case graphically. We will suppose ABCD to be a rectangle whose sides AB and AD contain a units and b units respectively. The area ABCD will then contain $a \times b$ units of area.

If however the side AB is too *short* by x units, and the side AD is too short by y units, both of the numbers x and y being very small compared with a and b, the true area is shewn in the figure by AFGE, where $BE = x$ units and $DF = y$ units. Thus the area ABCD is too small by the sum of the figures BCKE, DFHC and CHGK.

But the figure CHGK is clearly very small compared with the other two figures BCKE and DFHC, and may be neglected. Thus



the error in the area is approximately equal to BCKE + DFHC, or $(a \times y + b \times x)$ units of area.

If the errors in the sides are equal, and each error is equal to x units, the error in the area will be $(a + b)x$ units.

Similarly, if the sides AB and AD are too *long* by x units and y units respectively, it may be shewn that the error in the area is again equal approximately to $(a \times y + b \times x)$ units. The student should draw a figure to suit this case and satisfy himself that this statement is true.

420. We will now consider the product $32{\cdot}41 \times 36{\cdot}37$, when both numbers are correct to 2 decimal places. We see that either number may be too large, or either may be too small, by an amount not greater than $\cdot 005$; and it is clear that the error in the product will be greatest when *both* are too large, or *both* too small. We will suppose that both are too small. Then

the true value of $32{\cdot}41$ cannot exceed $32{\cdot}415$,

and the true value of $36{\cdot}37$ cannot exceed $36{\cdot}375$;

$\therefore$ the true value of the product cannot exceed

$$(32{\cdot}41 + {\cdot}005)(36{\cdot}37 + {\cdot}005);$$

$\therefore$ the true value of the product cannot exceed

$$32{\cdot}41 \times 36{\cdot}37 + 32{\cdot}41 \times {\cdot}005 + 36{\cdot}37 \times {\cdot}005 + ({\cdot}005)^2.$$

But $({\cdot}005)^2$ is very small compared with the other quantities, so that the *error* in the product will approximately be not greater than $(32{\cdot}41 \times {\cdot}005 + 36{\cdot}37 \times {\cdot}005)$, or $\cdot 3439$. Thus the **limits of error** are $\pm {\cdot}344$ nearly.

By actual multiplication we find that $32{\cdot}41 \times 36{\cdot}37 = 1178{\cdot}7517$. Hence the true product must lie between $1179{\cdot}0957$ and $1178{\cdot}4077$; therefore the units' figure may be either **8** or **9**, and accuracy is

possible only as far as the tens' figure. The product, correct to the tens' figure, is 1180.

The process is similar when the factors are correct to *different* numbers of places.

421. In the general case we have

$$(a+x)\times(b+y)=ab+ay+bx+xy,$$

and $$(a-x)\times(b-y)=ab-ay-bx+xy.$$

Hence, if in two quantities a and b there are possible small errors of x and y respectively, the possible error in the product of a and b must be approximately not greater than $(ay+bx)$, *i.e.* the **limits of error** are $\pm(ay+bx)$.

Example i. *The sides of a rectangular field are 37·46 m. and 52·34 m. respectively, each side being correct to 2 decimal places. To what extent can the calculated area be relied upon? What is the possible error per cent. in the area?*

The error in the area will not exceed

$\pm(37{\cdot}46\times{\cdot}005+52{\cdot}34\times{\cdot}005)$ sq. metres, or $\pm$ ·45 sq. metres nearly.

The whole area $=37{\cdot}46\times52{\cdot}34$ sq. m. nearly $=1960$ sq. m. roughly;

$$\therefore \text{ the error per cent. in the area} = \pm\frac{{\cdot}45}{1960}\times100 = \pm\ {\cdot}02 \text{ nearly.}$$

Example ii. *A plate of iron is 137 cm. long, 643 mm. wide and 43·1 mm. thick, each of the measurements being correct to 3 figures; if the specific gravity of iron is 7·68, correct to 3 figures, find how many figures of the calculated weight in grams may be trusted.*

Volume of plate $=137\times64{\cdot}3\times4{\cdot}31$ c.cm.;

$\therefore$ Weight of plate $=137\times64{\cdot}3\times4{\cdot}31\times7{\cdot}68$ grams.

We will calculate the limits of error at each stage of the multiplication.

The limits of error for 137 are $\pm$ ·5,
and ..... 64·3 ... $\pm$ ·05;
$\therefore$ ..... the product are
$\pm(137\times{\cdot}05+64{\cdot}3\times{\cdot}5)$, *i.e.* ±39.

The limits of error for 8809·1 are ±39,
and ..... 4·31 ... $\pm$ ·005;
$\therefore$ ..... the product are
$\pm(8809\times{\cdot}005+4{\cdot}31\times39)$, *i.e.* ±212 nearly.

The limits of error for 37967 are ±212,
and ..... 7·68 ... $\pm$ ·005;
$\therefore$ ..... the product are
$\pm(37967\times{\cdot}005+212\times7{\cdot}68)$, *i.e.* ±1818 nearly.

64·3	
13 7	
643	
1929	
450	·1
8809	·1
	4·31
35236	·4
2642	·73
88	·091
37967	·221
7	·68
265770	
22780	
3037	
291587	

Thus the weight lies between 289769 grams and 293405 grams. Consequently the answer can be given correct only to **two** significant figures; therefore the weight of the plate $=290$ Kg. *correct to two figures.*

(iv). Division.

422. The principles involved in this case are the same as those already discussed in the case of *multiplication*. We will therefore examine at once the general case, and proceed to a numerical example.

The quotient of a divided by b is the value of the fraction $\frac{a}{b}$. If a and b may be too large, or too small, by quantities x and y respectively, it is clear that the greatest error in the value will occur when the quotient is $\frac{a+x}{b-y}$ or $\frac{a-x}{b+y}$. In the latter case the possible error

$$= \frac{a}{b} - \frac{a-x}{b+y} = \frac{ab+ay-ab+bx}{b(b+y)} = \frac{ay+bx}{b^2} \text{ nearly};$$

and in the former case it will be found that the possible error is the same. The **limits of error** are therefore $\pm\frac{ay+bx}{b^2}$ nearly.

If divisor and dividend are correct to the same number of places, x and y are equal to one another, and the **limits of error** are $\pm\frac{a+b}{b^2}\times x$.

Example. *The length of one side of a rectangle was found by measurement to be 25·7 m. to the nearest decimetre; and the area was found by calculation to be 10 a. 55 ca., to the nearest centiare. What is the length of the other side, and to what degree of accuracy can it be calculated?*

The length of the other side $= \frac{1055}{25{\cdot}7}$ m. $= 41{\cdot}05$ m. to 2 places.

The limits of error for 1055 are $\pm{\cdot}5$,

and ..... $25{\cdot}7$ ... $\pm{\cdot}05$;

$\therefore$ ..... the quotient are $\frac{\pm(1055\times{\cdot}05+25{\cdot}7\times{\cdot}5)}{(25{\cdot}7)^2}$,

i.e. $\pm\frac{65{\cdot}6}{660}$ nearly, *i.e.* $\pm{\cdot}1$ nearly.

Therefore the length of the other side lies between 40·95 m. and 41·15 m., and is therefore 41 *metres*, to the nearest metre.

EXAMPLES. 186.

1. Eight sums of money, expressed in pounds and decimals of a pound to 3 places, are added together; what are the limits of error in the result?

2. If two sums of money are decimalized correctly to 3 places, shew that the error in their difference can never exceed $\frac{1}{2}d$.

3. Five sums of money are decimalized by the method of multiplying the pence by 4 and the shillings by 5, and they are then added together. Find the limits of error in the result.

4. Three quantities, each correct to two decimal places, are added together; what are the limits of error in the sum?

5. A quantity, correct to 3 decimal places, is multiplied by 24; what are the limits of error in the result?

Find how many figures in the following products and quotients agree with the true value, and to how many figures the results can be found 'correct,' assuming that each quantity is correct to as many figures as are given.

6. $3{\cdot}34 \times 5{\cdot}71$.
7. $52{\cdot}31 \times 75{\cdot}05$.
8. $374{\cdot}3 \times 5{\cdot}75$.
9. $342{\cdot}71 \times 17{\cdot}8$.
10. $567{\cdot}84 \times 25{\cdot}125$.
11. $846{\cdot}812 \times 15{\cdot}6241$.
12. $3{\cdot}72 \div 1{\cdot}5$.
13. $37 \div 5{\cdot}21$.
14. $3{\cdot}721 \div {\cdot}126$.
15. $846{\cdot}5 \div {\cdot}0124$.
16. $576{\cdot}5 \div 25{\cdot}75$.
17. $93{\cdot}642 \div 3{\cdot}712$.

Find, to the nearest correct figure, the value of the following expressions, the numbers being correct as far as the figures given.

18. $\dfrac{35{\cdot}75 \times {\cdot}17}{3{\cdot}76}$.
19. $\dfrac{345 \times 26{\cdot}7}{2{\cdot}45}$.
20. $\dfrac{175{\cdot}32 \times 15{\cdot}7}{26{\cdot}54}$.

21. The sides of a rectangle are given as 5·873 metres and 2·485 metres. To how many correct figures can the area be calculated?

22. The sides of a rectangle are known to be 6·34 m. (to two places) and 4·275 m. (to three places). What are the limits of error in the calculated area?

23. By how much per cent. may the area of a field be wrong, if its sides are found by measurement to be 25 chains and 18 chains, to the nearest chain?

24. The volume of a substance is found by measurement to be 2367 c.cm., to the nearest cubic centimetre, and the specific gravity is given to be 8·5, correct to one decimal place; what may be the error in the calculated weight?

25. The sides of a rectangular field are measured, with possible errors of $\pm{\cdot}2$ per cent. in each case. By how much per cent. may the calculated area be wrong?

26. The area of a circle is πr^2. If $\pi = 3{\cdot}1416$, correct to 4 places, and $r = 3{\cdot}57$ metres, correct to 2 places, between what limits must the true area lie?

27. If the area of a field is 3546 sq. m., to the nearest sq. m., and the length of one side is known to be 57·4 m., to the nearest decimetre, between what limits must the other side lie?

28. If the volume of a tank is 65 cu. ft., to the nearest cubic foot, and the depth is 3 ft. 1 in., to the nearest inch, find as accurately as possible the area of the base, and state the limits of error.

29. If a cubic foot of water weighs 62·426 lb., correct to 3 places, and 1 foot = 30·480 cm., correct to 3 places, to how many significant figures can the number of grams in a pound be determined?

30. A plate of iron, 125 cm. long, 654 mm. wide and 45·6 mm. thick, weighs 290·5 Kg. Find the specific gravity of iron. If the figures are correct as far as given in the question, to how many figures can the value of the specific gravity be relied upon?

(b). Approximate Values of Infinite Series, and Approximate Fractional Forms.

423. There are certain quantities whose values can be expressed as the sum of an infinite number of fractions, each of which can be derived from the preceding fraction by some obvious law. Such expressions are called **infinite series.** The *exact* values of such quantities cannot be found by arithmetical processes, but, by taking a sufficient number of terms, we can find the values to as many decimal places as we please. When dealing with expressions of this kind, the pupil must *never attempt to add together the terms in their fractional form.* Each term must be expressed in the form of a decimal, and the number of terms taken must be such that the last of these terms has at least as many zeros between the decimal point and the first significant figure as there are decimal places retained in the working. The following example will illustrate the method to be employed.

Example. *Find the value of*

$$1+\frac{1}{1.2}+\frac{1}{1.2.3}+\frac{1}{1.2.3.4}+\ldots$$

so that the error may be less than ·00000005.

The value of the first term is 1. We find the value of the second term by dividing the value of the first term by **2**; the value of the third term by dividing that of the second by **3**; the value of the fourth term by dividing that of the third by **4**; and so on. We continue taking the terms in

succession until we arrive at one from which we obtain a decimal with 9 zeros between the decimal point and the first significant figure. Here we stop, as the values of the subsequent terms will not, in general, affect the value of the result to 7 decimal places.

Thus we have $1 = 1{\cdot}0000000\,|\,00$;

$\therefore \quad \frac{1}{1.2} = {\cdot}5000000\,|\,00$;

$\therefore \quad \frac{1}{1.2.3} = {\cdot}1666666\,|\,66$;

$\therefore \quad \frac{1}{1.2.3.4} = {\cdot}0416666\,|\,66$;

$\therefore \quad \frac{1}{1.2.3.4.5} = {\cdot}0083333\,|\,33$;

$\therefore \quad \frac{1}{1.2.3.4.5.6} = {\cdot}0013888\,|\,88$;

$\therefore \quad \frac{1}{1.2.3.4.5.6.7} = {\cdot}0001984\,|\,12$;

$\therefore \quad \frac{1}{1.2.3.4.5.6.7.8} = {\cdot}0000248\,|\,01$;

$\therefore \quad \frac{1}{1.2.3.4.5.6.7.8.9} = {\cdot}0000027\,|\,55$;

$\therefore \quad \frac{1}{1.2.3.4.5.6.7.8.9.10} = {\cdot}0000002\,|\,75$;

$\therefore \quad \frac{1}{1.2.3.4.5.6.7.8.9.10.11} = {\cdot}0000000\,|\,25$;

$\therefore$ the next term $= {\cdot}0000000\,|\,02$;

and $\therefore$ by addition, the expression $= 1{\cdot}7182818$ within ·00000005.

NOTE.—If the number of terms which have to be taken be very large, it may be necessary in the working to retain more than two additional figures, in order to obtain the correct carrying figure.

424. There are some quantities usually given in decimal form to a certain number of places, whose values can be approximately expressed as the sum or difference of simple vulgar fractions. By employing these approximate equivalents we can escape much tedious labour, and yet obtain a required result to as great a degree of accuracy as is necessary, or perhaps possible.

The ratio of the circumference of a circle to its diameter, for example, is 3·14159265..., and is commonly denoted by π. The decimal does not terminate, but to 4 decimal places it is equal to $3 + \frac{1}{7} - \frac{1}{7\,0\,0}$. Instead of multiplying by 3·1415926 ... we may therefore multiply by the terms of the expression $3 + \frac{1}{7} - \frac{1}{7\,0\,0}$ in succession. In the following example we will shew how this approximation may be used.

Example. *The diameter of a circle is 536·69 metres; find the circumference to the nearest metre.*

Rough approximation. The result must lie between 500 m. × 3 and 600 m. × 4, *i.e.* between 1500 m. and 2400 m.

The circumference of the circle is equal to 536·69 m. multiplied by π, and we will obtain this product by using the terms 3, $+\frac{1}{7}$, and $-\frac{1}{800}$, as in the margin.

	m.
N =	536·69
∴ 3N =	1610·07
∴ $\frac{1}{7}$N =	76·67
	1686·74
∴ $-\frac{1}{800}$N =	·67
	1686 metres.

Hence the circumference of the circle is 1686 metres.

To *divide* by π we may use the approximate value

$$\frac{1}{\pi} = \frac{1}{3} - \frac{1}{100} - \frac{1}{200} - \frac{1}{(200)^2}.$$

425. Approximate fractional equivalents such as those for π and $\frac{1}{\pi}$ are frequently useful in changing from the British to the Metric system of measurement.

In Art. 399 we stated that 1 acre = ·404671 hectares. It will be found that, to 5 significant figures, this may be expressed as

$$1 \text{ acre} = (1 + \tfrac{1}{100})\{\tfrac{4}{10} + \tfrac{4}{6000}\} \text{ hectares.}$$

It will be found also that, to 4 decimal places,

$$1 \text{ lb.} = (1 - \tfrac{1}{10})(\tfrac{1}{2} + \tfrac{4}{1000}) \text{ Kg.}$$

We will use these approximations in the following examples.

Example i. *The area of a field is 375 acres; find the area in hectares, to 4 significant figures.*

Rough approximation. 1 Ha. = $2\frac{1}{2}$ ac. nearly;

∴ 1 ac. = $\frac{2}{5}$ Ha.;

∴ 375 ac. = 150 Ha. nearly.

We have

1 acre = $(1 + \frac{1}{100})\{\frac{4}{10} + \frac{4}{6000}\}$ Ha.

Thus the area is 151·8 hectares.

N =	375
∴ $\frac{4}{10}$N =	150·0
∴ $\frac{4}{6000}$N =	·25
∴ S =	150·25
∴ $\frac{1}{100}$S =	1·5025
∴ area =	151·8 Ha.

Example ii. *Find the value of 224 lb. in Kg., to 4 significant figures.*

Rough approximation. 1 Kg. = $2\frac{1}{5}$ lb. nearly;

∴ 1 lb. = $\frac{5}{11}$ Kg.;

∴ 224 lb. = 102 Kg. nearly.

We have

1 lb. = $(1 - \frac{1}{10})(\frac{1}{2} + \frac{4}{1000})$ Kg.

Thus 224 lb. = 101·6 Kg.

N =	224
$\frac{1}{2}$N =	112
$\frac{4}{1000}$N =	·896
S =	112·896
$-\frac{1}{10}$S =	11·290...
∴ 224 lb. =	101·6 Kg.

EXAMPLES. 187.

Find the value of the following series, correct to 5 decimal places.

1. $\frac{1}{8}+\frac{1}{8^2}+\frac{1}{8^3}+\frac{1}{8^4}+\dots.$

2. $\frac{1}{1.2}+\frac{1}{1.2.3}+\frac{1}{1.2.3.4}+\dots.$

3. $1+\frac{1}{3}+\frac{1}{3.5}+\frac{1}{3.5.7}+\dots.$

4. $\frac{2}{5}+\frac{4}{5.10}+\frac{8}{5.10.15}+\frac{16}{5.10.15.20}+\dots.$

5. $\frac{1}{1.4.7}+\frac{1}{4.7.10}+\frac{1}{7.10.13}+\frac{1}{10.13.16}+\dots$ to 3 places.

6. $4\left\{\frac{1}{5}-\frac{1}{3}\cdot\frac{1}{5^3}+\frac{1}{5}\cdot\frac{1}{5^5}-\frac{1}{7}\cdot\frac{1}{5^7}+\dots\right\}$
$$-\left\{\frac{1}{239}-\frac{1}{3}\cdot\frac{1}{239^3}+\frac{1}{5}\cdot\frac{1}{239^5}-\dots\right\}.$$

7. If $\frac{1 \text{ yard}}{1 \text{ m.}}=1-\frac{1}{10}+\frac{1}{70}+\frac{1}{10000}$, express in metres, to 4 significant figures, the value of (i) 1 chain, (ii) 56 yds. 2 ft., (iii) 72 yds. 9 in.

8. If $\frac{1 \text{ litre}}{1 \text{ pint}}=2-\frac{1}{4}+\frac{1}{100}+\frac{1}{1300}$, express in pints, to 4 significant figures, the value of (i) 37·5 l., (ii) 5 Hl. 7 l., (iii) 9 l. 7 cl.

9. If $\frac{1 \text{ sq. metre}}{1 \text{ sq. yard}}=1+\frac{2}{10}-\frac{4}{1000}+\frac{1}{30000}$, find, to 4 significant figures, the value of (i) 87·5 sq. m., (ii) 8 a. 7 ca., (iii) 5 Ha. 7 a. 5 ca.

10. If $\frac{1 \text{ kilogram}}{1 \text{ lb.}}=\left(1+\frac{1}{10}\right)\left\{2+\frac{4}{1000}+\frac{2}{10000}\right\}$, express in pounds, to 4 significant figures, the value of (i) 26 Kg., (ii) 3 q. 57 Kg., (iii) $\frac{1}{4}$ Kg.

11. If the area of a circle be πr^2, and $\pi=3+\frac{1}{7}-\frac{1}{800}$, find, to 4 significant figures, the area of a circle whose radius is 3·57 ft.

12. If the circumference of a circle be 31·275 metres, find the value of the radius, to the nearest centimetre.

13. If $\sqrt{2}=7\left\{\frac{2}{10}+\frac{2}{10^3}+\frac{3}{10^5}+\frac{5}{10^7}\right\}$ to 6 decimal places, find, to 5 decimal places, the length of the diagonal of a square whose side is (i) 5·34256 mi., (ii) 1·34675 Km.

(c). Approximate Square Roots.

426. When we have to extract a square root to a large number of decimal places, and we have found it to one more than half this number, we may find the remaining digits by dividing the last remainder by the last divisor as in Art. 232. But since we cannot rely upon the correctness of the *last* digit thus obtained, we should determine two more digits than we want.

Example. *Find the square root of* 321·73025 *to* 12 *places.*

sq. rt. = 17·936840580213

(i)

1	321·73025		
	1		
27	221		
	189		
349	3273		
	3141		
3583	13202		
	10749		
35866	245350		
	215196		
358728	3015400		
	2869824		
3587364	14557600		
	14349456		
358736805	20814400	00	
	17936840	25	
3587368108	2877559	7500	
	2869894	4864	
358736811602	7665	26360000	
	7174	73623204	
3587368116041	490	5273679600	
	358	7368116041	
35873681160423	131	790556355900	
	107	621043481269	
	24	169512874631	

sq. rt. = 17·936840580213

(ii)

1	321·73025	
	1	
27	221	
	189	
349	3273	
	3141	
3583	13202	
	10749	
35866	245350	
	215196	
358728	3015400	
	2869824	
3587364	14557600	
	14349456	
35,8,7,3,6,8,0	20814400	
	17936840	
	2877560	
	2869894	
	7666	
	7175	
	491	
	359	
	132	
	107	
	25	

It will be seen that the figures to the left of the vertical line, if in each line we make the usual allowance for the first rejected figure, are nearly what we should have got by an approximate division as in Art. 232. The shortened work appears as in (ii). The work may be shortened still further by writing down *remainders only* as in the abbreviated method of Art. 35.

427. In order to understand the meaning of an approximate square root, the pupil must understand clearly the meaning of the operations which he is performing. In the example in the preceding Article, after the second figure of the square root has

been found, there is a remainder 32·73025. This means that 17^2 is less than 321·73025 by 32·73025. Similarly, giving the figures their proper place values, which during the operation are somewhat obscured, we see that

$(17{\cdot}9)^2$ is less than 321·73205 by 1·32025,
$(17{\cdot}93)^2$ ..... 321·73205 by ·245350,
$(17{\cdot}936)^2$ ..... 321·73205 by ·030154,

and finally

$(17{\cdot}936840580213)^2$ ..... 321·73205 by ·000000000024.

It is clear that the differences between the original number and the squares of the successive approximations continually diminish. We therefore conclude that the further we carry on the process, the nearer we shall get to a number whose square will be *equal* to the original number.

It should be noticed that after the last significant figure of the original number has been brought down, each of the succeeding dividends will end in 0. If then the operation does not terminate when the last significant figure of the dividend has been used, it will never terminate, because no number ending with any digit from 1 to 9, when squared, can produce a number ending in 0. Hence we see that by carrying on the process, as given for numbers which have an exact square root, we can *approximate as nearly as we please* to the square root of *any* other number, but we can never find its *exact value*.

428. When we require the approximate square root of a fraction, we may proceed in one of the ways indicated in the following examples.

Example i. *Find the square root of* $18\frac{5}{9}$.

$$\sqrt{18\tfrac{5}{9}}=\sqrt{\frac{167}{9}}=\frac{\sqrt{167}}{3}=\frac{1}{3}\sqrt{167}.$$

We now find $\sqrt{167}$ to a certain number of places, and divide it by 3

Example ii. *Find the square root of* $\frac{7}{2}$.

(i) $\sqrt{\frac{7}{2}}=\sqrt{\frac{14}{4}}=\frac{\sqrt{14}}{2}=\frac{1}{2}\sqrt{14}=$ etc.;

or (ii) $\sqrt{\frac{7}{2}}=\sqrt{3{\cdot}5}=$ etc.

429. If the square root be required within a certain fraction we can proceed as in the following example.

Example. *Find to the nearest eighth of an inch the side of a square whose area is $5\frac{1}{3}$ sq. inches.*

The side of the square $= \sqrt{\frac{16}{3}}$ in. If $\frac{x}{8} = \sqrt{\frac{16}{3}}$, then $\frac{x^2}{64} = \frac{16}{3}$;

$$\therefore\ x^2 = \frac{1024}{3} = 341\tfrac{1}{3};$$

$\therefore\ x = 18$, to the nearest integer;

$\therefore$ the side of the square $= \frac{18}{8}$ in. $= 2\frac{1}{4}$ inches approximately.

EXAMPLES. 188.

Find, to 3 decimal places, the square root of

1.	2.	2.	5.	3.	12.	4.	13.	5.	1·6.
6.	·4.	7.	·5.	8.	·7.	9.	·002.	10.	$4\cdot\dot{5}$.
11.	$3\cdot\dot{2}5\dot{6}$.	12.	$2\cdot3\dot{6}\dot{1}$.	13.	$\cdot005\dot{2}\dot{1}$.	14.	$5\cdot0\dot{1}3\dot{2}$.		
15.	$\cdot02\dot{9}8\dot{1}$.	16.	3.	17.	64·34.	18.	10.	19.	·001728.
20.	$\frac{3}{10}$.	21.	$17\frac{23}{100}$.	22.	$\frac{7}{8}$.	23.	$\frac{4}{7}$.	24.	$\frac{8}{11}$.
25.	$\frac{478}{549}$.	26.	$4\frac{5}{21}$.	27.	$10\frac{17}{92}$.	28.	$21\frac{35}{117}$.		

29. Find, to the nearest inch, the side of a square whose area is 14 sq. feet.

30. Find the square root of $\frac{65}{7}$ within $\frac{1}{3}$.

31. Find, to the nearest foot, the side of a square whose area is 3 sq. yds. 7 sq. ft. 72 sq. in.

32. Find, to the nearest centimetre, the length of the diagonal of a square whose side is 31 metres.

33. If the diagonal of a square be 3 ft. long, find the length of one side, correct to ·0005 of an inch.

34. If the diagonal of a square be 384 metres, find, to the nearest centimetre, the length of one side.

(d). Surds.

430. To find the value of such expressions as

$$\frac{1}{\sqrt{3}-1},\ \frac{2}{\sqrt{3}-\sqrt{2}},\ \frac{\sqrt{3}}{\sqrt{5}+\sqrt{3}},\ \frac{\sqrt{3}+\sqrt{2}}{\sqrt{5}-\sqrt{2}},\ \text{etc.},$$

the student must notice the following statements, which are true whatever numbers the letters may stand for:

(i) $\sqrt{a}\times\sqrt{a}=a$, or $(\sqrt{a})^2=a$.

(ii) $\sqrt{a}\times\sqrt{b}=\sqrt{a\times b}$.

(iii) $\sqrt{a^2\times b}=a\times\sqrt{b}$

(iv) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})=a-b$.

(v) $(\sqrt{a}+\sqrt{b})^2=a+b+2\sqrt{ab}$, or $\sqrt{a}+\sqrt{b}=\sqrt{a+b+2\sqrt{ab}}$.

(vi) $(\sqrt{a}-\sqrt{b})^2=a+b-2\sqrt{ab}$, or $\sqrt{a}-\sqrt{b}=\sqrt{a+b-2\sqrt{ab}}$.

Expressions such as $\sqrt{2}$, $\sqrt{7}$, $\sqrt[3]{10}$, in which the root cannot be found exactly, are called **Surds**.

Example i. *Find the value of* $\dfrac{1}{\sqrt{3}-1}$.

We first free the denominator from surds. We do this by multiplying numerator and denominator by $\sqrt{3}+1$. The denominator thus becomes $3-1$ by (iv). Hence

$$\frac{1}{\sqrt{3}-1}=\frac{\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}+1)}=\frac{\sqrt{3}+1}{3-1}=\frac{\sqrt{3}+1}{2}=1\cdot366\ldots.$$

NOTE.—The operation of freeing a denominator from surds is called **rationalizing**.

Example ii. *Find the value of* $\dfrac{2}{\sqrt{3}-\sqrt{2}}$.

We multiply numerator and denominator by $\sqrt{3}+\sqrt{2}$, and we get

$$\frac{2}{\sqrt{3}-\sqrt{2}}=\frac{2(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}=\frac{2(\sqrt{3}+\sqrt{2})}{3-2}=2(\sqrt{3}+\sqrt{2})=\text{etc.}$$

We must now find the values of $\sqrt{3}$ and $\sqrt{2}$, substitute them, and simplify the result as in Example i.

Example iii. *Find the value of* $\dfrac{\sqrt{3}}{\sqrt{5}+\sqrt{3}}$.

$$\frac{\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\frac{\sqrt{3}\times(\sqrt{5}-\sqrt{3})}{5-3}=\frac{\sqrt{15}-3}{2}=\text{etc.}$$

We need only the value of $\sqrt{15}$.

Example iv. *Find the value of* $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$.

$$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\frac{(\sqrt{3}+\sqrt{2})(\sqrt{5}+\sqrt{2})}{5-2}=\frac{\sqrt{15}+\sqrt{6}+\sqrt{10}+2}{3}=\text{etc.}$$

We need the values of $\sqrt{15}$, $\sqrt{6}$, and $\sqrt{10}$.

Example v. *Find the value of* $\sqrt{7+2\sqrt{10}}$.

We have $\sqrt{7+2\sqrt{10}}=\sqrt{5+2+2\sqrt{5\times2}}=\sqrt{5}+\sqrt{2}$ from (v).

NOTE.—In these cases the term involving the surd must always be expressed with 2 only outside the root; we have then to find, by inspection, two numbers which, when added together, will give the rational number 7, and which, when multiplied together, will give the number 10 under the root.

Example vi. *Find the value of* $\sqrt{4+\sqrt{15}}$.

We have

$$\sqrt{4+\sqrt{15}}=\sqrt{\frac{8+2\sqrt{15}}{2}}=\sqrt{\frac{5+3+2\sqrt{5\times 3}}{2}}=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{2}}=\tfrac{1}{2}\sqrt{10}+\tfrac{1}{2}\sqrt{6}=\text{etc.}$$

NOTE.—The values of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and $\sqrt{6}$ will be useful in working out the examples in the next set;—

$$\sqrt{2}=1{\cdot}41421356\ldots,\quad \sqrt{5}=2{\cdot}23606797\ldots,$$
$$\sqrt{3}=1{\cdot}73205080\ldots,\quad \sqrt{6}=2{\cdot}44948974\ldots.$$

EXAMPLES. 189.

Find, to 3 decimal places, the value of

1. $\dfrac{1}{\sqrt{2}-1}$.
2. $\dfrac{1}{\sqrt{3}-1}$.
3. $\dfrac{1}{\sqrt{5}+1}$.
4. $\dfrac{1}{\sqrt{6}+1}$.
5. $\dfrac{1}{\sqrt{5}+2}$.
6. $\dfrac{1}{\sqrt{6}-2}$.
7. $\dfrac{1}{\sqrt{3}-\sqrt{2}}$.
8. $\dfrac{1}{\sqrt{3}+\sqrt{2}}$.
9. $\dfrac{1}{\sqrt{5}+\sqrt{3}}$.
10. $\dfrac{2}{\sqrt{3}+1}$.
11. $\dfrac{2}{\sqrt{3}-1}$.
12. $\dfrac{2}{\sqrt{5}-1}$.
13. $\dfrac{2}{\sqrt{7}-\sqrt{5}}$.
14. $\dfrac{3}{\sqrt{5}+\sqrt{2}}$.
15. $\dfrac{2}{\sqrt{6}-\sqrt{2}}$.
16. $\dfrac{\sqrt{5}}{\sqrt{5}-1}$.
17. $\dfrac{\sqrt{3}}{\sqrt{3}-\sqrt{2}}$.
18. $\dfrac{\sqrt{5}}{\sqrt{5}+\sqrt{3}}$.
19. $\dfrac{\sqrt{5}}{3-\sqrt{2}}$.
20. $\dfrac{\sqrt{3}}{4-\sqrt{3}}$.
21. $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$.
22. $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$.
23. $\dfrac{\sqrt{5}+1}{\sqrt{5}-1}$.
24. $\dfrac{\sqrt{5}-2}{\sqrt{2}+1}$.
25. $\dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$.
26. $\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$.
27. $\dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{3}+1}$.
28. $\dfrac{\sqrt{3}+1}{\sqrt{5}+\sqrt{2}}$.
29. $\dfrac{\sqrt{3}+\sqrt{5}}{\sqrt{5}-\sqrt{2}}$.
30. $\dfrac{\sqrt{3}+\sqrt{7}}{\sqrt{11}-\sqrt{5}}$.
31. $\sqrt{5+2\sqrt{6}}$.
32. $\sqrt{8+2\sqrt{7}}$.
33. $\sqrt{12+8\sqrt{2}}$.
34. $\sqrt{\dfrac{2+\sqrt{3}}{2}}$.
35. $\sqrt{\dfrac{\sqrt{3}+1}{\sqrt{3}-1}}$.
36. $\sqrt{\dfrac{2\sqrt{7}-2\sqrt{3}}{2\sqrt{7}+2\sqrt{3}}}$

XLIX. GRAPHS.

Graphical Representation.

431. Squared paper is particularly useful in the representation of the results of observation, such as the variations of temperature and of the height of the barometer, and such fluctuations as occur in records of statistics. It must not be supposed however that this method of *graphical* representation is due to the squared paper; the paper merely makes it easier for us to apply the method, the sides of the small squares serving as measures and so enabling us to dispense with the too frequent use of a measuring instrument.

Example i. *To show the variations of maximum temperature on the successive days of June,* 1902.

The lowest maximum temperature during the month was 55° and the highest was 81°.

In the diagram we will suppose the distance between two successive horizontal parallels to represent 1 degree of temperature and the distance between two successive vertical parallels to represent 1 day.

We will suppose also that the lowest horizontal parallel represents a temperature of 50°, this being lower than the lowest maximum recorded during the month.

The left-hand vertical line we suppose to represent June 1, on which day the recorded maximum was 74°. We show this by marking on it a point at a distance of **24** divisions above the 50° line.

On June 2, the maximum was 71°. The second vertical represents June 2, and on it we mark a point at a distance of **21** divisions above the 50° line.

On June 3, the maximum was 77°. On the third vertical we therefore mark a point at a distance of **27** divisions above the 50° line.

On the remaining days of the month the numbers of degrees in the recorded maxima were successively 64, 66, 59, 61, 57, 56, 58, 61, 56, 55, 55, 60, 61, 62, 63, 73, 63, 71, 70, 74, 79, 69, 71, 77, 80, 81 and 81.

On the verticals for June 4, June 5, June 6, etc., we mark points at distances of **14**, **16**, **9**, **11**, etc., divisions respectively above the 50° line. We now join consecutive points by straight lines, the first point being joined to the second, the second to the third, the third to the fourth and so on. The resulting broken line is called a **graph**, and shows very clearly the fluctuations of the maximum daily temperature during the month.

Example ii. *To show graphically the sums to which* £100 *will amount at the end of* 5, 10, 15, 20 *and* 25 *years, at Simple and Compound Interest respectively, at* 3 % *in both cases; the following data being used:*

Years	0	5	10	15	20	25
Amount at Simple Interest	£100	£115	£130	£145	£160	£175
Amount at Compound Interest	£100	£116	£134	£156	£181	£209

We will take *two* horizontal divisions on the squared paper to represent each year, and *one* vertical division to represent £1.

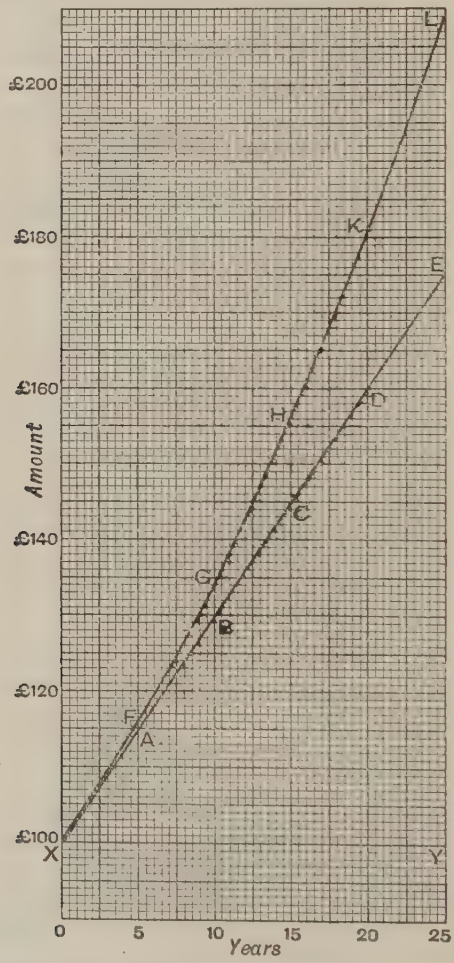
We will suppose X to be a point on the left-hand vertical at a distance of 100 divisions above the zero point (which is not shown in the figure). The height of the point X will then represent the principal, *i.e.* £100.

We omit that part of the diagram which would be below this, since it is of no importance as regards the question under consideration.

On the lowest horizontal line we count off the divisions in groups of ten, and we mark the points obtained successively with the numbers 5, 10, 15, 20, 25, which represent years.

(i) The amounts of **Simple** Interest for 5, 10, 15, 20 and 25 years are £15, £30, £45, £60 and £75 respectively. On the verticals through the numbered points we take points A, B, C, D, E, at distances of 15, 30, 45, 60 and 75 divisions respectively above the horizontal through X. These

distances from the horizontal through X represent the increments due to Simple Interest; and the distances of A, B, C, D, E, from the horizontal through *zero* represent the sums to which £100 will amount in 5, 10, 15, 20, 25 years respectively.



The points A, B, C, D, E lie on a straight line, which is the graph required in this case; it shows the amounts of £100 at *Simple* Interest.

(ii) In the case of **Compound** Interest we take points F, G, H, K, L, on the verticals numbered 5, 10, 15, 20, 25 respectively, at distances from the

horizontal through X equal respectively to **16, 34, 56, 81, 109** divisions. If we now join the points X, F, G, H, K, L, by a continuous line, which will be slightly curved, this curved line will be the **graph** required in this case; it will show the amounts of £100 at *Compound* Interest.

432. The student will see that the three graphs which we have obtained in the two preceding examples are very different in form. The first is an irregular broken line, the second is a straight line and the third is a continuous curve.

The maximum temperature recorded on any day has but little to do with the day of the month. It does not increase uniformly as the month grows older, but is determined mainly by physical and other conditions. There will not *necessarily* be any change at all from day to day, and, when there is such a change, we cannot tell beforehand whether there will be an increase or a decrease of temperature. The points therefore fall irregularly. The graph, in fact, merely represents usually certain changes which have taken place, and it cannot be used for any other purpose; we cannot, for example, use it to estimate what the temperatures were in the previous month, or what they would be in July.

In the cases considered in Example ii., however, we know that when the rate per cent. and the principal are given, the *amount* depends only on the *time*, whether the interest be Simple or Compound. In the former case the increase is both regular and uniform; in the latter case it is regular, but it is not uniform.

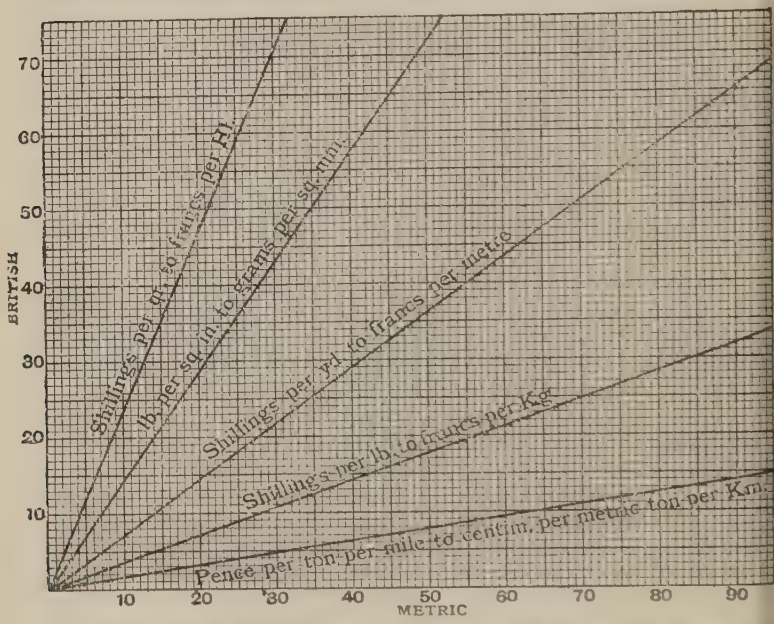
In the former case, when we obtain the points A, B, C, D, E and find that they lie on a straight line, we may conclude that all points determined in the same manner would lie on the same straight line, and we can determine, by means of the graph, the sum to which the Principal will amount at the end of *any* time. To find the amount of £100, for example, in 22 years at 3 % simple interest, we first count 44 divisions horizontally from X, and we then count upwards until we reach the point at which the 44th vertical cuts the graph. This happens at the 66th division above the line XY, and we therefore conclude that £166 is the sum to which £100 will amount in 22 years. Moreover the straight line XE may be produced, and be then used to determine the amount of £100 at the end of a time greater than the 25 years to which the diagram extends.

In the case of Compound Interest we might use the graph XFGHKL in a similar way to find the amount of £100 for *any* intermediate period *between* 0 years and 25 years, but, as we are not sure of the form of the curve between the successive points,

the results obtained would be only approximate. And, as we know nothing at all of the form of the curve beyond L, we cannot produce the curve, and so make use of it to find the amount of £100 for a period *greater* than 25 years.

We have dwelt upon these cases thus fully in order to shew some of the principles which govern the drawing of graphs, and which regulate the use of the graphs when they are drawn.

433. When two quantities are so related that one bears a constant ratio to the other, the graph which shews this relation



will always be a straight line. This principle is used in the construction of the diagram on page xvii, in which are shewn the relations between British measures and the corresponding measures in the Metric System. For example, if a metres $= b$ yards, then

$$\frac{1 \text{ yard}}{1 \text{ metre}} = \frac{a}{b};$$

but we know that 915 metres = 1000 yards nearly;

$$\therefore \frac{1 \text{ yard}}{1 \text{ metre}} = \frac{915}{1000}; \quad \therefore \frac{a}{b} = \frac{915}{1000} = \frac{183}{200}.$$

Hence we determine one point on the 'yards to metres' graph by measuring 183 spaces horizontally and 200 spaces vertically, and we then join that point to the origin. How to use the graph when drawn was explained on page xiii.

In the preceding diagram we have drawn similar graphs shewing the connections between other British and Metric measures. They are based upon the following relations:

(i) 100*s.* per yard is equivalent to 138 f. per metre.
(ii) 100*s.* per pound ..... 278 f. per kilogram.
(iii) 100*s.* per qr. (capacity) ..... 43 f. per hectolitre.
(iv) 100*d.* per ton per mile ..... 642 c. per metric ton per Km.
(v) 100 lb. per sq. inch ..... 70 g. per sq. mm.

The student should verify these statements by working out each case for himself. The graphs themselves can be used for purposes of conversion in the manner explained on page. xiii.

EXAMPLES. 190.

1. The price of an ounce of silver on Jan. 1st in each of the ten years (1891-1900) was 45, 40, 36, 29, 30, 31, 28, 27, 27, 28 pence respectively. Draw a graph shewing these changes in value.

2. The mean temperature on the first day of each month shewed, on an average of 50 years, the following values:—Jan., 37°; Feb., 38°; March, 40°; April, 45°; May, 50°; June, 57°; July, 62°; Aug., 62°; Sept., 59°; Oct., 54°; Nov., 46°; Dec., 41°. Draw a graph to represent these variations.

3. The temperature taken every two hours one day, from midnight to midnight, shewed 41°·0, 40°·8, 40°·7, 39°·5, 40°·8, 44°·5, 48°, 51°·2, 53°, 46°·5, 46°·3, 46°·7, and 47°·4. Draw a graph to shew the variation of temperature throughout the day, and estimate the temperature at 3 p.m.

4. The distances one can see at given heights above the earth's surface are given approximately in the following table:

Height in feet	10	50	70	100	150	200	300	400	500
Distance in miles	4·2	9·4	11	13·2	16·2	18·7	22·9	26·5	29·6

Draw a graph shewing the connection between the two quantities, and estimate the distances one can see at heights of 130 feet and 260 feet respectively.

5. The vertical heights to which a body rises when projected vertically upwards with velocities of 8, 16, 32, 40, 64, 80 feet per second, respectively, are 1, 4, 16, 25, 64, 100 feet. Draw a graph shewing the variation in height; and estimate the height to which a body will rise if projected with a velocity of 56 ft. per second.

6. When the readings on a Centigrade thermometer are 5, 10, 15, 20, 30, 50, 80 degrees, respectively, the corresponding readings on a Fahrenheit thermometer are 41, 50, 59, 68, 86, 122, 176 degrees. Illustrate graphically the connection between the two scales. Express 140° F. in Centigrade.

7. Six 'reputed quarts' make one gallon, and 1 gall. = 4·54346 litres. Draw a graph for converting reputed quarts into litres. Hence read off the values of 5, 14, 21 reputed quarts in litres.

8. Draw a graph for converting pence per cwt. per mile to centimes per quintal per kilometre.

9. The following table shews the volume of the same quantity of water at different temperatures.

Temperature in Centigrade	4°	20°	30°	40°	50°	60°	70°	80°	90°	100°
Volume	1	1·002	1·004	1·008	1·012	1·017	1·022	1·028	1·036	1·043

Draw a graph shewing the connection between volume and temperature, and estimate the volume at 65° C.

10. The distances through which a body would fall freely in certain times are given in the following table:

Time in seconds	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
Distances in feet	1	4	9	16	36	64	100	144

Draw the graph, and estimate the distance through which a body will fall in $2\frac{1}{4}$ seconds.

11. The times of oscillation of a simple pendulum 1, 4, 9, 16, 25, 36 feet long are 1·1, 2·2, 3·3, 4·4, 5·6, 6·7 seconds respectively. Draw the graph, and estimate the time of oscillation of a pendulum 12 feet long.

12. The areas of circles whose radii are 1, 2, 3, 4, 5 cm. are 3·14, 12·6, 28·3, 50·3, 78·5 sq. cm. respectively. Prove, by drawing a graph, that the ratio of the area to the *square* of the radius is constant, and find the value of the constant ratio.

13. The notes emitted by organ pipes 1 ft., 1 ft. 3 in., 2 ft., 2 ft. 6 in., 2 ft. 9 in., 3 ft. 4 in., 5 ft., 10 ft. in length are represented by the numbers 550, 440, 275, 220, 200, 165, 110, 55, respectively. Draw a curve shewing the connection between the note and the length of the pipe; and estimate the length of a pipe which will give a note represented by (i) 512, (ii) 256.

14. The following table gives, to the nearest shilling, the rate of premium to secure £100 payable at 60 or at death :

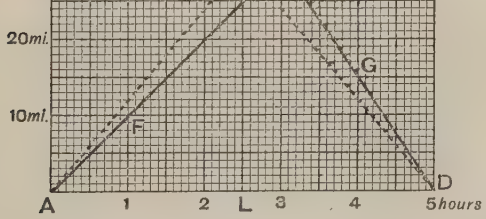
Age next birthday	20	25	30	35	40	45	50
Annual Premium	£2. 8*s.*	£2. 16*s.*	£3. 6*s.*	£4. 2*s.*	£5. 4*s.*	£7. 3*s.*	£11

Draw a graph shewing the relation between the premium and the age of the person. Estimate the premium to be paid by persons 27 and 37 years old respectively.

Applications to Problems.

434. We will now illustrate the use of graphical methods by applying them to a few problems.

Example 1. *A man cycles from* A *to* B *at* 10 *miles an hour, and returns from* B *to* A *at* 15 *miles an hour. Find his average speed. Also, if he takes* 5 *hours to go there and back, find the distance from* A *to* B.



Since the distance which the man travels at a given rate is proportional to the time the journey takes, it follows that the graph which represents the relation between distance and time is a straight line.

We will take 10 horizontal divisions to represent one hour, and 1 vertical division to represent a mile. Distances from A, the starting point, are then represented by distances above the horizontal through A.

Since the man travels at 10 miles an hour, we will take a point F on the vertical marked 1, at a distance of 10 divisions above the horizontal

through A, and we will join AF and produce it. The line AF is the *graph* for the outward journey.

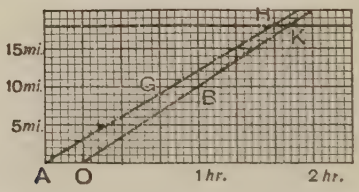
The man is back again at A in 5 hours, which is represented by a point D, on the horizontal through A, at a distance of 50 divisions from A. He rides back at 15 miles an hour; we will therefore take a point G, on the vertical marked 4, at a distance of 15 divisions above AD. It follows that GD is the *graph* for the return journey.

The graphs AF and DG meet at a point C on the 30th horizontal line above AD. Hence it follows that the distance from A to B is 30 miles.

If the man had travelled at a uniform rate throughout, and had taken the same time for the whole journey as he actually did, he would have taken half that time to go and half to return. Consequently, if L be the middle point of AD, and H be the point at which the vertical through L meets BC, it is plain that if he travelled uniformly at the *average* speed, the graph would be AHD. Examining this graph we see that AH meets the vertical marked 1 at a point 12 divisions above AD. Therefore the average rate is 12 miles per hour.

Example ii. *At a paper chase the hares had* 20 *minutes start, and ran the whole distance at the rate of* 9 *miles per hour. One of the hounds, who ran the same course at* 10 *miles per hour, arrived* 8 *minutes after the hares. How long was the run?*

Let each of the horizontal divisions represent 4 minutes, and each vertical division one mile. As in Example i. the graphs are straight lines.



The hares had 20 minutes *start*; we therefore take A at a distance of 5 divisions, which represent 20 minutes, measured to the *left* of O. On the 15th vertical line from A we now take G, 9 vertical divisions above AO. We then join AG, which is consequently the graph of the *hares*.

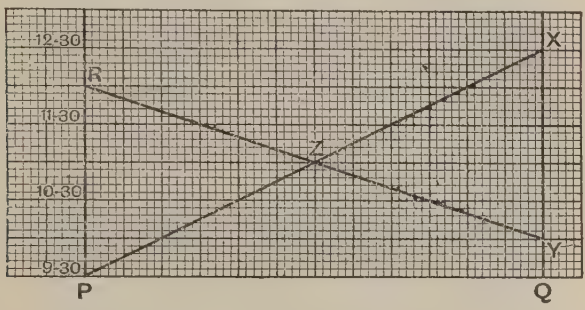
On the 15th vertical line from O we take B, 10 vertical divisions above AO. We then join OB, which is consequently the graph of *one of the hounds*. Examining these lines, we see that they are exactly two horizontal divisions apart, *i.e.* 8 minutes, when they cross the 18th horizontal line at H and K; and as H and K are 18 divisions above AO, it follows that the length of the course was 18 miles.

Example iii. A *started to walk from* P *to* Q *at* 9.30 *a.m. At* 11.0 *a.m. he met* B, *who had started to walk from* Q *to* P *at* 10.0 *a.m. If* A *arrives at* Q *at* 12.30 *p.m., when will* B *arrive at* P?

We will measure the time by vertical divisions, 10 divisions representing one hour; and we will measure the distances by horizontal divisions. We may take any convenient horizontal length to represent the distance from P to Q.

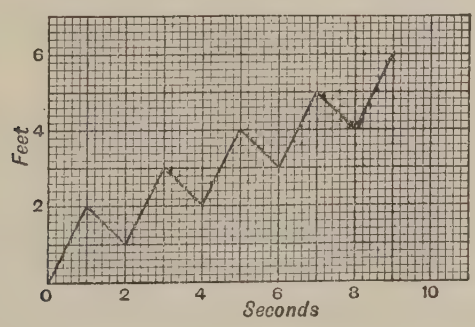
We will now take X on the vertical through Q such that QX contains 30 divisions, and therefore represents 3 hours. If PX be joined, it follows that PX is the graph of A.

We see now that PX cuts the 11.0 a.m. line at Z; therefore Z is a point on the graph of B.



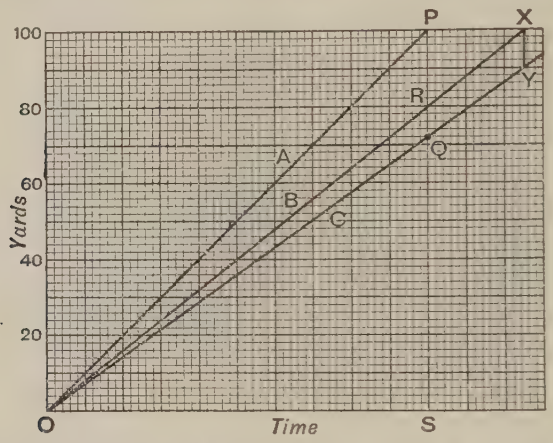
But B starts from Q at 10.0 a.m.; therefore the point Y on the vertical through Q, and at a distance of 5 divisions from Q, is also a point on B's graph. B's graph is consequently the line YZ; and this line, when produced, meets the vertical through P at the point R, at a distance of 25 divisions from P. Hence it follows that B will arrive at P at 12.0 noon.

Example iv. *A monkey, climbing up a greased pole, ascends 2 ft. and slips down 1 foot in alternate seconds, until he reaches the top of the pole. If the pole be 6 feet high, how long will it take him to reach the top?*



We will take 5 vertical divisions to represent one foot, and 5 horizontal divisions to represent one second. It will be clear then that the graph is the broken line shewn in the figure. By inspecting the graph and counting the number of horizontal spaces we see that the time taken is 9 seconds.

Example v. *In a* 100 *yards race* A *beats* B *by* 20 *yards, and* B *beats* C *by* 10 *yards. How many yards start can* A *give* C *in* 100 *yards so that they may run a dead heat?*



We will take one vertical division to represent 2 yards and we will measure the times by horizontal divisions. We will suppose OS to represent the time in which A can run 100 yards. Then it will be evident that A's graph is the line OP.

If we take R, 10 vertical divisions (representing 20 yards) below P, we see that OR is B's graph.

Let OR cut the '100 yards' line in X, and through X draw a vertical line cutting the 90 yards line in Y; then OY is C's graph.

The line OY cuts SP at a point Q which is 14 vertical divisions (representing 28 yards) below P. Therefore A would beat C by 28 yards in 100 yards; which means that, if A gives C 28 yards start, they will run a dead heat.

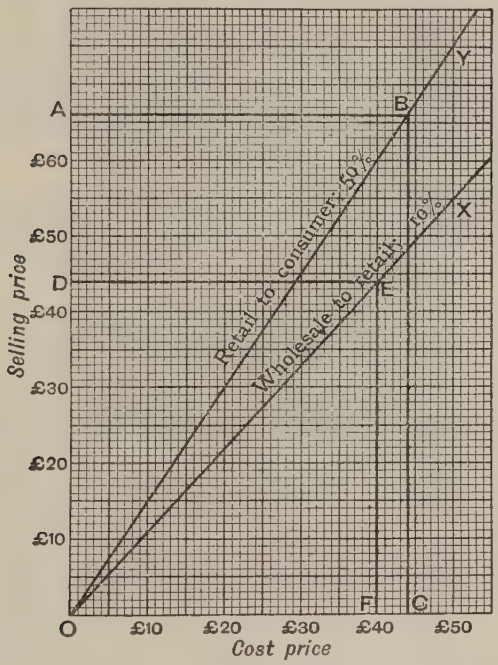
Example vi. *A wholesale dealer gets a profit of* 10 % *when he sells to a retail dealer; and the retail dealer gets a profit of* 50 % *when he sells to a consumer. What is the cost to the wholesale dealer of an article for which the consumer pays* £66?

When there is a profit of 10 %, the selling price is to the cost price as 110 is to 100, *i.e.* as 55 is to 50. Thus, if we represent the cost price by horizontal divisions and the selling price by vertical divisions, and take each division to represent £1, the graph shewing the relation between the cost to the wholesale dealer and the retail dealer's buying price is got by joining the point X to O; the point X being obtained by counting 50 divisions horizontally and 55 divisions vertically.

Similarly the graph shewing the relation between the retail dealer's buying and selling prices is got by joining Y to O; the point Y being obtained by counting 50 divisions horizontally and 75 divisions vertically.

Taking a point A, 66 divisions above O, we see that OA will represent £66. the *price paid by the consumer.*

We then draw the horizontal through A to meet the 'retail to consumer' graph at B. Then AB, or OC, will represent the *buying price of the retail dealer.*



We next take the vertical distance OD, equal to AB, and we draw the horizontal DE to meet the 'wholesale to retail' graph at E. Then DE, or OF, will represent the *cost to the wholesale dealer.*

OF contains 40 divisions; hence the cost to the wholesale dealer is £40.

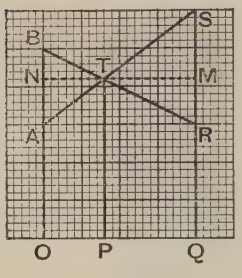
Example vii. *Three teas, whose prices per lb. are respectively* 1*s.* 3*d.*, 2*s.* 1*d. and* 2*s.* 6*d., are to be taken two at a time and mixed in the same proportion, so that the resulting mixtures are of equal value. How may they be mixed?*

We will take one vertical division to represent 1*d.* Then if OA be taken equal to 15 and OB equal to 25 vertical divisions, it follows that OA will represent the price of the 1*s.* 3*d.* tea, and OB the price of the 2*s.* 1*d.* tea.

On any other vertical line QRS we now take QR equal to 15, and QS equal to 30, vertical divisions. QR will then represent the price of the 1*s.* 3*d.* tea, and QS the price of the 2*s.* 6*d.* tea.

Let AS and RB meet at T, and draw the horizontal NTM through T.

It is then clear, from the figure, that AS is divided by the horizontals into equal parts, of which, by counting, we see that AT contains 6, and TS contains 9. Consequently, $\frac{ST}{AT}=\frac{9}{6}=\frac{3}{2}$.



We see too that SM contains 9 and that AN contains 6 vertical divisions, therefore $2SM=3AN$; and therefore $2SQ+3OA=5PT$.

If then we mix 2 lb. of 2*s.* 6*d.* tea with 3 lb. of 1*s.* 3*d.* tea, the price of one lb. of the mixture will be represented by PT, and is therefore 1*s.* 9*d.*

Similarly it can be seen that $\frac{RT}{BT}=\frac{3}{2}$, and therefore that PT represents the price per lb. of a mixture of 3 lb. of the 2*s.* 1*d.* tea and 2 lb. at 1*s.* 3*d.*

Thus the teas can be mixed in the proportion of 2 to 3 in the ways described; and in each case the price per lb. of the mixture will be 1*s.* 9*d.*

Example viii. *Four people wish to be conveyed as quickly as possible to a place* 16 *miles distant. They can walk* 4 *miles per hour, and they have a motor car, which can go* 20 *miles per hour, but only holds two besides the driver. Two of them set off on foot: the car takes the other two a certain distance, sets them down to continue on foot, and then returns to bring the first two. At what point must the car set down those it took first, in order that the four may arrive at the destination at the same instant? Find also the time taken to do the journey.*

Let one horizontal division represent 2 minutes and 3 vertical divisions represent 1 mile.

We take the point P by measuring 30 horizontal divisions, representing 1 hour, and 60 vertical divisions, representing 20 miles, and we join OP. Then OP is the direction of the graph of the motor car at the start.

We now take a point Q on SP such that Q is 12 vertical divisions, representing 4 miles, above OS; then OQ is the graph of the two who are walking.

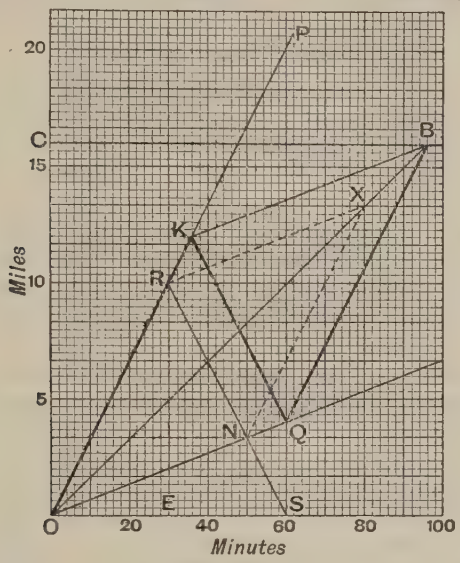
Since the distance is to be covered as quickly as possible, those who are walking will walk at 4 miles per hour, and the motor car will travel at 20 miles per hour both going and returning. Hence the graph of the motor car when it is returning will be in the *same direction* as RS, since OE = ES.

Let RS cut OP and OQ in R and N. Draw RX parallel to OQ and NX parallel to OP.

If the motor car set down the first pair at R, and take up the other pair at N, all four will reach X in the same time.

Produce OX to cut at B the horizontal line BC, which is 48 divisions, representing 16 miles, above OS. Draw BK and BQ parallel to OQ and OP respectively. Then if the car set down the first pair at K and take up the others at Q, all four will reach B at the same time.

By inspecting the figure, it will be seen that (i) CB contains 48 divisions, representing 96 minutes, (ii) K is 36 divisions above OS, representing 12 miles, and (iii) Q is 12 divisions above OS, representing 4 miles.



Thus the motor sets down the first pair when it has gone 12 miles, picks up the second pair when they have gone 4 miles, and the whole journey occupies 1 hr. 36 min.

EXAMPLES. 191.

(To be solved graphically.)

1. What per cent. of 6*s.* 8*d.* is 4*s.* 4*d.*?

2. Express each of the following numbers as a percentage of the highest number of the series:—70, 85, 45, 100, 60, 145, 115.

3. If £80 amounts to £92 in 5 years at Simple Interest, find the rate per cent. Also find the amount of £40 in 10 years at the same rate per cent.

4. Find the amount of £120 in 3 years at $2\frac{1}{2}\%$ simple interest. What per cent. of the principal is the interest?

5. Draw a graph shewing the relation between the cost price and the selling price, when goods are sold at a gain of 15%. *Read off from this graph* the selling prices of articles, which cost 3*s.* 4*d.* and £3 respectively; and the cost prices of articles which are sold for 2*s.* $10\frac{1}{2}$*d.* and 17*s.* 3*d.* respectively.

6. A man sets out at 9.30 a.m. to walk to a certain place, which is 10 miles distant. He walks out at the rate of 4 miles per hour. He rests for 45 minutes on his arrival at the place, and he then walks back, arriving home at 3.25 p.m. Find at what rate he walked back.

7. A train leaves A for B at 9.15 a.m. and travels at the rate of 30 miles per hour. At 9.35 a.m. a second train starts, and travels at the rate of 35 miles an hour. If both trains arrive at B at the same time, find the distance from A to B, and the time each train takes.

8. A walks completely round a rectangular area, whose sides are 22·5 metres and 54 metres, in the same time that B takes to walk across it from one corner to the opposite corner and back. How much further does A walk than B? Compare their rates of walking.

9. In what proportion must teas, costing 1*s.* 1*d.* and 2*s.* 6*d.* per lb. respectively, be mixed so that the mixture may be worth 1*s.* 6*d.* per lb.?

10. Teas, costing 1*s.* 2*d.* and 2*s.* 5*d.* per lb. respectively, are mixed in a certain proportion. Teas, costing 11*d.* and 2*s.* 9*d.* per lb. respectively, are mixed in the same proportion, and the values of the two mixtures are equal. Find the proportion in which the mixtures are made and the value per lb.

11. In a 100 yds. race A beats B by 9 yards, and in 100 yds. C beats B by 8 yards. If A's time for the hundred yards is $10\frac{2}{5}$ sec., what are B's and C's times?

12. A walks from P to Q, a distance of 10 miles, in $2\frac{1}{2}$ hours. He stays at Q for $1\frac{1}{2}$ hours, and then returns to P at the same rate. B walks from P to Q and back, without stopping, at the rate of 3 miles per hour. If both started at the same time, when and where will A overtake B on the return journey?

13. A train leaves P at 10.15 a.m. for Q, which is 90 miles distant, and travels at the rate of 60 miles per hour. At 10.42 a.m. a train, which travels at the rate of 45 miles per hour, leaves Q for P. When and where will the trains meet?

14. A starts at 8 a.m. to walk from P to Q, a distance of 30 miles. At noon he meets B, who started from Q for P at 7.30 a.m. If A reaches Q at 6 p.m., when will B reach P?

15. A man travels 60 miles in 3 hours, partly by rail and partly by coach; but if he had gone all the way by rail he would

have ended his journey an hour sooner, and have saved $\frac{2}{5}$ of the time he was on the coach; how far did he go by coach?

16. A man travels 2, 4, 6, 8, ... miles per day on successive days. In how many days will he have travelled 156 miles? How many miles does he travel on the last day?

17. A man marks his goods $33\frac{1}{3}\%$ above the cost price, but allows a discount of 10% on the marked price for cash. What is the cost price of an article for which he receives 12*s.*? What is his real gain per cent.?

18. A man walks out to a certain place at the rate of 11 miles in 4 hours and returns at the rate of $8\frac{1}{4}$ miles in 2 hours. Find his average rate of walking for the whole journey.

19. A man can row a certain distance with the stream in 2 hours; but it takes him 6 hours to row the same distance against the stream. How long would he take to row the same distance in still water?

20. A certain sum of money will amount to £69 in 5 years, and to £87 in 15 years. Find the sum of money and the rate per cent. simple interest.

21. Using the data given in Example ii page 377, find in how many years the amount of £100, at Compound Interest at 3%, will be the same as the amount of £130 at 3% Simple Interest. Estimate the amount to the nearest £.

22. The average speed of a train for a certain number of hours was 60 mi. per hr.; but, owing to an accident, only 12 miles were covered in the next hour, and the average was thus reduced to 52 mi. per hr. For how many hours had the train been travelling before the accident?

23. A cricketer, who had a batting average of 20, made 48 runs in his next innings. After this innings his average was 24. How many innings had he before the last one?

L. CUBE ROOT AND HIGHER ROOTS.

435. The cube of a number of *one* digit may consist of 1, 2 or 3 digits; for example, $2^3=8$, $3^3=27$, $5^3=125$; but each *additional* digit in a number produces *three* additional in the cube: for example, $9^3=729$ and $91^3=753571$. If then we mark off the digits of a number in periods of *three*, beginning at the right hand, just as in square root we mark them off in periods of *two*, the number of periods will be the same as the number of

digits in the cube root. The left-hand period may consist of 3 digits, or of 2, or of only 1.

The arrangement which we give below for the extraction of a cube root is called **Horner's method.** It will be sufficient for our purpose if we consider a numerical example, and briefly state the steps of the process.

Example i. *Find the cube root of* 50653.

			cube root =	3 7
1	0	0		50,653
	3	9		27
	3	9		23653
	3	18		23653
	6	2700		
	3	679		
	90	3379		
	7			
	97			

It is convenient to use four columns, beginning the first column with 1, the second with 0, the third with 0, and the fourth with the number 50653 whose cube root is required. We next mark off the digits in the fourth column in periods of three, beginning at the right.

(i) The first period to be considered is 50. The integer next less than the cube root of this is 3. We therefore place the digit 3 in the root-place as the first digit of the required cube root.

(ii) We next multiply the 1 in the first column by the root-digit 3, and add the product to the second column. The result is 3. We now multiply this 3 by the root-digit 3, and add the product to the third column. The result is 9. We lastly multiply this 9 by the root-digit 3, and subtract the product from the period 50 in the fourth column. The result is 23.

(iii) We multiply the 1 in the first column again by the root-digit 3, and add the product to the second column. The result is 6. We multiply this 6 by the root-digit 3, and add the product to the third column. The result is 27. We multiply the 1 in the first column again by the root-digit 3, and add the product to the second column. The result is 9.

(iv) After the 9 in the second column we place a 0, making 90: and after the 27 in the third column we place 00, making 2700: and to the remainder 23 in the fourth column we annex the second period 653, making 23653.

(v) The 2700 now in the third column is the *trial divisor* for the determination of the next root-digit. Dividing 23653 by 2700 the quotient is 8; but, when the second and third columns are completed by using the digit 8 in the same way as the digit 3 was used in (ii), it will be found that 8 is too great; we therefore try 7.

(vi) We now use the root-digit 7 just as we used the root-digit 3 in (ii). We multiply the 1 in the first column by 7, and add the product to the second column. The result is 97. We multiply this 97 by the root-digit

7, and add the product to the third column. The result is 3379. Lastly we multiply this 3379 by the root-digit 7, and subtract the product from the 23653 in the fourth column. There is no remainder. The process is therefore at an end, and 37 is the exact cube root required.

NOTE A.—It should be noticed particularly that when we operate with a new root-digit, we operate in the **first** stage upon all **three** columns (second, third and fourth); in the **second** stage upon only **two** (second and third) and in the **third** stage upon **one** only (second).

NOTE B.—In the preparation of the second and third columns it will be found much better to perform the various multiplications mentally, and to put down only the results of the additions (Art. 32). The written work will then appear as below. When the student has had a little practice he will find that he may safely omit the lines marked with an asterisk (*).

			cube root = 3 7
1	0	0	50,653
	3	9	27
	6	27	23653
	*90	*2700	23653
	97	3379	

Example ii. *Find the cube root of* 91125.

			cube root = 4 5
1	0	0	91,125
	4	16	64
	4	16	27125
	4	32	27125
	8	4800	
	4	625	
	120	5425	
	5		
	125		

Example iii. *Find the cube root of* 193100552.

			cube root = 5 7 8
1	0	0	193,100,552
	5	25	125
	10	75	68100
	*150	*7500	60193
	157	8599	7907552
	164	9747	7907552
	*1710	*974700	
	1718	988444	

Example iv. *Find the cube root of* 27543608.

			cube root = 3 0 2
1	0	0	27,543,608
	3	9	27
	6	2700	543608
	902	271804	543608

Example v. *Find the cube root of* 179301192791869.

			cube root = 5 6 3 8 9
1	0	0	179,301,192,791,869
	5	25	54301
	10	75	3685192
	156	8436	847645791
	162	9408	85838719869
	1683	945849	
	1686	950907	
	16898	95225884	
	16906	95361132	
	169149	9537635541	

436. The reason for the method described above is contained in the formulæ

$$(b+a)^3 = b^3 + (3b^2 + 3b \times a + a^2) \times a,$$

$$(c+b+a)^3 = c^3 + (3c^2 + 3c \times b + b^2) \times b + \{3(c+b)^2 + 3(c+b) \times a + a^2\} \times a.$$

The divisors are here of the form $3x^2 + 3x \times y + y^2$, where x is that part of the root which has already been obtained. In order to show that these divisors are obtained by the process, we will suppose that N is the number whose cube root is required, that c is the first part of the cube root and that b is the second part. The first steps in the process will then appear as below :—

1	0	0	N	$\lfloor c+b$
	c	c^2	c^3	
	$2c$	$3c^2$	M	
	$3c$	$+3cb$		
	$+b$	$+\ b^2$		

We thus see that to determine the second part b of the cube root, this process gives us $3c^2$ for a *trial divisor*, and $3c^2 + 3cb + b^2$ for the actual divisor. The process thus gives us what the formula shows to be necessary.

EXAMPLES. 192.

Find the cube root of

1.	2197.	2.	4913.	3.	6859.
4.	79507.	5.	205379.	6.	226981.
7.	571787.	8.	912673.	9.	1030301.
10.	1295029.	11.	1442897.	12.	2571353.
13.	7645373.	14.	11089567.	15.	22188041.
16.	25153757	17.	28934443.	18.	58863869.

19. 97972181. 20. 186169411. 21. 214921799.
22. 265847707. 23. 329939371. 24. 454756609.
25. 34677868581. 26. 126354865832.
27. 1790515088207. 28. 12189231526744.

Cube Root of a Decimal or of a Fraction.

437. The method of finding the cube root of an integer is of course applicable also to decimals, but one or two special points must be noticed; (i) we mark off the periods, right and left, *beginning at the decimal point*; (ii) we put a decimal point in the root immediately before using the first period in the decimal part; (iii) we annex *zeros* to the decimal part, if necessary, so that the digits may form an exact number of periods; and we may then add as many more periods of zeros as we please; (iv) if the cube root of an integer does not come out exactly, we can approximate to it by placing a decimal point to the right of the units digit and annexing zeros, and so continuing the operation.

Example i. *Find the cube root of* ·000000000008.

Marking off the periods we have ·000,000,000,008.

Since the number begins with a decimal point and three periods consisting wholly of zeros, the cube root begins with ·000. The next period gives the digit 2. Hence the complete cube root is ·0002.

Example ii. *Find the cube root of* ·000027543608.

We have already found (Art. 435, Ex. iv.) that the cube root of 27543608 is 302, hence

$$\sqrt[3]{\cdot 000{,}027{,}543{,}608} = \cdot 0302.$$

Example iii. *Find the cube root of* 1338·273208.

			cube root = 1 1 · 0 2
			1,338,·273,208
1	0	0	1
	1	1	338
	2	3	331
	30	300	7273208
	31	331	7273208
	32	363	
	330	36300	
	3300	3630000	
	3302	3636604	

We insert the point in the root immediately before the digit which arises from the use of the period 273. In all other respects we may disregard the point in the given number, after the periods have once been marked off.

438. If both the numerator and the denominator of a fraction are perfect cubes, the cube root of the fraction is a fraction which

has for its numerator the cube root of the original numerator, and for its denominator the cube root of the original denominator. Thus :—

$$\sqrt[3]{\tfrac{1331}{1728}} = \frac{\sqrt[3]{1331}}{\sqrt[3]{1728}} = \tfrac{11}{12}.$$

439. If the denominator is a perfect cube, but the numerator is not, we proceed thus :—

$$\sqrt[3]{\tfrac{3}{64}} = \frac{\sqrt[3]{3}}{\sqrt[3]{64}} = \frac{\sqrt[3]{3}}{4} = \text{etc.}$$

We now find the cube root of 3 to a sufficient number of places, and divide by 4.

If the denominator is *not* a perfect cube, as in $\frac{3}{4}$, we may proceed in one of the following ways :—

(i) $\sqrt[3]{\tfrac{3}{4}} = \sqrt[3]{\tfrac{6}{8}} = \dfrac{\sqrt[3]{6}}{2} = \text{etc.}$, or (ii) $\sqrt[3]{\tfrac{3}{4}} = \sqrt[3]{\cdot 75} = \text{etc.}$

EXAMPLES. 193.

Find the cube root of

1.	·008.	2.	·027.	3.	·064.
4.	·512.	5.	·729.	6.	1·331.
7.	1·728.	8.	4·096.	9.	9·261.
10.	12·167.	11.	474·552.	12.	551·368.
13.	1367·631.	14.	10·218313.	15.	12·812904.
16.	66·923416.	17.	206·425071.	18.	444194·947.
19.	·000405224.	20.	2·268747144.	21.	2326203·125.
22.	126732·947167.	23.	512·768384064.	24.	18·962.

Find, to 3 decimal places, the cube root of

25.	11.	26.	2·9.	27.	$\frac{9}{64}$.	28.	$\frac{11}{512}$.
29.	$\frac{4}{7}$.	30.	$\frac{3}{13}$.	31.	$\frac{213}{1728}$.	32.	$\frac{184}{4713}$.

Applications of Cube Root.

440. If the length, breadth, and height of a rectangular solid are all equal to one another, the solid is a *cube*, and its volume = (length)^3. Conversely, if a rectangular solid be a cube and its volume be known, we may determine an edge by

expressing the volume as a number of simple units, cubic feet or cubic yards for example, and finding the *cube root* of this number.

Example. *A cube contains* 1520 *cu. ft.* 1512 *cu. in.; find its edge.*

Vol. of cube $= (1520 \times 1728 + 1512)$ cu. in. $= 2628072$ cu. in. $= 2^3 \times 3^3 \times 23^3$ cu. in.,

$\therefore$ length of edge $= 2 \times 3 \times 23$ in. $= 138$ in. $= \underline{11\frac{1}{2} \text{ ft.}}$

EXAMPLES. 194.

1. A cubical tank will hold 597160402·461 cu. in. of water; what is the distance round it?

2. A pile of wood is $21\frac{1}{3}$ ft. long, 6 ft. high, and 4 ft. wide; if it were made into a cubical pile, how high would it be?

3. Find the length of the edge of a cube which contains

(i) 450 cu. ft. 1088 cu. in. (ii) 1520 cu. ft. 1512 cu. in.
(iii) 3605 cu. ft. 64 cu. in. (iv) 41063 cu. ft. 1080 cu. in.
(v) 120066 cu. yds. 10 cu. ft. (vi) $151\dot{7}0\dot{3}$ cu. yds.

4. A square block of stone, 2 ft. thick, contains 5 cu. ft. 24 cu. in.; what is the length of one side of the square?

5. A rectangular block of granite is 10 ft. 6 in. long, 8 ft. 2 in. wide, and 4 ft. deep; find the length of an edge of a cube of the same solid content.

6. A cube contains 13634·789869 cu. metres; find the area of one of its faces.

7. A cubical cistern will hold 2000 cu. ft. of water; find the length of an edge to the nearest $\frac{1}{10}$ of an inch.

8. How many inches, to three decimal places, are there in the diagonal of a cube whose edge measures one foot?

9. A cube contains 19683 c.cm. Find the length of an edge and also of a diagonal, to 3 decimal places.

10. The edges of a rectangular solid are respectively 25, 54, and 160 feet. Find, to 3 decimal places, the lengths of the edge and of the diagonal of a cube which has the same volume.

11. How many cu. feet of stone have been dug from a quarry whose length, breadth, and depth are each 15 ft. 1·5 in.? When 64 times as much stone has been obtained from the quarry, and it is still a perfect cube, what will be its depth?

12. The content of a cubical box is 103 cu. ft. 470 cu. in.; find, to the nearest penny, the cost of painting the inside at $9\frac{3}{4}d.$ per sq. ft. excluding the lid.

13. What will be the cost of lead at $6\frac{1}{4}d.$ per lb., $1\frac{1}{2}$ lb. to the sq. foot, to line an open cubical box containing $15\frac{5}{8}$ cu. ft.?

Higher Roots.

441. We can see that 5 is the *fourth* root of 5^4, and also that $5^4 = 5 \times 5 \times 5 \times 5 = 5^2 \times 5^2$. Thus 5^2 is the square root of 5^4, and 5 is the square root of 5^2. Hence 5 is the square root of the square root of 5^4: and, generally, the *fourth* root of any number is *the square root of its square root.*

Again, 5 is the *sixth* root of 5^6; and we know that

$$5^6 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^2 \times 5^2 \times 5^2;$$

hence 5^2 is the *cube* root of 5^6; also 5 is the *square* root of 5^2: hence the *6th root* of a number *is the square root of its cube root.*

Similarly, we may find any root such as the 8th, 12th, 16th and 18th, whose index is the product of 2's only, or of 3's only, or of 2's and 3's.

442. To obtain higher roots there are methods similar to those for square root and cube root, but they are not used. We can sometimes obtain a higher root by separating the number into prime factors. The matter will be considered further in the section on logarithms.

Example. *Find the sixth root of* 2985984.

$$\sqrt[6]{2985984} = \sqrt[6]{2^{12} \times 3^6} = 2^2 \times 3 = 12.$$

EXAMPLES. 195.

What is the

1. 4th root of 104976?
2. 4th root of 1500625?
3. 4th ..... $50\frac{4}{81}$?
4. 5th ..... 7776?
5. 6th ..... 2985984?
6. 6th ..... 4826809?
7. 6th ..... 11390625?
8. 8th ..... 214358881?
9. 12th ..... 13841287201?
10. 18th ..... 3814697265625?

LI. PROPERTIES OF RECURRING DECIMALS.

443. In Arts. 246 and 247 we stated and illustrated the process by which a vulgar fraction equivalent to a given recurring decimal may be obtained. We will now consider the matter in a more general form, and will first take the case of a *pure* circulator, *i.e.* a decimal in which *all* the figures recur.

We will suppose Q to denote the group of digits in the recurring period, and that the number of these digits is q. We will

also denote the value of the decimal by x. Then, in accordance with this notation,

$$x = \cdot\dot{\mathrm{Q}} = \cdot\mathrm{QQQQ}\ldots$$

Since there are q digits in the group represented by Q, it follows that if we multiply the decimal by 10^q, the effect is to move the decimal point q places to the right, *i.e.* to a position between the first Q and the second Q. Thus

$$x = \cdot\mathrm{QQQQ}\ldots, \text{ and } x \times 10^q = \mathrm{Q}\cdot\mathrm{QQQQ}\ldots.$$

Hence,
$$x \times (10^q - 1) = \mathrm{Q},$$
$$\therefore\ x = \frac{\mathrm{Q}}{10^q - 1}.$$

But 10^q is represented in the ordinary notation by 1 followed by q successive 0's, just as 10^4 is represented by 1 followed by 4 0's; therefore $10^q - 1$ will be a number represented by q successive 9's.

Hence we get the common rule (Art. 246) that a *pure* circulator is equivalent to a vulgar fraction, which has, for its numerator, the number formed by the digits of the period, and, for its denominator, the number formed by as many 9's as there are digits in the period.

444. We will next consider the case of a *mixed* recurring decimal, and we will suppose P to denote the group of digits in the non-recurring part, and Q the group of digits in the recurring period. (In the decimal $\cdot 320745821$, P would stand for 3207, and Q for 45821.)

We will suppose also that p is the number of the digits represented by P, and that q is the number of the digits represented by Q. (In $\cdot 320745821$, p is 4 and q is 5.) We will also denote the value of the decimal by x.

Thus
$$x = \cdot\mathrm{P}\dot{\mathrm{Q}} = \cdot\mathrm{PQQQ}\ldots.$$

Since there are p digits in P, it follows that, if we multiply x by 10^p, we thereby move the decimal point p places to the right, *i.e.* to a position between P and the first Q. Thus

$$x \times 10^p = \mathrm{P}\cdot\mathrm{QQQ}\ldots.$$

Again, since there are p digits in P, and q digits in Q, it follows that there are $(p+q)$ digits in the number represented, in our notation, by PQ; therefore, multiplying the previous result by 10^q,

$$x \times 10^{p+q} = \mathrm{PQ}\cdot\mathrm{QQQ}\ldots;$$

$\therefore$ by subtraction, $x \times (10^{p+q} - 10^p) = \mathrm{PQ} - \mathrm{P}$,

$$i.e. \quad x = \frac{\mathrm{PQ} - \mathrm{P}}{10^{p+q} - 10^p} = \frac{\mathrm{PQ} - \mathrm{P}}{10^p(10^q - 1)}.$$

Now 10^p is represented in the ordinary notation by 1 followed by p successive 0's; and $10^q - 1$ is represented by q successive 9's. Thus the denominator of the above result is the product of two numbers, one of which is represented by q 9's, and the other by 1 followed by p 0's. The denominator is consequently a number expressed by q 9's followed by p 0's. Hence we see that a mixed recurring decimal is equivalent to a vulgar fraction, which has, for its **numerator**, the difference between the number formed by all its digits to the end of the first period and the number formed by the digits which do not recur; and, for its **denominator**, the number formed by as many nines as there are recurring digits, followed by as many zeros as there are non-recurring digits.

445. It follows from Art. 444 that, whenever a vulgar fraction can be expressed in such a form as to have for its denominator a number expressed by n successive 9's, the fraction will be equivalent to a pure circulator having n digits in its period. Now the only way in which the *form* of a fraction can be altered, without changing its value, is by multiplying or dividing both numerator and denominator by the same number; if then we can find the number, expressed by the least possible number of successive 9's, of which the original denominator is a factor, we can state the least number of digits in the period of the equivalent recurring decimal. There is no general method of solving this problem, but it can be shewn that if the denominator of a fraction be n, where n is any prime except 2 and 5, the number of figures in the period of the equivalent decimal is $n - 1$, or a factor of $n - 1$.

This rule is based upon the property of numbers that $10^{n-1} - 1$ is a multiple of n, when n is any prime except 2 and 5. It is clear that $10^{n-1} - 1$ is a number which would be represented by $n - 1$ successive 9's; but, by the theorem just stated, n is a factor of this number; hence the fraction can, by choosing a suitable multiplier for numerator and denominator, be converted into one which has for its denominator $n - 1$ successive 9's. The equivalent recurring decimal will therefore have $n - 1$ digits in its period.

446. In certain cases the period of $(n - 1)$ figures can be separated into two or more sets of figures, the digits in each set being exactly the same as those in any other set. For example,

$$\tfrac{1}{13} = \cdot\dot{0}76923076923\dot{0}, \quad \tfrac{1}{11} = \cdot\dot{0}909090909\dot{0},$$

$$\text{and } \tfrac{1}{41} = \cdot\dot{0}2439024390243902439024390243902439024390\dot{2}43\dot{9}.$$

In the first case, the set of 12 figures can be separated into 2 identical sets of 6 figures each, viz. 076923 and 076923; in the second case, the set of 10 figures can be separated into 5 identical sets of 2 figures each, viz. 09, 09, 09, 09 and 09; and in the third case, the set of 40 figures can be separated into 8 identical sets of 5 figures, each set being 02439.

Instead of the longer periods of 12, 10, and 40 figures respectively, we may therefore use the smaller periods 076923, 09, and 02439; and we may therefore write

$$\tfrac{1}{13} = \cdot\dot{0}76923\dot{}, \quad \tfrac{1}{11} = \cdot\dot{0}\dot{9}, \quad \text{and} \quad \tfrac{1}{41} = \cdot\dot{0}2439\dot{}.$$

We have shown that n must divide into a number expressed by $n-1$ successive 9's, but it may happen to divide into a number expressed by a smaller number of 9's, say m. If this should be the case, it is clear that the number expressed by m successive 9's must be a factor of the number expressed by $(n-1)$ successive 9's; hence m must be a factor of $n-1$.

447. We will next consider briefly the converse question:—what denominators will lead to pure recurring decimals with periods of 1, 2, 3, 4, ... figures?

The decimal, which has *three* figures in its period, is clearly equivalent to a vulgar fraction whose denominator is 999, *i.e.* $3 \times 3 \times 3 \times 37$. Hence the denominators to be considered are the factors 3, 3, 3 and 37, and any combination of them, but we may exclude 3 and 3×3, because we know that in these cases the periods reduce to 1 figure: the actual denominators, which produce a period of 3 figures, are therefore

3^3, 37, 3×37, $3^2 \times 37$ and $3^3 \times 37$, *i.e.* 27, 37, 111, 333 and 999.

Similarly, periods of 5 figures may result from fractions whose denominators are the factors of, or any combination of the factors of, 99999 or $3 \times 3 \times 41 \times 271$; but we must exclude the numbers 3 and 3^2, because in these cases we know that the periods reduce to one figure. The required denominators are therefore

$$41,\ 271,\ 3 \times 41,\ 9 \times 41,\ 3 \times 271,\ 9 \times 271,\ 41 \times 271,$$
$$3 \times 41 \times 271 \text{ and } 9 \times 41 \times 271,$$

i.e. 41, 271, 123, 369, 813, 2439, 11111, 33333 and 99999.

448 There is another interesting peculiarity of recurring decimals, which we may notice here, and which is called the **Complementary Law.** The law may be stated thus:—If a frac-

tion is equivalent to a pure circulator, having $2n$ digits in its period, and if the denominator be either prime or the product of primes, none of which are separately capable of giving a period of n digits, then the last n digits of the period may be found by subtracting each of the first n digits successively from 9.

The proof of this theorem belongs to the Theory of Numbers; the following examples will, however, illustrate its meaning. We know that $\frac{1}{7} = \cdot\dot{1}42857\dot{}$; and we see that 8, 5 and 7 are respectively, 9 – 1, 9 – 4 and 9 – 2. Again $\frac{2}{13} = \cdot\dot{1}53846\dot{}$, and we see that 8, 4 and 6 are respectively, 9 – 1, 9 – 5 and 9 – 3. Thus in both these cases the digits in the second half of the period can be found from those in the first half, by subtracting each of these in turn from 9.

Again it will be found that $\frac{1}{259} = \cdot\dot{0}03861\dot{}$, $\frac{1}{407} = \cdot\dot{0}02457\dot{}$ and $\frac{1}{91} = \cdot\dot{0}10989\dot{}$, each period containing 6 figures. The last decimal clearly obeys the complementary law, but the first and second do not. This is due to the fact that both the first and the second denominators, 259 and 407, have a prime factor 37, and that $\frac{1}{37}$ is a pure circulator with a period of 3 figures. The third denominator, 91, is however the product of 7 and 13, and we know that the decimals equivalent to $\frac{1}{7}$ and $\frac{1}{13}$ are *not* pure circulators of 3 figures. The conditions are therefore satisfied in the case of $\frac{1}{91}$, but not in the case either of $\frac{1}{259}$ or of $\frac{1}{407}$.

The following example illustrates the way in which the law may be used.

Example. *Convert $\frac{1}{17}$ into a recurring decimal.*

The complementary law holds in this case, since (i) 17 is prime and (ii) the number of digits in the period is even, being 16 or a factor of 16.

By actual division we find that $\frac{1}{17} = \cdot 058823529\ldots$.

Since 2, 4 and 8 are all factors of 16, the law might begin to act at the 2nd, 3rd, 5th or 9th digit. We see that it begins at the 9th digit and not before. The remaining digits can now be written down; thus

$$\frac{1}{17} = \cdot\dot{0}58823529411764\dot{7}.$$

Addition, Subtraction and Multiplication.

449. A recurring decimal may be expressed in more than one form, according to the digit with which we choose to begin the period. Thus in $\cdot\dot{5}1\dot{4}3\dot{}$, which is the same as $\cdot 5143143143143\ldots$, we may consider the period to be either 143 or 431 or 314, and we may write the decimal as either $\cdot\dot{5}1\dot{4}3\dot{}$, or $\cdot\dot{5}1\dot{4}3\dot{1}$, or $\cdot 514\dot{3}1\dot{4}$.

Example i. *Add together* $5\cdot76\dot{2}\dot{2}$, $\cdot5\dot{4}\dot{9}$, $1\cdot20\dot{4}$.

We first alter the decimals so that the periods may begin at the same, *i.e.* at the 3rd, place in all. They thus become $5\cdot762\dot{6}\dot{2}\dot{6}$, $\cdot54\dot{9}5\dot{4}$, $1\cdot20\dot{4}$.

If we now carry on each decimal to the 8th place, it will be seen that the 9th column after the decimal point will be the same as the 3rd, the 10th the same as the 4th, etc.: and again, the 15th the same as the 3rd and 9th, the 16th the same as the 4th and 10th, and so on: hence the *columns* will recur in periods of six. It follows therefore that the digits at the foot of these columns will recur; that is, the decimal part of the sum will also be periodic, and its period will consist of six digits.

$5\cdot76262626\ldots$
$\cdot54954954\ldots$
$1\cdot20444444\ldots$

It will moreover be seen that the number of columns that recur, and therefore the number of digits in the periodic part of the result, is the L.C.M. of the numbers of digits in the three given periods.

Next we notice that the sum of the 8th column must be increased by a number carried to it from the sum of the 9th. In order therefore that the 8th place in the result may be correct, we carry on each decimal to *two* places more; but the 9th and 10th places may be ignored when the addition has been performed. Adding the decimals, we find that the sum is $7\cdot51\dot{6}62025$.

$5\cdot76262626$	26
$\cdot54954954$	95
$1\cdot20444444$	44
$7\cdot51\dot{6}62025$	

Example ii. *Add together* $2\cdot546\dot{2}\dot{6}$, $3\cdot251$ *and* $\cdot34\dot{7}\dot{2}$.

We make all the periods begin at the 4th place, and proceed as in Ex. i.

$2\cdot546246246$	24
$3\cdot251$	
$\cdot347272727$	27
$6\cdot144\dot{5}18973$	

NOTE.—It must be remembered that (i) the sum of any number of recurring decimals is also a recurring decimal, and (ii) the number of digits in its period is the L.C.M. of the numbers of digits in the given periods.

A result obtained as above may sometimes be put in a simpler form. If the digits of its period are all 9's, for example, the result reduces to a decimal which terminates.

450. To subtract one recurring decimal from another we proceed on the same principle as in addition.

Example i. *Subtract* $2\cdot46\dot{2}1\dot{6}$ *from* $9\cdot2\dot{1}\dot{4}$.

We replace the 2nd decimal by $9\cdot2\dot{1}4\dot{2}$, so that its period may begin at the 2nd place.

There are 4 digits in one period, and 3 in the other: hence there will be 12 in the periodic part of the result. The result is $6\cdot7520479975925$.

$9\cdot2\dot{1}42142142142\dot{1}\dot{4}\dot{2}$	14
$2\cdot46\dot{2}1662166216\dot{2}\dot{1}\dot{6}$	62
$6\cdot7520479975925$	

451. If the nature of a recurring decimal is understood, it will be clear that if we multiply a recurring decimal by a whole number or by a terminating decimal the product is also a recurring decimal, and that its period contains the same number of digits.

Example i. *Multiply* $3\cdot1\dot{5}3\dot{2}$ *by* 7.

As in addition, we carry the decimal two places beyond the end of the period, in order to ensure the correctness of the last digit retained. The product is $22\cdot0\dot{7}2\dot{7}$.

$$\begin{array}{r|l} 3\cdot1\dot{5}3\dot{2} & 53 \\ 7 & \\ \hline 22\cdot0\dot{7}2\dot{7} & \end{array}$$

Example ii. *Multiply* $1\cdot8\dot{4}\dot{7}$ *by* $6\cdot3185$.

We multiply by the digits in succession as in Ex. i., but, instead of *writing down* what we get from the two digits beyond the period, it will be better to determine *mentally* how much must be carried;—this being the only purpose which the extra digits serve. Thus we get (*a*). We now add these lines together, but to do this correctly we extend each line by repeating the digits of its period. The whole operation will then appear as in (*b*), and the required product is $11\cdot6732\dot{6}9\dot{1}$.

(*a*)

$$\begin{array}{r|l} 1\cdot8\dot{4}\dot{7} & 47 \\ 6\cdot3185 & \\ \hline 11\cdot0\dot{8}\dot{4} & \\ \cdot554\dot{2} & \\ \cdot018\dot{4}\dot{7} & \\ \cdot014779 & \\ \cdot0009\dot{2}3\dot{7} & \\ \hline 11\cdot673\,269\dot{1} & \end{array}$$

(*b*)

$$\begin{array}{r|l} 1\cdot8\dot{4}\dot{7} & 47 \\ 6\cdot3185 & \\ \hline 11\cdot084\,8484 & 84 \\ \cdot554\,2424 & 24 \\ \cdot018\,4747 & 47 \\ \cdot014\,7797 & 97 \\ \cdot000\,9237 & 37 \\ \hline 11\cdot673\,269\dot{1} & \end{array}$$

EXAMPLES. 196.

Simplify each of the following expressions, giving the answer as a circulating decimal:

1. $\cdot\dot{1}42857\dot{1} + \cdot\dot{7}14285\dot{7}$.
2. $2\cdot\dot{1}88\dot{1} + \cdot0\dot{8} + 3\cdot\dot{1}\dot{6}$.
3. $193\cdot0\dot{9}\dot{0} + \cdot4\dot{0}7\dot{1} + 35\cdot13 + 76\cdot\dot{5}$.
4. $\cdot\dot{6}4\dot{8} + \cdot0\dot{1}\dot{6} + 3\cdot5\dot{6}$.
5. $6\cdot5\dot{6}\dot{3} + 1\cdot\dot{5}\dot{6} + \cdot0\dot{1}\dot{8}$.
6. $174\cdot\dot{1} - 143\cdot0\dot{2}\dot{9}$.
7. $36\cdot6571428\dot{1} - 23\cdot6\dot{9}\dot{1}$.
8. $10\cdot\dot{1}0\dot{1} - \cdot3\dot{5}06\dot{1}$.
9. $1\cdot08\dot{2} \times 365$.
10. $7\cdot367\dot{4} \times 560$.
11. $\cdot00\dot{7}0\dot{4} \times 834$.
12. $\cdot523\dot{8}0\dot{9} \times 7\cdot62$.
13. $\cdot0\dot{3}2147\dot{2} \times \cdot00182$.
14. $\dfrac{\cdot3\dot{7}14285}{5\cdot2}$.
15. $\dfrac{\cdot\dot{0}\dot{4}}{\cdot769230}$ of $\dfrac{91\cdot\dot{7}4\dot{7}}{5\cdot0\dot{9}\dot{7}}$.
16. $\dfrac{5\cdot1183}{\cdot0141}$ of $22\cdot\dot{2}$ of $\cdot0\dot{9}$ of $\cdot2\dot{3}\dot{4}$.
17. $\dfrac{3\cdot302083}{16\cdot510416} + \dfrac{6\cdot\dot{6} \times \cdot375}{1\cdot857142 \text{ of } \cdot538461 \text{ of } \cdot75} + \dfrac{2\cdot7\dot{7}\dot{2}}{11\cdot\dot{0}\dot{9}}$.

Convert the following fractions into circulating decimals; examine whether they do or do not obey the complementary law, and explain why:

18. $\dfrac{1}{13}$.
19. $\dfrac{1}{1111}$.
20. $\dfrac{1}{143}$.
21. $\dfrac{1}{803}$.
22. $\dfrac{1}{2329}$.
23. $\dfrac{1}{N}$ is equivalent to a pure circulator. If N be prime, find its possible values when the least numbers of figures in the period are (i) 1, (ii) 2, (iii) 3, (iv) 4, (v) 5, (vi) 6 respectively.

LII. ELEMENTARY MENSURATION.

(a) Areas of Plane Figures.

452. A **triangle** is a figure bounded by three straight lines. Any side of the triangle may be called its **base**; the perpendicular dropped upon the base from the opposite angle is called the **altitude** or **height** of the triangle; the sum of the three sides is called the **perimeter**. To find the **area** of a triangle we have two rules:—

(i) Given base and altitude:—**Multiply the base by the altitude, and divide the product by 2**: *i.e.* area $= \frac{1}{2} \times$ base $\times$ altitude.

Example i. *The base of a triangle is* 16 *feet, and its altitude is* 10 *feet; find its area.*

$$\text{Area} = \tfrac{1}{2} \times 16 \times 10 \text{ sq. ft.} = 80 \text{ sq. ft.}$$

Example ii. *The area of a triangle is* 60 *sq. inches, and the height is* 10 *inches; find the base.*

$$\text{Area} = \tfrac{1}{2} \times \text{base} \times \text{height};$$

$$\therefore \text{ base} = \frac{2 \times \text{area}}{\text{height}} = \frac{2 \times 60}{10} \text{ inches} = 12 \text{ inches.}$$

(ii) Given the three sides:—**Multiply half the perimeter by the product of the remainders obtained on subtracting each side from it, and take the square root of the result.**

If the sides be represented by a, b and c, and $s = \frac{1}{2}(a+b+c)$, the rule may be stated thus:—

$$\text{area} = \sqrt{s(s-a)(s-b)(s-c)}.$$

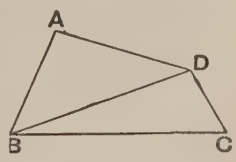
Example. *The sides of a triangle are* 13, 14 *and* 15 *feet, respectively; find the area.*

Here $a=13$, $b=14$, $c=15$; $\therefore$ $s = \frac{1}{2}(13+14+15) = 21$;

$$\therefore \ s-a=8, \ s-b=7, \text{ and } s-c=6;$$

$$\therefore \text{ Area} = \sqrt{21 \times 8 \times 7 \times 6} \text{ sq. ft.} = \sqrt{7^2 \times 3^2 \times 4^2} \text{ sq. ft.} = 84 \text{ sq. ft.}$$

453. A **quadrilateral** figure is a figure which is contained by four straight lines. The area can be found by adding together the areas of the two triangles into which the figure can be divided by a straight line joining opposite corners. Thus



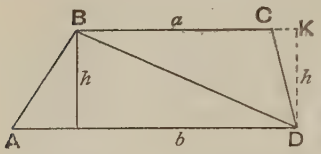
the area of ABCD
= the area of ABD + the area of BCD.

A straight line which joins two opposite corners of a quadrilateral figure is called a **diagonal**.

454. A **trapezoid** is a figure bounded by four straight lines, two of which are parallel.

The area of the trapezoid ABCD = area of ABD + area of BCD

$$= \tfrac{1}{2}h \times b + \tfrac{1}{2}h \times a$$
$$= \tfrac{1}{2}(b + a) \times h.$$



Hence, to find the area of a trapezoid, we **multiply half the sum of the parallel sides by the perpendicular distance between them.**

Example. *Find the area of a field in the shape of a trapezoid, when its parallel sides measure* 25 *chains and* 15 *chains, and the perpendicular distance between them is* 13 *chains.*

No. of *sq. chains* in area $= \frac{1}{2}(25 + 15) \times 13 = \frac{40}{2} \times 13 = 260$;

$\therefore$ area = 260 sq. chains = 26 acres.

455. The following particular cases of trapezoids may be noticed.

(i) A **parallelogram** is a figure bounded by four straight lines, such that opposite sides are equal and parallel.

Any side may be taken as **base.** The **height** is the perpendicular distance between the base and the side parallel to it.

To find the area of a parallelogram we **multiply the base by the perpendicular height.**

Example. *The base of a parallelogram is* 18 *ft., and the height is* 10 *ft.; find the area.*

Area = base × height = 18 × 10 sq. ft. = 180 sq. ft.

(ii) When the parallelogram has **all** its sides equal, but its angles are not right angles, it is called a **rhombus.**

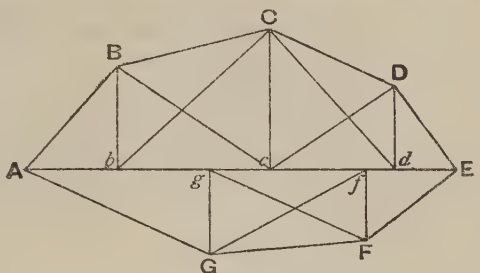
The area of a rhombus may be found by the rule given for a parallelogram, and it may also be found by taking **half the product of the diagonals.**

Example. *Find the area of a rhombus whose diagonals measure* 14 *ft. and* 16 *ft. respectively.*

Area = $\frac{1}{2}$ × 14 × 16 sq. ft. = 112 sq. ft.

456. To find the area of a figure ABCDEFG contained by four, or more, straight lines, we proceed as follows.

We join any two corners, say A and E, and we call this line AE the **base line**. From the corners B, C, D, F, G we draw Bb, Cc, Dd, Ff, Gg at right angles to the base line, and we join Bc, Cb, Cd, Dc, Fg, and Gf. From Art. 452 we know that



$\triangle Bbc = \triangle BCb$;

$\therefore$ fig. $ABCb = \triangle ABc$;

similarly, fig. $bCDc = \triangle bCd$;

$\therefore$ area of fig. $ABCDE = \triangle ABc + \triangle bCd + \triangle cDE$.

In a similar way it may be shewn that

area of fig. $AGFE = \triangle AGf + \triangle gFE$.

The perpendiculars Bb, Cc, etc., are called **offsets**.

It should be noticed that the triangle **bCd** has Cc for its altitude, and for its base the distance **bd**. This distance, being equal to Ad − Ab, is the difference between the distances of **b** and **d** from A. The base of the triangle is therefore the distance between those two offsets which are on the same side of the base line and are adjacent to the altitude of the triangle. This is true as regards each of the triangles whose sum is equal to the area of the figure considered.

Hence, if we have the measures of the distances of **b, g, c, f, d** and E from A, and of the offsets from these points to the corners of the figure, we can calculate the areas of the figures ABCDE and AGFE by the method of Art. 452. The sum of these two areas is the area of the whole figure ABCDEFG.

As an example we will suppose that the distances of **b, g, c, f, d,** E from A are 5, 8, 10, 13, 15, 18 chains, respectively, and that the offsets measured from **b, g, c, f, d** are 4, 5, 8, 3·1, 3 chains, respectively.

The measurements are usually entered in a *field-book* in the manner shewn in the margin. The numbers in the central column denote the distances of **b, g, c, f, d,** E respectively from A. The numbers to the *left* of the central column denote the lengths of the offsets from **b, c, d** to B, C, D respectively, on

	Chains.	
	To E	
	18	
D 3	15	
	13	3·1 F
C 8	10	
	8	5 G
B 4	5	
	from A	

the *left* hand side of the base line, *looking from* A *to* E. The numbers to the *right* of the central column denote the lengths of the offsets on the *right* hand side of the base line, *i.e.* from g and f to G and F respectively.

The portion of the entry which refers to the figure ABCDE is here shewn separately in the margin; from what we have said it will be seen that the triangles, whose altitudes are 4, 8 and 3 chains, have for their bases 10 – 0, 15 – 5, and 18 – 10, *i.e.* 10, 10 and 8 chains, respectively;

	Chains.
	To E
	18
D 3	15
C 8	10
B 4	5
	from A

∴ area of ABCDE

$= \frac{1}{2}(4 \times 10 + 8 \times 10 + 3 \times 8)$ sq. ch. $= \frac{1}{2}(144)$ sq. ch. $= 72$ sq. ch.

Similarly, from the portion of the entry which refers to AGFE, it will be seen that the triangles, whose altitudes are 5 and 3·1 chains, have for their bases 13 – 0 and 18 – 8, *i.e.* 13 and 10 chains, respectively;

Chains.	
To E	
18	
13	3·1 F
8	5 G
from A	

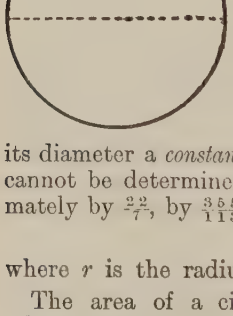
∴ area of AGFE

$= \frac{1}{2}(5 \times 13 + 3{\cdot}1 \times 10)$ sq. ch. $= \frac{1}{2}(96)$ sq. ch. $= 48$ sq. ch.;

∴ area of ABCDEFG

$= 72$ sq. ch. $+ 48$ sq. ch. $= 120$ sq. ch. $= 12$ acres.

457. A **circle** is a plane figure, bounded by one line called **the circumference**, and is such that all straight lines drawn to the circumference from a certain point called the **centre** are equal to one another.



A line drawn through the centre, and terminated at both ends by the circumference, is called a **diameter** of the circle.

A straight line drawn from the centre of a circle to the circumference is half a diameter, and is called a **radius**.

The circumference of any circle bears to its diameter a *constant ratio* denoted by the symbol π. This ratio cannot be determined exactly, but may be represented approximately by $\frac{22}{7}$, by $\frac{355}{113}$, or by 3·1416. Thus

$$\text{circumference} = 2\pi r,$$

where r is the radius of the circle.

The area of a circle may be determined by either of the following rules:—

(i) **Area** $= \pi r^2 =$ **sq. of radius** $\times 3{\cdot}1416$.

(ii) **Area** $= 2\pi r \times \frac{r}{2} =$ **circumference** $\times \frac{1}{2}$ **radius.**

Example. *The diameter of a circle is* 8 *feet; find the circumference and the area.*

Circumference $= 3{\cdot}1416 \times$ diam. $= 3{\cdot}1416 \times 8$ ft. $= 25{\cdot}1328$ ft.;
and area $= (\text{rad.})^2 \times 3{\cdot}1416 = 4^2 \times 3{\cdot}1416$ sq. ft. $= 50{\cdot}2656$ sq. ft.

EXAMPLES. 197.

1. Find the area of a triangle when the altitude is 15 feet, and the length of the base is 50 feet.

2. Find the area of a triangle whose sides are 25, 20 and 15 ft.

3. The sides of a triangle are 39, 42 and 45 yards: what will be the altitude, if the side 45 yds. long be taken as base?

4. The sides of a triangle are 10, 8 and 12 ft. If one side of a right-angled triangle of the same area be $5\sqrt{7}$ feet, what will be the length of the other side?

5. Find the area of a field which is in the form of a parallelogram, one side being 30 chains long, and the perpendicular distance between that and the opposite side being 55 chains.

6. The cost of carpeting a room in the form of a parallelogram is £8. 1*s.* 6*d.*, at the rate of 1*s.* $2\frac{1}{4}d.$ per sq. foot. If the perpendicular distance between two opposite sides of the room be 8 ft., what is the length of those sides?

7. A gardener planted 55 seedlings in a bed which was in the form of a parallelogram, allowing 4 sq. ft. for each. Find the perpendicular distance between two parallel sides, each of which measures 20 feet.

8. The diagonals of an equilateral four-sided field are 80 chains and 120 chains respectively; what will be its assessment at the rate of 10*s.* per acre?

9. The area of a rhombus is 96 sq. ft.; if one diagonal be 12 ft., what will be the length of the other?

10. A field is in the form of a trapezoid; the two parallel sides measure 130 yds. and 140 yds. respectively, and the perpendicular distance between them is 136 yds.; find the value of the field at £20 per acre.

11. A courtyard is in the form of a trapezoid. The two parallel sides are 36 yards and 24 yards, and the perpendicular

distance between them is 15 yards. Find the cost of paving it at £13. 10*s.* per 100 sq. yds.

12. In a 4-sided field, one of the diagonals measures 320 yds. and the sum of the perpendiculars on this diagonal from the opposite corners is 250 yds. How many trees will the field contain, assuming that each requires an area of 16 sq. yds.?

13. A diagonal of a 4-sided field is 132 yds. and the perpendiculars on it from the opposite corners are 81 yds. and 84 yds. respectively; find the rent of the field at 10*s.* per acre.

14. The sides AB, BC, CD, and AD of a field ABCD are respectively 20, 22, 28 and 24 feet; the distance AC is 30 ft.; find the area of the field.

15. The longest straight line which could be laid in a circular field measures 226 feet; what will be the cost of fencing such a field at the rate of 9*d.* for every 10 feet; the ratio of the circumference to the diameter being taken as $\frac{355}{113}$?

16. Find the cost of weeding a circular field whose radius is 10 chains; the cost of weeding being 1*s.* for every 2 sq. chains.

17. Find the cost of paving a circular courtyard, whose circumference is 75 feet, at the rate of 2*s.* 6*d.* per square foot.

18. How many plants will there be in a circular bed whose outer edge measures 30 in., allowing 4 sq. in. for each plant?

19. The base of a parallelogram is 100 ft. and its altitude is 25 ft. Find the side of a square of the same area.

20. Find the length of the diagonal of a square whose area is equal to that of a trapezoid; the two parallel sides of the latter being 66 yds. and 114 yds. respectively, and the perpendicular distance between them being 40 yds.

21. Find, correct to two decimal places, the side of a square which is equal in area to a triangular field whose sides are 222, 350, 352 yards respectively.

Calculate the areas of the fields whose dimensions are given as follows:

22.

Yards.		
To D		
84		
65	40 E	
C 30 52		
B 26 28		
15	20 F	
from A		

23.

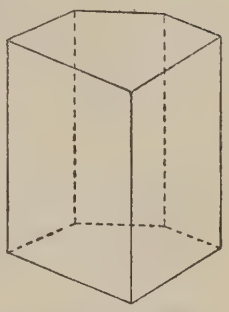
Chains.		
To E		
21		
D 9 16		
13	7 F	
C 8 10		
B 4 6	5 G	
from A		

24.

Metres.	
To G	
70	
48	36 F
42	30 E
35	42 D
32	27 C
20	15 B
from A	

(b). Volumes of certain solids.

458. We will next consider rules for determining the volume of (i) a right prism, (ii) a right cylinder, (iii) a right cone, and (iv) a sphere.



(i) A **right prism** is a solid figure bounded by plane surfaces, such that two which are opposite to one another are equal, similar and parallel, and all the others are rectangles.

Volume = area of base × height.

Example. *A prism, 2 ft. 3 in. high, stands on a triangular base; if the lengths of the sides of the base be 3 ft. 4 in., 3 ft. 3 in., and 2 ft. 1 in., find the volume of the prism.*

Sum of sides of base = 8 ft. 8 in. = 104 in.; $\therefore\ s = 52$;

$\therefore$ area of base $= \sqrt{52 \times 12 \times 13 \times 27}$ sq. in.

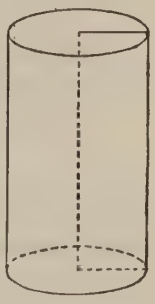
$= \sqrt{13^2 \times 2^4 \times 3^4}$ sq. in.

$= 13 \times 4 \times 9$ sq. in.

$= 468$ sq. in.;

height of prism = 27 in.;

$\therefore$ volume of prism $= 468 \times 27$ cu. in. $= 7\frac{5}{18}$ cu. ft.



(ii) A **right cylinder** is a solid figure, described by the revolution of a rectangle about one side which remains fixed. The ends of the cylinder are circles.

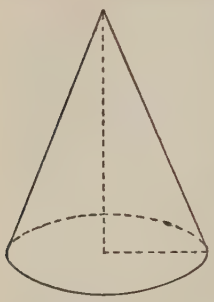
Volume = area of base × height $= \pi r^2 \times h$,

where r stands for the radius of the circular base, and h for the height of the cylinder.

Example. *Find the cubic content of a cylindrical well, 14 feet in diameter and 20 feet deep.*

Vol. of well $= \frac{22}{7} \times 7^2 \times 20$ cu. ft. $= 22 \times 7 \times 20$ cu. ft. $= 3080$ cu. ft.

(iii) A **right cone** is a solid figure described by the revolution of a right-angled triangle about one of the sides containing the right angle. The fixed straight line about which the triangle revolves is the **axis** of the cone. The base of the cone is a circle.



In the case of a **pyramid** the base may be any plane rectilineal figure. In both cases **Volume** $= \frac{1}{3} \times$ **area of base** $\times$ **height**;

$$\therefore \text{ Volume of cone} = \tfrac{1}{3} \times \pi r^2 \times h,$$

where h is the height of the cone and r is the radius of the base.

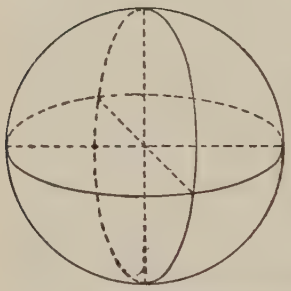
Example. *The diameter of a conical dome is* 10 *feet, and its height is* 12 *feet. How many cubic feet of air does the dome contain?*

Diameter of dome = 10 feet; $\therefore$ radius of dome = 5 feet;

$\therefore$ volume of dome $= \frac{1}{3} \times 3{\cdot}1416 \times 5^2 \times 12$ cu. feet

$= 3{\cdot}1416 \times 100$ cu. feet

$= 314{\cdot}16$ cu. feet.



(iv) A **sphere** is a solid figure described by the revolution of a semicircle about the bounding diameter.

$$\text{Volume of sphere} = \tfrac{1}{6} \times \pi d^3 = \tfrac{4}{3}\pi r^3,$$

where d and r are the lengths of diameter and radius respectively.

Example. *Find the weight of a solid ball of iron,* 6 *in. in diameter, supposing that a cu. foot of iron weighs* 450 *lb.*

Volume of ball $= \frac{4}{3} \times 3{\cdot}1416 \times 3^3$ cu. in. $= 36 \times 3{\cdot}1416$ cu. in.;

$$\therefore \text{ weight of ball } = \frac{36 \times 3{\cdot}1416}{1728} \times 450 \text{ lb.} = 29{\cdot}4525 \text{ lb.}$$

459. The volume of any solid bounded by two parallel planes can be obtained by finding the *average* area of sections parallel to these planes, and multiplying this area by the height of the solid, the height being the perpendicular distance between the bounding planes. The average area, or the area of the average section, is found by **Simpson's Rule**, which states that

Area of average section

$= \frac{1}{6}$**(sum of areas of end sections + 4 $\times$ area of mid-section).**

In all the cases which we have considered above, and in some others also, this Rule will give the volume correctly; but in the case of those solids, whose volumes cannot be found by a simple method, the result given by the Rule is only approximately correct.

It should be noticed that the area of the average section is not always that of the mid-section.

(c). Areas of the Surfaces of certain Solids.

460. We may find the surface-areas of the cylinder, cone and sphere by the following rules; the symbols used having the meanings previously assigned to them.

Area of curved surface of a cylinder

$$= \text{circumference of base} \times \text{height}$$
$$= 2\pi r \times h.$$

Area of each end $= \pi r^2$.

Area of curved surface of a cone

$$= \tfrac{1}{2} \times \text{circumference of base} \times \text{slant height}$$
$$= \tfrac{1}{2} \times 2\pi r \times l.$$

Area of base $= \pi r^2$.

Area of surface of a sphere $= \pi \times (\text{diameter})^2 = 4\pi r^2$.

A **zone** of a sphere is that portion of the surface of a sphere which is intercepted between two parallel planes. If these planes be perpendicular to the axis of a cylinder which circumscribes the sphere, the curved surfaces cut off from sphere and cylinder are equal in area. Hence,

Area of curved surface of zone of sphere $= 2\pi r \times h$,

where r is the radius of the sphere, and h is the distance between the parallel planes.

EXAMPLES. 198.

(*The value of π is to be taken as* 3·1416, *unless another value is given.*)

1. Find the volume of a prism 4 feet high, the area of the base being 15 sq. feet.

2. Find the height of a prism whose volume is 180 cu. ft., and the area of whose base is 20 sq. ft.

3. The height of a triangular prism is 9 ft. and its volume is 108 cu. ft. Find the area of the triangular base; and find the

length of a side of the base if the perpendicular dropped upon it from the opposite angle measures 4 ft.

4. The sides of the base of a triangular prism are 10, 8 and 12 inches respectively, and the height of the prism is 16 inches. What is the volume of the prism?

5. Find how many cu. ft. of earth must be dug out in order to make a well 14 ft. in diameter and 50 ft. deep. ($\pi = \frac{22}{7}$.)

6. Water is poured into a cylindrical reservoir, 20 ft. in diameter, at the rate of 400 gallons a minute; find the rate at which the water in the reservoir rises. (1 gallon = 277·25 cu. in.)

7. Find the depth of a cylindrical pit containing 4712·4 cu. ft. of air, the diameter of the pit being 20 ft.

8. Find the cost of painting the curved surface of a cylindrical vessel 2 ft. 6 in. high and $22\frac{3}{5}$ inches in diameter: the cost of painting 100 sq. inches being 2*d*. ($\pi = \frac{355}{113}$.)

9. Find the area of the complete surface of a solid cylinder 5 in. in diameter and 6 inches high.

10. Find the volume of a cone 6 ft. in perpendicular height, the diameter of the base being 4 feet.

11. Find the weight of water contained in a conical glass, 6 in. deep and 5 in. in diameter, if a cu. foot of water weighs 1000 oz.

12. Find the area of the slant surface, and of the base, of a cone 8 in. high and 6 in. in diameter. ($\pi = \frac{22}{7}$.)

13. Find the volume of a sphere 6 inches in diameter; and find the cost of painting the outside of such a sphere at the rate of 2*d*. for every 25 sq. inches.

14. Find the diameter of a sphere whose volume is $179\frac{2}{3}$ cu. inches. ($\pi = \frac{22}{7}$.)

15. A solid ball, 4 in. in diameter, weighs 2 lb.; find the weight of a ball whose diameter is 12 in.

16. The area of the base of a cylindrical vessel is 59·319 sq. inches and its height is 8 inches; what would be the height of a cube of the same volume?

17. The sides of the base of a triangular prism are 51, 40 and 13 inches, and the height is 18 inches; find the edge of a cube which is of the same volume as the prism.

18. The diameter of a sphere is 6 feet; what will be the edge of a cube of the same volume?

LIII. LOGARITHMS.

461. We propose in this section to explain briefly the use of logarithms, by means of which the operations of multiplication and division are reduced to little more than simple addition and subtraction. All calculations which depend upon such operations are thus very much simplified.

462. The **logarithm** of a number to a given **base** is the **index** of the **power** to which the base must be raised in order to produce the number.

Thus if $n = b^l$, the index l is called the *logarithm* of the number n to the *base* b. For example,

$$100 = 10^2, \quad 32 = 2^5, \quad \text{and} \quad 625 = 5^4;$$

$\therefore$ the **logarithm** of **100** to the **base 10** is **2**; or $\log_{10} 100 = 2$;

the **logarithm** of **32** to the **base 2** is **5**; or $\log_2 32 = 5$;

and the **logarithm** of **625** to the **base 5** is **4**; or $\log_5 625 = 4$.

Any number may serve as the *base* of a system of logarithms, but we are chiefly concerned with the system used in practical calculations. This has the number **10** for its base, and the logarithms are called *common* logarithms. When we are dealing with common logarithms, we need not therefore state the base each time; we may write simply log 100, log 43, log 129, etc., instead of $\log_{10} 100$, $\log_{10} 43$, $\log_{10} 129$, etc.

463. We know that $10 = 10^1$; it therefore follows from the definition that

$$\log 10 = 1.$$

Similarly, $\log 100 = 2$, $\log 1000 = 3$, $\log 10000 = 4$, etc.

464. It will be seen from the definition, that a logarithm is simply an index. Logarithms have therefore the same properties as indices and obey the same laws. The laws of indices are

$$b^m \times b^n = b^{m+n}, \quad \text{.....(i)}$$

$$b^m \div b^n = b^{m-n}, \quad \text{.....(ii)}$$

and $$(b^m)^n = b^{mn}. \quad \text{.....(iii)}$$

If now $x = b^m$ and $y = b^n$, it follows that

$$\log_b x = m \quad \text{and} \quad \log_b y = n;$$

and from the laws (i), (ii) and (iii) respectively we see that

$xy = b^m \times b^n = b^{m+n}$; hence $\log_b(xy) = m + n = \log_b x + \log_b y$,

$\frac{x}{y} = \frac{b^m}{b^n} = b^{m-n}$; hence $\log_b\left(\frac{x}{y}\right) = m - n = \log_b x - \log_b y$,

and $x^n = (b^m)^n = b^{mn}$; hence $\log_b(x^n) = nm = n\log_b x$.

The three *laws of indices* lead therefore to the following three **laws of logarithms**:—

(i) *The logarithm of a* **product** *is equal to the* **sum** *of the logarithms of the factors.*

(ii) *The logarithm of a* **quotient** *is equal to the logarithm of the dividend* **minus** *the logarithm of the divisor.*

(iii) *The logarithm of any* **power** *of a number is equal to the logarithm of the number* **multiplied** *by the index of the power.*

The following examples will shew the application of the laws.

Example i. $\log 35 = \log(7 \times 5) = \log 7 + \log 5$.

Example ii. $\log 105 = \log(3 \times 5 \times 7) = \log 3 + \log 5 + \log 7$.

Example iii. $\log 5\frac{2}{3} = \log \frac{17}{3} = \log 17 - \log 3$.

Example iv. $\log 10\frac{1}{2} = \log \frac{21}{2} = \log 21 - \log 2 = \log 3 + \log 7 - \log 2$.

Example v. $\log 81 = \log 3^4 = 4 \log 3$.

EXAMPLES. 199.

Prove the following statements:

1. $\log 15 = \log 3 + \log 5$.
2. $\log 55 = \log 5 + \log 11$.
3. $\log 2\frac{1}{3} = \log 7 - \log 3$.
4. $\log 49 = 2 \log 7$.
5. $\log 5 = \log 10 - \log 2$.
6. $\log 64 = 6 \log 2$.
7. $\log 88 = 3 \log 2 + \log 11$.
8. $\log 2\frac{2}{3} = 3 \log 2 - \log 3$.
9. $\log 33\frac{1}{3} = 2 \log 10 - \log 3$.
10. $\log 66\frac{2}{3} = 3 \log 2 + 2 \log 5 - \log 3$.

465. The meaning assigned to a^n in Art. 28, namely, that $a^n = a \times a \times a \times \ldots$ to n factors, is not intelligible unless n is an integer. The expression $2^{\frac{1}{2}}$, for example, could not mean $2 \times 2 \times \ldots$ to $\frac{1}{2}$ a factor. It is necessary therefore to decide what meanings shall now be assigned to such expressions as $a^{\frac{1}{2}}$, a^{-2}, $a^{\frac{3}{2}}$, etc., and it is agreed that their meanings shall be such that the three index laws stated in Art. 464 may still hold.

In accordance therefore with this agreement

$$a^{\frac{1}{n}} \text{ is equivalent to } \sqrt[n]{a},$$

$$a^{\frac{m}{n}} \text{ ..... } \sqrt[n]{a^m},$$

$$a^{-n} \text{ ..... } \frac{1}{a^n},$$

and $$a^0 \text{ ..... } 1.$$

Thus $2^{\frac{1}{3}} = \sqrt[3]{2}$, $2^{\frac{3}{2}} = \sqrt{2^3}$, $2^{-2} = \frac{1}{2^2}$, $2^0 = 1$.

NOTE.—Since $10^0 = 1$, it follows that **log 1 = 0.**

466. The logarithm of a number such as 53 is that value of l which makes 10^l equal to 53. The value can be calculated by algebraical processes to as many decimal places as we please, but it is not generally either a whole number or a decimal which terminates.

The index l increases as the number of which it is the logarithm increases, but the index and the number do not increase in the same proportion.

The integral part of a logarithm is called its **characteristic**, and the decimal part its **mantissa**.

467. We know that $1 = 10^0$ and $10 = 10^1$; hence for any number lying between 1 and 10, *i.e.* for any number which has **one** digit in its *integral* part, the logarithm lies between 0 and 1, and is therefore **a pure decimal.**

Again, since $10 = 10^1$ and $100 = 10^2$, it follows that for any number lying between 10 and 100, *i.e.* for any number which has **two** digits in its *integral* part, the logarithm lies between 1 and 2, and is therefore equal to **1 + a pure decimal.**

Similarly, for any number which lies between 100 and 1000, *i.e.* for any number which has **three** digits in its *integral* part, the logarithm lies between 2 and 3, and is therefore equal to **2 + a pure decimal.**

Generally, a number which has n digits in its *integral* part lies between 10^{n-1} and 10^n; its logarithm is therefore equal to **(n − 1) + a pure decimal.**

It should now be clear that the integral part, or the characteristic, of a logarithm depends only on the *number of digits* in the integral part of the number of whose logarithm it is a part, and not upon the digits themselves; and that it can be determined

at sight by the following rule:—The **characteristic** of the logarithm of a number **greater than unity** is **one less than the number of digits in the integral part** of the number. Thus

the characteristic of log 532·75 is 2,

..... log 5·3275 is 0,

and ..... log 532750 is 5.

468. If a and b be any two numbers such that a is less than b, it follows that $\frac{a}{b}$ is less than unity; and since $\log b$ is greater than $\log a$, and $\log \frac{a}{b} = \log a - \log b$, we see that $\log \frac{a}{b}$ is negative. Thus the **logarithm of a number less than unity is negative.** It is found convenient however to keep the mantissa always positive, and to attach the *minus* sign to the characteristic only. Thus instead of $-\cdot30103$, it is customary to write $\bar{1}\cdot69897$, which means $-1+\cdot69897$.

469. We know that $1 = 10^0$ and $\cdot1 = \frac{1}{10} = 10^{-1}$; hence, for any number between 1 and ·1, *i.e.* for any number which has *no zero* between the decimal point and the first significant figure, it follows that the logarithm lies between 0 and -1, and is therefore equal to **-1 + a pure decimal**, *i.e.* to **$\bar{1}$ + a pure decimal.**

Again, since $\cdot1 = \frac{1}{10} = 10^{-1}$ and $\cdot01 = \frac{1}{10^2} = 10^{-2}$, it follows that for any number between ·1 and ·01, *i.e.* for any number which has *one zero* between the decimal point and the first significant figure, the logarithm lies between -1 and -2, and is therefore equal to **$\bar{2}$ + a pure decimal.**

Generally, if there be n zeros between the decimal point and the first significant figure, the number lies between 10^{-n} and $10^{-(n+1)}$; its logarithm therefore lies between $-n$ and $-(n+1)$, and is equal to **$-(n+1)$ + a pure decimal.**

Thus, if a number be **less than unity**, the **characteristic** of its logarithm is **negative**, and is numerically equal to **one more than the number of zeros** between the decimal point and the first significant figure. For example,

the characteristic of log ·5732 is $\bar{1}$,

..... log ·00732 is $\bar{3}$,

and ..... log ·0000723 is $\bar{5}$.

470. If l be the logarithm of a number n, we know that

$$n = 10^l;$$

hence

$$n \times 10^x = 10^l \times 10^x = 10^{l+x};$$

$$\therefore \log (n \times 10^x) = l + x.$$

If, therefore, we multiply a number n by any power of 10, say 10^x, we obtain the logarithm of the product by adding x to the logarithm of n; consequently the logarithms of n and $n \times 10^x$ differ only by the integer x, and have therefore *the same mantissa*. Similarly, since

$$\log (n \div 10^x) = \log (10^l \div 10^x) = \log 10^{l-x} = l - x,$$

we see that the logarithms of n and $n \div 10^x$ differ only by the integer x, and therefore have *the same mantissa*.

But the multiplication or the division of a number by a power of 10 does not alter the significant figures of the number or their relative positions; it has the effect only of annexing 0's at the end or at the beginning of the number, or of moving the decimal point. Hence we see that *if two numbers are formed by the same digits, in the same order, their logarithms have the same mantissa*. Thus, for example, if

$$\log 32710 = 4{\cdot}5146805,$$

it follows that

$$\log 327{\cdot}1 = 2{\cdot}5146805, \quad \log 327100 = 5{\cdot}5146805,$$
$$\log {\cdot}3271 = \bar{1}{\cdot}5146805, \quad \log {\cdot}003271 = \bar{3}{\cdot}5146805.$$

EXAMPLES. **200.** *Oral.*

State the characteristics in the following cases :—

1.	$\log 25$.	**2.**	$\log 451$.	**3.**	$\log 200$.	**4.**	$\log 7234$.
5.	$\log {\cdot}5$.	**6.**	$\log 1{\cdot}37$.	**7.**	$\log 245{\cdot}3$.		
8.	$\log {\cdot}02$.	**9.**	$\log 37000$.	**10.**	$\log {\cdot}00123$.		
11.	$\log {\cdot}00032$.	**12.**	$\log 367{\cdot}542$.	**13.**	$\log 32{\cdot}567$.		
14.	$\log \frac{13}{3}$.	**15.**	$\log \frac{175}{6}$.	**16.**	$\log \frac{1}{256}$.		

471. The student will at first find some difficulty in dealing with negative characteristics. He should therefore master the method of using them before proceeding to the application of logarithms to practical work.

Ex. i. *Multiply* $\bar{5}{\cdot}6289$ *by* 3.

$$\bar{5}{\cdot}6289 \times 3 = -5 \times 3 + {\cdot}6289 \times 3$$
$$= -15 + 1{\cdot}8867$$
$$= \bar{14}{\cdot}8867.$$

Ex. ii. *Divide* $\bar{5}{\cdot}6289$ *by* 3.

$$\bar{5}{\cdot}6289 \div 3 = (-5 + {\cdot}6289) \div 3$$
$$= (-6 + 1{\cdot}6289) \div 3$$
$$= \bar{2}{\cdot}5430.$$

The intermediate steps are here written down merely to shew how the results are arrived at, but after a little practice the operations should be performed mentally.

EXAMPLES. 201.

Add together

1. $\bar{3}\cdot8980$, $\bar{5}\cdot6785$, $\bar{7}\cdot9105$.
2. $\bar{1}\cdot2085$, $\bar{4}\cdot0013$, $6\cdot1423$.

Subtract

3. $\bar{7}\cdot6901$ from $\bar{9}\cdot2036$.
4. $\bar{3}\cdot6292$ from $\bar{1}\cdot6654$.

Multiply

5. $\bar{3}\cdot2084$ by 3, 5, 10.
6. $\bar{6}\cdot9847$ by 4, 9, 48.

Divide, to 4 decimal places,

7. $\bar{6}\cdot6214$ by 7, 9, 11.
8. $\overline{19}\cdot0852$ by 10, 16, 25.

Logarithmic Tables.

472. For assistance in practical work the logarithms of all numbers of 5 digits have been calculated, and are tabulated in books of Mathematical Tables. The mantissae are given as far as a certain number of decimal places, as the case may be, but no characteristics are given, because these can always be determined by inspection.

Suppose now that we wish to find the logarithm of 486·5. We look in the Tables for the 5-figure number which has the same digits, *i.e.* the number 48650. We find assigned to it the number 6870828, which means that ·6870828 is the mantissa of the logarithm of any number composed of the digits 4, 8, 6, 5, 0, and in that order. In the case of 486·5, the characteristic is clearly 2; hence

$$\log 486\cdot5 = 2\cdot6870828.$$

473. If we require the logarithm of a number with 6 or 7 significant figures, we can calculate it by means of data obtained from the Tables. Suppose, for example, that we require the logarithm of 578·2643. From the Tables we find that to the number 57826 belongs the mantissa ·7621232, and also that the "*difference for* 1" is 75. This means that the mantissa for the next number 57827 is greater than that for 57826 by ·0000075. The process is then as follows:—

$\log 57826 = 4\cdot7621232,$

the difference for ·4 = 30, (since $75 \times \cdot4 = 30$)

and for ·03 = 2; (since $75 \times \cdot03 = 2\cdot25$)

$\therefore \log 57826\cdot43 = 4\cdot7621264;$

$\therefore \log 578\cdot2643 = 2\cdot7621264.$

In this example we have used what is called the **Principle of Proportional Parts.** This principle is that *when the change in a number is small, the increase or the decrease of the number and the increase or the decrease of the logarithm are proportional to one another.*

474. In the converse problem, the determination of the number to which a given logarithm belongs, we employ the same principle.

Suppose, for example, that we wish to ascertain the number of which $\bar{3}\cdot5775120$ is the logarithm. On referring to the tables we do not find the mantissa $\cdot5775120$ at all, but we find two successive mantissae between which it lies. We find that the mantissa next less than $\cdot5775120$ is $\cdot5775033$, which belongs to the number 37801. We find also that the "*difference for* 1" is 115, which means that the mantissa for 37802 is greater by $\cdot0000115$ than that for 37801. We see too that the given mantissa $\cdot5775120$ is greater than $\cdot5775033$ by $\cdot0000087$. The process is then as follows:—

The given mantissa $= \cdot5775120$,
and $\log 37801 = 4\cdot5775033$.
But diff. for $\cdot7 = 81$, (since $115 \times \cdot7 = 80\cdot5$)
and diff. for $\cdot05 = 6$; (since $115 \times \cdot05 = 5\cdot75$)
$\therefore \log 37801\cdot75 = 4\cdot5775120$;
$\therefore \log \cdot003780175 = \bar{3}\cdot5775120$;
$\therefore$ the required number $= \cdot003780175$.

NOTE.—We may also proceed thus:—

$115 =$ difference for 1;
$\therefore 87 = \ldots\ldots\ldots\ldots \frac{87}{115}$
$= \ldots\ldots\ldots\ldots \cdot756 \ldots$;
$\therefore$ the required number $= \cdot0037801756 \ldots$.

475. In the next examples the student will require tables of 7-figure logarithms.

EXAMPLES. 202.

Find, to 7 decimal places,

1. $\log 6\cdot80915$, 2. $\log 2\cdot100483$, 3. $\log 86301753$,
4. $\log 3\cdot141592$, 5. $\log \cdot0004329747$.

Find, to 7 decimal places, the value of x in the following cases.

6. $\log x = \cdot6870891$, 7. $\log x = 2\cdot9283642$,
8. $\log x = 1\cdot8469892$, 9. $\log x = \bar{2}\cdot3822111$.

476. In using 4-figure logarithms the principle of proportional parts is not employed, accuracy to 4 significant figures being all that is possible. The following is an extract from such a table.

LOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6 7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6

The first two figures of the number are given in the column on the extreme left; the third figure, which may be 0, 1, 2, 3, 4, 5, 6, 7, 8 or 9, is given at the head of one of the succeeding columns; the mantissa corresponding to the three figures is given in the column headed by the third figure of the number. Thus the mantissa for 583 is ·7657, and that for 567 is ·7536. In the right-hand columns are given the numbers to be **added** to the mantissa when the given number has a fourth figure; thus the mantissa for 583 is ·7657, and therefore the mantissa for 5838 is ·7657 + ·0006, *i.e.* ·7663. Similarly the mantissa for 567 is ·7536, and that for 5676 is ·7536 + ·0005, *i.e.* ·7541. The characteristics are written down by inspection as explained before.

Tables of **anti-logarithms**, *i.e.* of the numbers to which a given mantissa belongs, are also given and are arranged on the same plan.

Applications.

477. We will work a few examples using a table of 4-figure logarithms.

Example 1. *Find, to* 3 *decimal places, the value of* $\sqrt[3]{36}$.

Let $$x = \sqrt[3]{36} = (36)^{\frac{1}{3}};$$

$$\therefore \log x = \tfrac{1}{3} \log 36 = \tfrac{1}{3} \times 1{\cdot}5563 = {\cdot}5188.$$

The Tables shew that the mantissa ·5188 belongs to all numbers whose significant digits are 3, 3, 0, 2;

$$\therefore \underline{x = 3{\cdot}302.}$$

Example ii. *Find, to 4 significant figures, the value of* $\sqrt[7]{\cdot00002675}$.

Let $x=\sqrt[7]{\cdot00002675}=(\cdot00002675)^{\frac{1}{7}}$;

then $\log x=\frac{1}{7}\log\cdot00002675=\frac{1}{7}\times\bar{5}\cdot4273=\bar{1}\cdot3468$.

Hence, from the Tables, we find that

$$x=\underline{\cdot2222}.$$

Example iii. *Find the value, to 4 significant figures, of* $\dfrac{(3\cdot937)^3\times1000}{1728\times16}$.

Let $x=\dfrac{(3\cdot937)^3\times1000}{1728\times16}$,

then $\log x=3\log3\cdot937+\log1000-\log1728-\log16$

$=\cdot5952\times3+3-3\cdot2375-1\cdot2041$

$=4\cdot7856-4\cdot4416$

$=\cdot3440$.

Hence, from the Tables, we find that

$$x=\underline{2\cdot208}.$$

Example iv. *Find in how many years a sum of money lent at compound interest will double itself at 3 % per annum.*

Let $n=$number of years required,

and £P=the original sum of money.

Then $R=\frac{100+3}{100}=1\cdot03$;

$\therefore$ amount of £P at the end of n years $=$£$P(1\cdot03)^n$.

Hence $P(1\cdot03)^n=2P$;

$\therefore\quad(1\cdot03)^n=2$;

$\therefore\quad\log(1\cdot03)^n=\log2$;

$\therefore\quad n\log(1\cdot03)=\log2$;

$\therefore\quad n\times\cdot0128=\cdot3010$;

$\therefore\quad n=\frac{3010}{128}=23\cdot5$ nearly;

$\therefore$ the sum of money will double itself in about $\underline{23\frac{1}{2}\text{ years}}$.

Example v. *The edge of a cube is 4·83 ft.; find the radius of a sphere whose volume is equal to that of the cube.* $\left(\pi=\frac{355}{113}.\right)$

Let r feet $=$radius of sphere;

then $\frac{4}{3}\pi r^3=$no. of cubic feet in the volume of the sphere;

$\therefore\quad\frac{4}{3}\pi r^3=(4\cdot83)^3$;

$\therefore\quad r^3=\dfrac{3\times(4\cdot83)^3}{4\pi}=\dfrac{339\times(4\cdot83)^3}{1420}$;

$\therefore\ 3\log r=\log339+3\log4\cdot83-\log1420$

$=2\cdot5302+3\times\cdot6839-3\cdot1523$

$=4\cdot5819-3\cdot1523$

$=1\cdot4296$;

$\therefore\quad\log r=\cdot4765$;

$\therefore\quad r=2\cdot995$; $\quad\therefore$ radius $=\underline{3\text{ ft. nearly}}$.

EXAMPLES. 203.

(The answers to the examples in this set are to be found correct to 4 significant figures, and the value of π is to be taken as $\frac{3}{1}\frac{5}{1}\frac{5}{3}$.)

Find the cube root of

1. 3. 2. 7894. 3. 3·175. 4. ·003679.

Find the value of

5. $·3010 \times 2·303$. 6. $(25·4)^3$. 7. $3957 \times 3·142$.

8. $\frac{774}{1393}$. 9. $\frac{10}{·1604}$. 10. $\frac{62·25}{·00559}$.

11. $\frac{1}{2} \times 32·18 \times (7·25)^2$. 12. $\sqrt[5]{6784}$. 13. $\sqrt[11]{38·97}$.

14. $\sqrt[4]{(·03816)^3}$. 15. $\sqrt{\left(\frac{113}{355}\right)^2}$. 16. $\frac{32·18}{(3·1416)^2}$.

17. $\frac{14·7}{2·205 \times 144 \times (2·54)^2}$. 18. $\frac{355}{113} \times \sqrt{\frac{39·37}{32·2}}$.

19. $35·37 \times \{(1·025)^{15} - 1\}$. 20. $\frac{(·41)^2 \times (7·801)^5 \times \sqrt[3]{413}}{\sqrt[7]{876500} \times (1·035)^4}$.

21. The area of a sector is $\frac{1}{2}r^2\theta$. Find the area when $r = 3\frac{1}{8}$ in and $\theta = 1·047$.

22. Find the radius of a circle whose area is equal to (i) a square foot; (ii) a square whose side is 5 feet.

23. A cube contains 3165 cu. in.; find the length of its edge.

24. Find the radius of a sphere, whose volume is 3·756 c.dm.

25. Find the amount of £567 in 25 years at $2\frac{1}{2}$ % compound interest.

26. Find the true ready-money value of £150 due in 10 years at 4 % compound interest.

27. At what rate per cent. compound interest will £465 amount to £778 in 12 years?

28. In what time will a sum of money double itself at 4 % compound interest?

29. The population of a town is 25670. Assuming that for the last 30 years the annual rate of increase has remained constant and been equal to 5 per 1000, what was the population 30 years ago?

30. The metal, contained in a cube whose edge is 13·75 in. and in a sphere whose diameter is 13·75 in., is moulded into a cylindrical bar 4 ft. long. Find the radius of the cross section of the bar.

EXAMPLES. 204.

(Four-figure logarithms are to be used.)

1. Find the square of the fifth root of 8.

2. What per cent. of $(14{\cdot}86)^2$ is 43·84?

3. Find the number of digits in 37^{38}.

4. Find a fourth proportional to 13·71, 14·68 and 0·003124.

5. Find a fourth proportional to 2·754, 9·381 and 15·65.

6. Find a mean proportional to 3·715 and 27·96.

7. Find a third proportional to the square and the square root of 3·976.

8. Find the product of the cube, and the cube root, of 1·397.

9. If $r = 17{\cdot}69$ and $\pi = \frac{3}{1}\frac{5}{1}\frac{5}{3}$, find the value of πr^2.

10. The population of a town in 1901 was 12689. If the rate of increase be 20 per cent. every 10 years, what will be the population in 1951?

11. The population of a town increases yearly at the rate of 24 per 1000. It is now 427893. What will it be 4 years hence?

12. Find the value of V, when $\mathrm{V} = \frac{4}{3}\pi r^3$, $r = 3{\cdot}715$, $\pi = \frac{3}{1}\frac{5}{1}\frac{5}{3}$.

13. If $s = \frac{1}{2}ft^2$, find f when $s = 289{\cdot}3$ and $t = 3{\cdot}875$.

14. Find the value of $\dfrac{\mathrm{M}v^2}{r}$, when $\mathrm{M} = 14$, $v = 263{\cdot}4$ and $r = 53{\cdot}47$.

Find, by logarithms, the approximate value of

15. 9^9.

16. $(0{\cdot}0329)^{\frac{4}{7}}$.

17. $(2{\cdot}718)^{3{\cdot}142}$.

18. $\sqrt[37]{37}$.

19. a^a, when $a = 3{\cdot}741$.

20. $\sqrt[3]{0{\cdot}006914} \times (1{\cdot}728)^5$.

21. $37{\cdot}67 \times (1{\cdot}055)^{17} - 37{\cdot}67$.

22. $(3{\cdot}142)^{\frac{4}{7}} \div \sqrt[5]{(197{\cdot}2)^2}$.

23. $\sqrt[5]{\dfrac{3{\cdot}142 \times (17{\cdot}36)^2 \times 9{\cdot}814}{2{\cdot}716 \div (1{\cdot}035)^7}}$.

24. $\dfrac{(81{\cdot}43)^2 \times \sqrt[5]{8{\cdot}713 \times 61{\cdot}48}}{\sqrt[3]{9{\cdot}184 \times 3{\cdot}142}}$.

25. Given $\log e = 0{\cdot}4343$, find the value of ae^c, when $a = 27{\cdot}63$ and $c = 2{\cdot}826$.

26. When $x = 17{\cdot}31$, find the value of $3x^2 - 14{\cdot}76x + 11{\cdot}21$.

27. The time, in seconds, of oscillation of a pendulum, length l cm., is $2\pi\sqrt{\frac{l}{g}}$, where $\pi = \frac{3}{1}\frac{5}{1}\frac{5}{5}$ and $g = 981{\cdot}2$. Find the time of oscillation of a pendulum 99·41 cm. long.

28. The diameter of a wrought-iron coupling bolt is given by the formula $\mathsf{D} = 0{\cdot}53 \times \sqrt{\frac{d^3}{n\mathsf{R}}}$. Find D, when $d = 3$, $n = 6$ and $\mathsf{R} = 4{\cdot}125$.

Solve the equations:

29. $e^x = 2{\cdot}718$, if $\log e = {\cdot}4343$.

30. $3{\cdot}792x = (14{\cdot}86)^2$.

31. $(1{\cdot}793)^x = 12{\cdot}41$.

32. $(1{\cdot}03)^x = 791{\cdot}5 \div 684{\cdot}6$.

33. $\sqrt[x]{39{\cdot}78} = (2{\cdot}541)^x$.

34. $\frac{x - 7{\cdot}641}{\sqrt[5]{{\cdot}01763}} = (9{\cdot}924)^2$.

35. $\log 3^{x-2} = 2{\cdot}4047$.

36. Between what powers of 10 does 2^{64} lie?

37. If the side of a cube be 8 inches, find the side of another cube exactly double the former in volume.

EXAMPLES. 205.

(Four-figure logarithms are to be used.)

Find, by logarithms, the sums to which the following will approximately amount at Compound Interest, the given rates being the rates per cent. per annum:

1. £225 in 4 years at 5 %.
2. £576 in 6 years at 4 %.
3. £836 in 5 years at 3 %.
4. £508 in 7 years at 4 %.
5. £935. 8*s.* in 6 years at $2\frac{1}{2}$ %.
6. £589. 12*s.* in 5 years at $3\frac{1}{2}$ %.
7. £356. 16*s.* in 4 years at 4 %.
8. £178. 15*s.* in 6 years at $2\frac{1}{2}$ %.
9. £306. 10*s.* in 5 years at $2\frac{1}{4}$ %.

10. £396. 5*s.* in 5 years at $4\frac{1}{2}\%$.

11. £390. 6*s.* in $4\frac{1}{2}$ years at $3\frac{1}{2}\%$.

12. £289. 10*s.* in $2\frac{1}{4}$ years at $4\frac{1}{2}\%$.

13. £251. 12*s.* in $2\frac{2}{3}$ years at $3\frac{3}{4}\%$.

14. £154. 10*s.* in 1 year at 4 % per annum, interest being added quarterly.

15. £615. 14*s.* in 4 months at 6 % per annum, interest being added monthly.

16. £366. 15*s.* in 2 years at $2\frac{1}{2}\%$ per annum for the first 6 months and $1\frac{1}{2}\%$ per annum for the remainder of the time, interest being added quarterly.

Find approximately how much, lent at Compound Interest, will amount to

17. £375 in 2 years at 5 % per annum.

18. £518. 5*s.* in 3 years at 4 % per annum.

19. £327. 10*s.* in 3 years at $2\frac{1}{2}\%$ per annum.

20. £861. 12*s.* in 3 years at $4\frac{1}{2}\%$ per annum.

21. £276. 18*s.* in $2\frac{1}{2}$ years at $2\frac{1}{2}\%$ for the first year and 3 % afterwards.

22. £513. 12*s.* in $3\frac{1}{2}$ years at $2\frac{3}{4}\%$ for the first year and $2\frac{1}{2}\%$ afterwards.

23. £278. 8*s.* in 3 years at 3 % for the first year, $3\frac{1}{2}\%$ for the second and 4 % for the third.

24. £371. 4*s.* in 1 year and 3 months at 4 % per annum, interest being added quarterly.

Find approximately the rate per cent. per annum, Compound Interest, at which

25. £375. 13*s.* 4*d.* will amount to £406. 10*s.* in 2 years.

26. £208. 10*s.* will amount to £237. 17*s.* in 3 years.

27. £243. 9*s.* will amount to £390. 5*s.* in 5 years.

28. £214. 7*s.* will amount to £228. 14*s.* in $1\frac{1}{2}$ years, interest being added half-yearly.

29. £620. 4*s.* will amount to £649. 18*s.* in $1\frac{1}{2}$ years, interest being added quarterly.

30. Find in how many years a sum of money will double itself at Compound Interest at $2\frac{1}{2}\%$ per annum.

31. Find in how many years a sum of money will treble itself at Compound Interest at 4% per annum.

32. Find in how many years a sum of money will double itself at 2% per annum, Compound Interest, interest being payable every three months.

33. A person borrowed £11000 for 2 months at 5% per annum. At the end of the time the interest was added on and the debt was renewed for another 2 months. This was continually repeated until, at the end of 2 years, the debt and the interest were paid. Find, approximately, the amount of the final payment.

NOTE.—Additional exercises in the use of logarithms may be provided from Examples 161–166, 193, 195, etc.

EXAMINATION PAPERS.

1. Express in words 90999090009; and in figures two thousand million four hundred thousand and sixty.

2. Multiply 810901 by 809101; and 1 ton 5 lb. by 449.

3. Divide 25600160001 by 159601; and £183. 12*s.* $1\frac{1}{2}d.$ by 58.

4. Reduce 28 tons 5 cwt. 1 qr. 1 lb. to pounds.

5. Express 75 cm. as the fraction of 6 Hm. 9 Dm. 3 m.

6. Divide $(3\frac{1}{7} - \frac{2}{5})$ by $(\frac{4}{49} \times 7\frac{7}{12})$; and ·00036 by ·006.

7. A field containing 18 ac. 36 per. is divided into allotments, each containing 1 ro. 14 per. How many allotments are there?

---

8. Add together—seventeen hundred and seventy, a hundred and seventy thousand and seventy, and seventeen million eight hundred and twenty-nine thousand: express the result in words.

9. Multiply 302050 by 702090, and by 99996; and divide 4·953 by ·1524.

10. A man earns 21 f. a week, and saves a fifth part of that sum every fortnight. Find how many weeks it will take him to save as much as he spends in 40 weeks.

11. The area of a county is 304639 acres. If the population be 207129, find to the nearest integer the number of people per sq. mile.

12. Find the number of quarters, bushels, etc., in 2559 pints.

13. Find the value of $(\frac{7}{55}$ of $\frac{1}{20}) \div (\frac{5}{33}$ of $\frac{2}{20})$, and of $\frac{2}{49}$ cwt. $+ \frac{3}{7}$ lb.

14. A mowed ·8675 of a field, and B ·125 of it. How much was left undone?

---

15. Reduce $\frac{4785}{12892}$ to its lowest terms.

16. Find the value of $\frac{1}{73}$ of £7. 18*s.* 2*d.*, and of $\frac{57}{244}$ of 219 f. 60 c.

17. What number multiplied by 86 will give the same product as 163 multiplied by 430?

18. Reduce $\dfrac{8}{1\frac{1}{4}}$ of 1*d.* to the decimal of £1.

19. £43. 6*s.* $1\frac{1}{2}d.$ was paid with equal numbers of pounds, shillings, pence and half-pence. How many coins of each kind were there?

20. After a length of 15·345 cm. has been cut as often as possible from a rod $12\frac{1}{4}$ m. long, what length remains?

21. What decimal multiplied by 125 will give the sum of $\frac{5}{8}$, $\frac{7}{16}$, $\frac{3}{4}$, ·09375, and 2·46?

---

22. Find the final remainder after subtracting 6734968 as many times as possible from 36082701.

23. Reduce $\frac{1569}{19356}$ and $\frac{33612}{22500}$ to their lowest terms.

24. Divide the sum of $9\frac{1}{4}$ and $3\frac{1}{5}$ by the difference between $1\frac{17}{18}$ and $4\frac{5}{8}$; and find the value of $\frac{4}{15}$ of £1. 19*s.* $8\frac{1}{4}d.$

25. Find the cost of $67\frac{3}{4}$ metres of cloth at 5 f. 40 c. per metre.

26. A lb. avoir. contains 7000 grains; how many grains are there in 2 oz. avoir.?

27. Determine the least number by which 272 must be multiplied in order to produce a number exactly divisible by 238.

28. Prize money to the amount of £192. 12*s.* was divided equally among a ship's crew, and each man's share was £2. 13*s.* 6*d.*; of how many did the crew consist?

---

29. What number is the same multiple of 5 that 148995 is of 9?

30. Find the least number that must be added to 57464 in order that the result may be exactly divisible by 429.

31. Find the least number which when divided by 9, 16, 42, 63, 14 or 72, will leave a remainder 1.

32. Add together $\frac{3}{7}$, $\frac{11}{87}$, $\frac{6}{133}$, and reduce the sum to the fraction of $\frac{5}{11}$.

33. Divide $8\frac{7}{8} - 7\frac{6}{7}$ by $12\frac{13}{14} - 11\frac{1}{4}$, and add the result to $\frac{5}{6} + \frac{9}{14} + \frac{36}{24}$.

34. How much must be taken from ·674 to leave $\frac{3}{8}$ of $\frac{5}{9}$ of $\frac{27}{45}$?

35. Find the difference between

$$\frac{3}{11} \text{ of £78. 16}s. \ 2\tfrac{1}{2}d. \text{ and £35. 14}s. \ 8\tfrac{1}{4}d. \div 1\tfrac{2}{3}.$$

---

36. Find the least number that must be added to 56438971 in order that the result may be exactly divisible by 4064.

37. Find the prime factors of 111540, 42330, 67392: and thence write down the L.C.M. of these numbers.

38. Find the G.C.M. of 8029 and 23791.

39. If 1 Km. = 1093·4 yds., what fraction of 49 miles is 7 Dm.?

40. If the product of $\frac{7}{8}$ and $2\frac{1}{4}$ be added to the sum of $21\frac{5}{7}$ and $3\frac{6}{11}$, by how much will the result differ from 100?

41. One kilogram of powder costs 6*s.*, and the charge for a gun is 4 grams; how many charges will 15*s.* worth of powder furnish?

42. How many times will a football turn round when kicked along the ground 140 yds., the ball being 30 in. round?

---

43. Find the L.C.M. of 77, 35, 165, and 1089.

44. Find the value of 23·8 Kg. of copper at 2 f. 5 c. per Kg.

45. How often must 312 be added to 4321 in order that the sum may be 49561?

46. Prize money to the amount of £2648. 18*s.* 9*d.* is to be divided equally among 357 men : find each man's share.

47. Find the value of $\frac{12}{91}$ of 13 guineas, and of $\frac{11}{47}$ of 289 m. 5 cm.

48. Divide $2+\frac{1}{2}-\frac{1}{3}$ by $2\frac{1}{3}-1\frac{1}{4}$, and shew by a diagram that $\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$.

49. A clock loses 5 seconds in every 24 minutes. At 10 p.m. on Sunday it is 19 minutes fast : when will it be at the right time exactly ?

---

50. Add together $\frac{3}{3}$, $\frac{4}{15}$, $\frac{5}{9}$, $\frac{7}{43}$; and divide the result by the difference between $\frac{2}{5}$ and $\frac{4}{15}$.

51. Find the least fraction which added to $\frac{29}{12}+\frac{15}{16}$ will make the result an integer.

52. Divide $\frac{21\frac{1}{4}}{9\frac{3}{7}}$ by $\frac{1}{2}$ of $\frac{6}{11}$, and express 510 g. as a fraction of 17 Kg.

53. Express $\frac{5}{8}1\frac{7}{25}$ and $\frac{5}{11}$ as decimals, and ·848 as a vulgar fraction.

54. A boy bought a knife for 2*s.* 6*d.*, and afterwards sold it for 9*d.* and 210 marbles. How much did he lose, supposing the marbles are worth 1*d.* per dozen ?

55. How many telegraph poles will be required for wires extending 73 mi. 3 fur. 4 ch., if the poles be 99 feet apart ?

56. Find all the whole numbers of which 222 is the L.C.M.

---

57. A debt collector collected £1. 0*s.* $1\frac{1}{2}d.$, £21. 1*s.* 6*d.*, £6. 8*s.* $0\frac{1}{4}d.$ and £1. 17*s.* $5\frac{1}{2}d.$ His pocket burst, the money was scattered and he picked up £30. 7*s.* $0\frac{1}{2}d.$ Did he lose any money ? If so, how much ?

58. A number diminished by $\frac{2}{3}$ of itself, when divided by 809, gives a quotient 327 and a remainder 456. What is the number ?

59. By what must $1550\frac{3}{16}$ be divided in order that the quotient may be $459\frac{1}{2}$?

60. Express $\frac{2}{5}\frac{7}{13}$ and $\frac{7}{7}\frac{1}{4}$ as decimals, and $\cdot\dot{4}3\dot{2}\dot{7}$ as a vulgar fraction.

61. Find to the nearest franc the cost of making a railway 27345 m. long ; the cost per kilometre being 303625 f.

62. What numbers of 4 digits each have 119 as their G.C.M., and 13923 as their L.C.M. ?

63. If 1 lb. = ·4536 Kg., find, to the nearest penny, the value of 1 ton of metal at 2*s.* 1*d.* per Kg.

---

64. Divide £146. 14*s.* $10\frac{1}{2}d.$ by 53, and 353 m. 91 cm. by 94.

65. Reduce $\frac{15}{119}$ and $\frac{18}{143}$ to a common *numerator*, and determine which is the greater.

66. Reduce $\frac{11}{12}\frac{5}{8}$ and $\frac{3}{7}$ to decimals.

67. Simplify $\frac{4}{7}+\frac{7\frac{3}{4}-6\frac{1}{2}}{2\frac{1}{3}\times 9}$.

68. A man whose weekly earnings are 17*s.* 6*d.* saves $\frac{1}{5}$ of that sum every fortnight; when will he have saved 50 guineas?

69. In any year shew that the same days of the month in March and November will fall on the same days of the week.

70. How often does the heart beat in a life of 75 years of 365 days each; supposing that the number of beats is 140 a minute during the first 3 years of life, 120 for the next 3, 100 for the next 6, 90 for the next 10, 75 for the next 28, 70 for the next 20, and 80 for the last 5?

---

71. Reduce 5 cwt. 3 qrs. 17 lb. 10 oz. to ounces, and 4037 half-pence to pounds.

72. Reduce $\cdot 02\dot{2}\dot{7}$ to a vulgar fraction.

73. Multiply ·47 by ·00432, and ·02057 by ·0039.

74. Divide 4·8 by ·0016, and 1126·08 by ·00276.

75. Find the value of 2·875 of a mile, and of an acre.

76. Find the greatest unit by means of which 23 hrs. 2 min. 36 sec. and 1 day 22 hrs. 8 min. 55 sec. can both be expressed as integers.

77. If a boy save 5 minutes altogether by coming late into school thrice a week, but gets 3 propositions to write out each time, which take him 5, 7 and 3 minutes respectively, what fraction of an hour does he lose in the week?

---

78. Find the value of $627^2 - 625^2$.

79. Add together ·00125, 14·473, 12500, ·0032; and subtract the sum from 13748·64.

80. From $\frac{1}{15} + \frac{2}{4}$ of $1\frac{1}{2}$ take $\frac{1}{12} + \cdot 3$ of $3\frac{1}{2}$.

81. Reduce £1. 14*s.* $9\frac{2}{4}d.$ to the decimal of 2 guineas, to 6 places.

82. Find the value of ·72645 of £1. 14*s.* 6*d.*, to the nearest penny.

83. Gunpowder is composed of $\frac{1}{10}$ sulphur, $\frac{3}{20}$ charcoal and $\frac{3}{4}$ nitre. How many lb. of each will be required to make $1\frac{1}{8}$ ton of powder?

84. From London to Exeter is 193 miles. How long would a man be in walking the distance at 3 miles an hour for 10 hours a day?

---

85. Multiply 56320 by 9995, in two lines, and by 125 in one line.

86. Divide £6868607. 16*s.* $4\frac{3}{4}d.$ by 781.

87. Divide $4\frac{7}{11}$ by $2\frac{3}{16}$, and ·0163917 by 3·51.

88. Reduce ·624 of 19*s.* $3\frac{1}{4}d.$ to pence, and ·21875 of 35 dol. 20 c. to cents.

89. A certain number, when divided by 323, leaves a remainder 125: what will be the remainder when the number is divided by 19?

90. Find, to the nearest square kilometre, the area of a railway 8·1 m. wide and 4372 Km. long.

91. Find, by Practice, the dividend on £4750 at 9*s.* $10\frac{3}{4}d.$ in the £.

---

92. Divide 92·3784 by ·623 to 3 places of decimals.

93. Find the value of 1·25 of £3. 6*s.* 8*d.*

94. Reduce 4 tons 3 cwt. 1 qr. 9 lb. to ounces.

95. Find by Practice the cost of 7 Hl. 85 l. at £2. 5*s.* 10*d.* per hectolitre.

96. Find the value of 5 cwt. 3 qrs. 21 lb. at £1. 5*s.* 8*d.* per cwt.

97. Find the income-tax on £356. 10*s.* at 5*d.* in the £.

98. If 69 lb. of salt cost 7*s.* $2\frac{1}{4}d.$, find the cost of 4 cwt. 1 qr. 7 lb.

---

99. How many times is £27. 15*s.* $7\frac{3}{4}d.$ contained in £1356. 6*s.* 9*d.*, and how much remains over?

100. Find, to the nearest centime, the value of 57·3456 Kg. at 3 f. 96 c. per kilogram.

101. Add together $3\frac{5}{14}$, $5\frac{4}{21}$, $\frac{11}{24}$ and $1\frac{1}{3}$ of $\frac{5}{56}$.

102. In a row of 25 houses, each house has 17 windows, each window 4 panes, and each pane measures 18 inches by 9 inches. What will be the cost of glazing all these windows at 6*d.* per sq. foot?

103. Reduce 64 f. 75 c. to the decimal of 185 f.

104. If $\frac{1}{18}$ of a certain sum of money be added to $\frac{1}{5}$ of the sum, the total is less than $\frac{1}{2}$ of the sum by 10 f. 1 c. What is the sum?

105. Find the cost of 227 tons 15 cwt. 21 lb. at £12. 13*s.* 4*d.* per ton.

---

106. Add together two million three hundred thousand and forty pence, eight hundred and eight thousand sixpences, and ten million one hundred and thirty-five thousand nine hundred and fifty shillings. Express the answer in pounds.

107. A man buys 497 sacks of potatoes for £339. 1*s.* $11\frac{3}{4}d.$ and sells 248 sacks for 17*s.* $9\frac{1}{2}d.$ per sack; the remainder he sells for 18*s.* $2\frac{1}{2}d.$ per sack. Find his total gain.

108. Find, to the nearest centime, the value of 100000 rupees in francs, 1 rupee being reckoned as worth 1*s.* 4*d.* and 25 francs as worth £1.

109. Find the cost of 16 oz. 6 dwt. 20 grs. at £3. 17*s.* 6*d.* per oz.

110. Reduce $\frac{1}{3}+\frac{3}{8}+\frac{2}{5}+\frac{5}{12}+\frac{3}{7}$ to a fraction in its lowest terms. Divide $\frac{2}{3}$ of $4\frac{1}{2}$ by $\frac{4}{13}$ of $3\frac{1}{4}$, and 12·9729 by 312·6.

111. Add together $\frac{5}{7}$ of a guinea, $\frac{2}{5}$ of a sovereign, $\frac{7}{20}$ of a crown, and $\frac{1}{4}$ of a shilling. Express the result as a fraction of $4\frac{7}{20}$ of £1.

112. If £11. 13*s.* 9*d.* be charged for paving $467\frac{1}{2}$ sq. yards, how many square yards could be paved for £61. 12*s.*?

---

113. How many guineas are there in 29274 half-crowns?

114. Two equal sums were divided, the one among 141 men and the other among a certain number of women; each man received 25*s.* and each woman 1*s.* 6*d.* less; how many women were there?

115. Find the value of a steel hammer weighing 225 tons, at the rate of $1\frac{3}{4}$ lb. for 3*d*.

116. Find, by Practice, the cost of 3329 articles at £2. 14*s*. $8\frac{1}{2}$*d*. each.

117. Reduce $\frac{1001}{1386}$ and $\frac{9555}{2695}$ to their lowest terms, and find their difference. Divide $3\frac{5}{14}+2\frac{4}{9}-4\frac{19}{21}$ by $3\frac{37}{63}$.

118. Add $\frac{21}{25}$ of $\frac{25}{49}$ of 8*s*. 2*d*. to ·0625 of £6. 13*s*. 4*d*.; and express the sum as a decimal of £2. 0*s*. 9*d*.

119. The length and breadth of a field are 4 fur. 8 ch. and 3 fur. 8 ch. 25 links respectively; find the area.

---

120. Find the G.C.M. of 149291688 and 2194920.

121. Divide £443716. 2*s*. 1*d*. into 817 equal parts.

122. Simplify $\dfrac{31\frac{1}{3}-22\frac{1}{5}}{11\frac{1}{5}-1\frac{1}{7}} \div 1\frac{49}{88}+2\frac{5}{12}$.

123. Convert $\frac{17}{37}$ and $\frac{3}{64}$ to decimals.

124. If one chain of wire weigh 99 lb., and one mile cost £27. 10*s*., how much would 8 cwt. 2 qr. 8 lb. of wire cost?

125. If 1 sq. m. = 1·196 sq. yds., find, to the nearest integer, the number of acres in a square kilometre.

126. Find the cost of 2 tons 11 cwt. 27 lb. at £3. 3*s*. 8*d*. per cwt., to the nearest penny.

---

127. The sum of £657. 9*s*. $9\frac{1}{2}$*d*. is divided among 68 persons, so that one receives 4 times as much as each of the others. Find the shares.

128. Simplify (i) $\dfrac{\frac{1}{2}\times\frac{5}{12}\times 1\frac{3}{5}}{\frac{3}{8}+\frac{1}{6}-\frac{1}{10}}$; (ii) $\dfrac{1}{2+\dfrac{1}{3+\frac{1}{5}}}$.

129. How many pieces, each ·00189 m. long, can be cut from a rod ·0976 m. long? What length remains?

130. Reduce 1 fur. 5 po. to the decimal of a mile.

131. Add together $\frac{3}{8}$ of 1*s*., $\frac{2}{7}$ of a guinea, $\frac{1}{12}$ of half-a-crown, $\frac{4}{15}$ of £1.

132. If £16. 6*s*. 3*d*. be divided among 7 men, 10 women, and 19 boys, so that a woman may receive as much as two boys, and a man as much as a boy and a woman together, how much will each man, woman, and boy receive?

133. Find, by Practice, the worth of a crop on 15 ac. 3 ro. 17 per. at £15. 13*s*. 4*d*. per acre.

---

134. If in a division sum the divisor be 8 times and the quotient 7 times the remainder, what is the dividend when the remainder is 452?

135. What must be the depth of a cistern, 8 ft. long and 4 ft. wide, which shall contain 400 gallons, if a gallon of water be supposed to weigh 10 lb., and a cubic foot of water to weigh 1000 ounces?

136. Shew that if the price in shillings of a cwt. of goods is multiplied by 3 and divided by 7, the result is the price in farthings of a lb. weight of the goods.

137. Simplify

$$\tfrac{1}{2} \text{ of } (\tfrac{1}{3}+\tfrac{1}{4}+\tfrac{1}{5})+7\times(\tfrac{1}{3\cdot5}+\tfrac{1}{3\cdot6})-\tfrac{1}{12}\div\left(\frac{11\frac{1}{2}}{15\frac{1}{3}}-\tfrac{1}{4}\right).$$

138. Divide ·0175 by 35, and deduce the value of $175\div\cdot00035$.

139. If 30 tons of coal at 20*s.* 6*d.* per ton be mixed with $22\frac{1}{2}$ tons at 24*s.* per ton, find the cost of 18 tons 6 cwt. of the mixture.

140. Find the greatest number which will divide 13956 and 14565 and leave a remainder 7 in each case.

---

141. A motor car travels for the first 20 min. at the rate of 6 Km. 126 m. per hour, then for 2 hr. 20 min. at the rate of 30 Km. per hour, and afterwards for 25 min. at the rate of 7 Km. 152 m. per hour. Find the total distance traversed by the car.

142. Divide 33·82289 by 66·4684, correct to 6 places.

143. Find the square root of 2401686049 and of 5186·160225.

144. A sum of £50. 7*s.* 5*d.* is to be divided equally among the boys of a certain school, and it is found that each boy will get a half-sovereign, a half-crown, a threepenny-piece and four pennies. How many boys are there in the school?

145. A gallon of water weighs 10 lb., and 1000 oz. of water measure one cu. foot. How many cu. feet of water will be required to fill a tank which holds 375 gallons?

146. Simplify $1\frac{4}{7}$ of $5\frac{4}{11}-1\frac{4}{2\cdot3}$ of $5\frac{4}{9}+1\frac{4}{1\cdot3}\frac{5}{}$ of $2\frac{4}{17}$.

147. Find the cost of 2 tons 11 cwt. 60 lb. of sugar at 15*s.* $5\frac{1}{2}$*d.* per cwt. What profit per lb. will be got by selling it at 8 lb. for 1*s.* $3\frac{1}{4}$*d.*?

---

148. A train is travelling at the rate of 35 miles an hour. How many feet does it travel in a second?

149. The live stock on a farm consists of a certain number of horses worth 60 guineas each, an equal number of pigs worth £2. 10*s.* each, 3 times as many cows worth £18. 10*s.* each, and 15 times as many sheep worth £1. 15*s.* each. The whole value of the live stock is £1030. 15*s.* How many are there of each kind?

150. A farmer has 1134 sheep and 630 lambs. He forms them into flocks, keeping sheep and lambs separate, and having the same number of animals in each flock. If these flocks are as large as possible, how many animals are there in each?

151. Divide $\dfrac{7\frac{1}{5}\times5\frac{1}{7}}{7\frac{1}{5}-5\frac{1}{7}}$ by $\dfrac{5\frac{1}{3}\times3\frac{1}{5}}{5\frac{1}{3}+3\frac{1}{5}}$.

152. A man leaves £5000 to his wife, and the remainder to be divided equally among his 4 children. It is found that each of his children has $\frac{1}{7}$ of the whole. How much did he leave altogether?

153. Find, by Practice, the cost of an iron bridge weighing 216 tons 12 cwt. 2 qrs. at £32. 17*s.* 2*d.* per ton.

154. A box 1·25 m. long, 85 cm. wide and 50 cm. deep, internal measurements, weighs 5 Kg. If it be filled with water, what is the total weight of the box and the water?

---

155. How many integers between 100 and 100,000 are perfect cubes?

156. If, in dividing a number by 336, the operation be performed by short division by employing the factors 6, 7, 8 in succession, and the several remainders be 1, 2, 3, what is the complete remainder?

157. If a person's income for the year 1904 be £37. 15*s.* per calendar month, and he spend at the rate of £1. 4*s.* 7½*d.* per day, how much will he have left at the end of the year?

158. If 44 yds. 1 ft. 9 in. of cloth cost £42. 7*s.* 9*d.*, what will be the cost of 74 yds. 11 in.?

159. Simplify $\frac{23}{36}\frac{76}{72}\frac{55}{85}$, and reduce 22 days 4 hrs. 35 min. 42 sec. to the fraction of 34 days 20 hrs. 56 min. 6 sec.

160. If £1 = 20·42 marks, find, to the nearest penny, the value of 1200 marks.

161. A picture frame 1 m. 5 cm. long and 74 cm. broad is made of wood 6 cm. wide. How many square centimetres of wood does it contain?

---

162. Add together $1\frac{3}{42}$, $\frac{5}{72}$, $\frac{17}{63}$, $\frac{7}{24}$, $\frac{9}{56}$; and divide ·13133 by 230, and by ·023.

163. Find the value of $\frac{3}{5}$ of 3 fur. 4 ch. 3 yds. 2 ft.

164. If £5 be divided among 6 men, 12 women and 17 boys, so that 2 men may receive as much as 5 boys, and 2 women as much as 3 boys, how much will each man, woman and boy receive?

165. Each copy of a newspaper, whose circulation is 242000 copies a day, has an area of 6 sq. feet; how many square miles will 16 weeks' (6 days a week) issue cover?

166. A grocer bought 2 tons 3 cwt. 3 qrs. of sugar for £61. 5*s.*; what would he give for 5 cwt. 1 qr. 16 lb.?

167. Find the depth of water in a tank which is 2·25 m. long, 1·34 m. wide and contains 1809 litres of water.

168. If the L.C.M. of 68, 170 and another number be 2380, and the G.C.M. of the three numbers be 17, what may the other number be?

---

169. Reduce 121 qrs. 1 bush. 1 pk. to quarts.

170. The length of a field is $357\frac{1}{2}$ metres and its area is 8 Ha. 86 a. 60 ca. Find the width.

171. Reduce to their simplest forms

(i) $\dfrac{\frac{1}{2}+\frac{1}{3}-\frac{1}{6}}{\frac{1}{2}\text{ of }\frac{1}{3}\text{ of }\frac{1}{6}}$; (ii) $\dfrac{2}{3+\dfrac{4}{5-\frac{6}{7}}}$.

172. What weight of copper is worth 723 f. 69 c., if 1 Kg. be worth 1 f. 29 c.?

173. Reduce $\frac{31}{57}$ of £2. 2*s.* 9*d.* to the vulgar fraction of £2. 2*s.* $7\frac{1}{2}d.$

174. Find the value of ·384375 of £5. 13*s.* 4*d.* to the nearest penny.

175. Find how much metal is worth £2. 2*s.* 9*d.*, having given that 2 tons 8 cwt. 3 qrs. 12 lb. is worth £1.

---

176. Multiply $4\frac{3}{14}$ of $6\frac{2}{9}$ of $\frac{25}{33}$ by $5\frac{1}{4}$ of $\frac{22}{59}$.

177. Express $2\frac{3}{16}$ of 104 lb. as the decimal of a ton.

178. What decimal of a sovereign is ·215 of £4. 16*s.*?

179. How long will a man take to walk 8 mi. 7 ch. 11 yds., if he walks at the uniform rate of 105 steps per minute and each step is 2 ft. 9 in. long?

180. Find the dividend on £8350. 12*s.* at 12*s.* 11*d.* in the £.

181. Find the cost of a rectangular field, 1 fur. 4 ch. long and 5 ch. broad, at £46. 6*s.* 8*d.* per acre.

182. If 625 ac. 1 ro. 10 per. of land cost £7878. 18*s.* 9*d.*, what is the price per acre?

---

183. Divide £9652. 6*s.* by 96, by short division; and 5 sq. ch. 404 sq. yds. 6 sq. ft. 57 sq. in. by 367.

184. The cost of 5 cwt. 2 qrs. 14 lb. of coffee is £56. 13*s.* 9*d.* If the whole is sold at 1*s.* $11\frac{1}{2}d.$ per lb., what is the total gain or loss?

185. Simplify $\dfrac{3\frac{7}{8}-4\frac{1}{3}+2\frac{1}{12}}{9\frac{1}{36}-7\frac{19}{27}}$.

186. Find the charge for fencing 2 mi. 3 fur. 170 yds. 1 ft. 6 in. of road at £671 per mile.

187. Two cogged wheels work together, there being 32 cogs on one and 36 on the other. If the former makes 128 revolutions per second, how often, during 24 hours, will the same cogs come together?

188. How many different integers besides itself and unity divide 9009 without remainder?

189. If it cost as much to feed 3 men as to feed 4 boys, and for 3 boys the cost is 19*s.* $2\frac{1}{4}d.$ per week, how much per week will it be for 51 men?

---

190. When 582167 is divided by a certain number, the quotient is 762 and the remainder 761. What is the number?

191. If $(\frac{2}{3}+\frac{1}{21})$ of an estate is worth £1003. 17*s.* 1*d.*, find the value of $(\frac{1}{3}+\frac{1}{15})$ of it.

192. If a metre be 39·37 in., find, to the nearest integer, the number of metres in 3 miles.

193. Given that Jan. 1st, 1904, was a Friday, find in what years in the last century Feb. 29th fell on a Sunday.

194. If 41 qrs. 2 bush. 3 pks. of wheat cost £86. 16*s.* 5¼*d.*, how much wheat can be bought for £136. 8*s.* 8¼*d.*?

195. The income-tax payable by the owner of 6 acres of land is equal to the rent which he receives for 605 sq. yds. What tax has he to pay on 45 acres of land let at a rent of £2. 4*s.* per acre?

196. If 2 cu. in. of mercury weigh 1 lb., and if 100 cu. in. of air weigh 31 grs., find how many miles high a column of air will be which has the same weight as, and stands on a base of the same area as, a column of mercury 29·388 in. high. (Answer correct to 3 places.)

---

197. Divide £219979. 6*s.* 4½*d.* by 841. Can a divisor or a multiplier be a concrete number?

198. In a distribution of alms each child had 1*s.* 3½*d.*, each woman twice as much, and each man thrice as much. The whole sum distributed was £930, and men, women and children were taken in equal numbers. How many persons were there in all?

199. Find the square root of $6{\cdot}24\dot{9}$, and of $15{\cdot}99\dot{9}$.

200. If $\frac{3}{5}$ of $\frac{1}{7}$ of a certain fraction lie between $\frac{9}{11}$ and $\frac{4}{5}$, between what numbers does the fraction itself lie?

201. Multiply 101·8679176 by 203·35491173, correct to 2 places.

202. Find the G.C.M. of 157 days 7 hrs. 4 min. 7 sec. and 243 days 2 hrs. 11 min. 49 sec.

203. Find the simple interest on £886. 10*s.* 9*d.* for $4\frac{1}{6}$ years at $5\frac{1}{3}$ per cent.

---

204. Simplify $1\frac{17}{154} \times 1\frac{29}{114} \times 2\frac{23}{30} \times 1\frac{29}{55}$.

205. If a cubic centimetre of air weighs ·001276 g., find, to the nearest cubic centimetre, the volume of 175·7 g.

206. From 2 cwt. take the sum of ·3125 of 6 cwt. and ·032 of 3 cwt. 2 qrs. 14 lb. 4 oz.

207. Find the value of 5 oz. 17 dwt. 12 grs. at £1. 7*s.* 6*d.* per oz.

208. A man pays £11. 13*s.* 4*d.* in income-tax, the tax being 5*d.* in the £: find his annual income.

209. By selling an article for £9. 12*s.* 6*d.* a tradesman loses $3\frac{3}{4}$ per cent.; at what price should he sell it to gain $2\frac{1}{2}$ per cent.?

210. Find, to the nearest penny, the simple interest on £481. 5*s.* for $2\frac{1}{4}$ yrs. at $3\frac{1}{4}$ %.

---

211. How many seconds were there in the 19th century?

212. Reduce $\frac{3}{20}$ of 4 guineas to the fraction of £7.

213. If $2\frac{3}{4} - 1\frac{5}{8}$ of an estate cost £440, what will $2\frac{1}{2}$ of $6\frac{5}{12}$ of it cost?

214. A labourer in charge of a horse-roller walks 99 miles, exclusive of turning, in rolling a field of 40 acres. Find the width of the roller, if he does not roll any part more than once.

215. A courtyard 25·25 m. long and 12·12 m. broad is to be paved with an exact number of square flags. How many must be used, if the flags are as large as possible? What size must each flag be?

216. If £20 gain £16 in trade in 5 months, what sum will gain £24 in 3 months at the same rate?

217. Find the simple interest on £4951. 6*s.* 8*d.* for $2\frac{1}{2}$ years at $3\frac{1}{2}$ per cent.

---

218. In 17 weeks 5 days 37 min. how many seconds are there?

219. Express 1·085 of £1. 16*s.* $4\frac{1}{2}d.$ as the decimal of $\frac{2}{3}$ of £14. 11*s.*

220. Divide $6\cdot\dot{3}\dot{0}$ by $14\cdot\dot{1}\dot{8}$; and $5\cdot7\dot{3}\dot{2}$ by $72\cdot\dot{1}6\dot{7}$, correct to 4 places.

221. What vulgar fraction of half-a-crown is equal to £$\cdot0\dot{1}\dot{2}$?

222. Find the value of 41 oz. 18 dwt. 18 grs. at £1. 10*s.* per oz.

223. A field, 150 yds. by 242 yds., is planted with potatoes in rows; the distance between every two rows is 15 inches. How many yards of potatoes are there in the field?

224. Find the simple interest on £6845. 15*s.* for $4\frac{1}{2}$ years at 5 %.

---

225. 7980 is the product of 3 consecutive integers; find their sum.

226. Find the value of ·8125 of a cwt.

227. Reduce $\frac{11}{560}$ to a decimal, and $\cdot2\dot{7}\dot{0}$ to a vulgar fraction.

228. Find the square root of 22530·01.

229. Find the cost of covering a room, 9 m. long and 6 m. 75 cm. wide, with carpet, 75 cm. wide, at 3*s.* 6*d.* per metre.

230. If £3. 10*s.* are the wages of 4 men for $6\frac{1}{2}$ days, find, to the nearest penny, the wages of 17 men for 4 weeks of $5\frac{1}{3}$ days each?

231. Find the simple interest on £451. 17*s.* 6*d.* for 4 yrs. at $2\frac{1}{4}$ %.

---

232. Simplify $4\frac{2}{9}\times6\frac{3}{7}\div\frac{2\frac{1}{2}}{7}$; and $\frac{17\frac{2}{7}}{9\frac{1}{11}}+2\frac{3}{7}\times(9\frac{1}{2}-2\frac{11}{17})-\frac{5}{1-\frac{3}{31}}$.

233. Add together $\frac{7}{9}$ of £1. 10*s.* $4\frac{1}{2}d.$, $\frac{4}{5}$ of £2. 8*s.* 4*d.*, and $\frac{3}{7}$ of half-a-guinea; and subtract the result from £5.

234. An English standard of deal contains 120 planks 12 ft. long, 9 in. wide and 3 in. thick. How many cubic feet of wood does it contain?

235. Find the square root of $\frac{5\cdot29}{7\cdot29}$ and of $20\frac{341}{841}$.

236. Find the cost of 32475 articles at £1. 13*s.* 6*d.* per dozen.

237. If 25 men can do a piece of work in 24 days, in how many days will fifteen men do it?

238. At what rate per cent. will £365 amount to £537. 9*s.* 3*d.* in 9 years at simple interest?

---

239. In a bag of coins there are equal numbers of sovereigns, half-sovereigns, crowns, half-crowns, florins, shillings, sixpences and threepenny pieces. The total value of the coins is £948. 15*s*. Find the number of each sort.

240. Simplify $3\frac{1}{3} - 2\frac{4}{5} - 4\frac{1}{20} + 343\frac{1}{30}$.

241. A bicycle wheel, 88 inches in circumference, makes 132 revolutions per minute. Express its speed in miles per hour.

242. A rent-charge is paid with $505\frac{1}{2}$ bush. of barley at 3*s*. $11\frac{1}{2}d$. per bushel, and 1455 bush. of oats at 2*s*. 9*d*. per bushel. Find its value.

243. A bowling green whose length is 10 times its breadth covers an area of 5 ac. 2 sq. ch. 260 sq. yds.; find its length.

244. If a man gain 4 % by selling articles at 10*s*. 10*d*. each, what price must he charge for 100 in order to gain £4?

245. Find the simple interest on £3713. 10*s*. for $1\frac{1}{4}$ yrs. at 7 %.

---

246. Reduce $\frac{64261}{68419}$ to its lowest terms.

247. Simplify $\frac{2\frac{5}{7}}{5} - 10\frac{5}{6} \div 11\frac{2}{3} + \frac{3}{7}$ of $3\frac{3}{4}$ of $3\frac{1}{5}$.

248. The yield of a gold mine increases from ·0043 to ·0048 per cent.; how much more gold does a ton of ore yield now than before?

249. Extract the square root of 36180225, and of ·000144.

250. Find the simple interest and amount of £225. 13*s*. 4*d*. for $2\frac{1}{2}$ years at $4\frac{3}{4}$ per cent. per annum.

251. If 27 men mow a field of 90 ac. in 7 days of 8 hours, how many men will be required to mow 200 ac. in 16 days of 10 hours?

252. In 1900-01 the national expenditure was £183,592,264. Of this the Army took £91,710,000, the Navy £29,520,000, and the Civil Services £23,500,000. Express each of these as a percentage of the whole, to the nearest integer.

---

253. Find the greatest number which is such that, when 12288, 19139 and 28200 are divided by it, the remainders are the same.

254. Reduce 2·0437 of $3\frac{1}{4}$ bushels to pints.

255. Add together ·0387 of 17 cwt. and 35·681 of 3 lb. 7 oz., and express the result in ounces.

256. Find the value of 3 cwt. 2 qrs. 20 lb. at £11. 13*s*. 4*d*. per ton.

257. If a man earns £4. 19*s*. 9*d*. in 21 days, in how many days could he earn £23. 15*s*.?

258. If 11 per cent. be lost by selling 230 yards of cloth for £85. 5*s*. 10*d*., at what price must it be sold per yard to gain 17 per cent.?

259. In what time will the simple interest on £2345 amount to £395. 14*s*. $4\frac{1}{2}d$. at $3\frac{3}{4}$ per cent?

---

260. Find the value of 34 cwt. 1 qr. $11\frac{3}{4}$ lb. at £4. 13*s.* 4*d.* per cwt.

261. A boy spent $\frac{3}{5}$ of his money, and then found that $\frac{3}{5}$ of the remainder was half-a-crown: what had he at first?

262. A beam, having a square section, is 9 ft. long, and weighs $3\frac{1}{2}$ cwt. A cubic foot of the substance of the beam weighs 32 lb. What is the thickness of the beam?

263. How much is gained or lost per cent. by buying oranges at 5 for 2*d.*, and selling half of them at 2 a penny and half at 3 a penny?

264. During 365 days 6 hrs. the Earth makes $366\frac{1}{4}$ revolutions about its axis. How long will it take to make 879 revolutions?

265. If 3 men can reap 8 acres in 5 days, working 8 hours a day, in how many days can 8 men working 12 hours a day reap 192 acres?

266. Find the simple int. on £7446. 12*s.* $2\frac{3}{4}$*d.* for $1\frac{3}{4}$ years at $4\frac{2}{7}$ %.

---

267. Find the value of $\cdot 7\dot{0}\dot{9}$ of 5 tons 3 cwt. 14 lb.

268. Simplify (i) $\dfrac{7{\cdot}5+3{\cdot}75-6{\cdot}375}{3{\cdot}75+2{\cdot}\dot{3}-4{\cdot}25}$; (ii) $\dfrac{(7{\cdot}346)^2-(2{\cdot}654)^2}{7{\cdot}346-2{\cdot}654}$.

269. If $6\frac{1}{4}$ tons of coal cost £6. 15*s.* 5*d.*, what will be the cost of 5 tons 3 cwt.?

270. If 5 men and 9 boys could do a piece of work in 17 days, in how many days could 9 men and 12 boys do it; the work of 2 men being equal to that of 3 boys?

271. A man sells goods for £11. 18*s.* $0\frac{1}{2}$*d.*, making a profit of 16 per cent.; what did he give for them?

272. Pipes A and B could fill a cistern in 3 min. and 5 min. respectively, and C could empty it in $7\frac{1}{2}$ min. In what time will the cistern be filled if A, B and C are all turned on together?

273. Find the simple interest on £2970. 16*s.* 8*d.* for 3 years 73 days at £3. 2*s.* 6*d.* per cent.

---

274. Two horses and a carriage cost together £250; one horse and the carriage cost £181. 13*s.* 7*d.*, the other horse and the carriage cost £155. 6*s.* 5*d.* What was the cost of each of the three?

275. Subtract $(3\frac{5}{7}-2\frac{2}{3}\frac{6}{5})\times\frac{\frac{5}{11}}{\frac{2}{14}}$ from $78\frac{1}{7}+23\frac{4}{11}$.

276. Find, by Practice, the cost of turning 453 ac. 3 ro. 15 per. of arable land into pasturage at £9. 17*s.* 4*d.* per acre.

277. Two ships travel due north and due east, respectively, from the same port, one at the rate of 18 Km. per hour, and the other at 13·5 Km. per hour. How far are they apart at the end of 6 hours?

278. If 10 sheep or 15 lambs can eat 40 bushels of turnips in 7 days, how long will it take 6 sheep and 18 lambs to eat 36 bushels?

279. Find the compound interest on £112. 10*s.* for 3 years at $3\frac{1}{3}$ %.

280. In 1901 the National Debt was £673,608,200. How much would be required to pay the interest on this for a year at $2\frac{3}{4}\%$? How much was saved by reducing the rate of interest to $2\frac{1}{2}\%$?

---

281. Subtract 2·8571 of an hour from ·0375 of a week, and express the result in minutes.

282. Simplify (i) $\dfrac{11\frac{1}{3}+5\frac{1}{6}-15\frac{1}{2}}{4\frac{1}{21}+7\frac{19}{91}+\frac{1}{13}-10}$; (ii) $\{\frac{1}{15}\times 1\frac{5}{7}+\frac{2}{49}\times 1\frac{2}{5}\}$ of $\dfrac{2\frac{2}{5}}{36\frac{6}{7}}$.

283. Find the square roots of 983, $\frac{2}{63}$ and $37\frac{7}{8}$, to 3 places.

284. Find the dividend on £5806. 10*s.* at 18*s.* $10\frac{1}{2}d.$ in the £.

285. A cistern is filled in $3\frac{1}{3}$ hours by a pipe, 18·75 sq. cm. in cross section, through which water flows at the rate of 10·24 Km. per hour. Find the volume of the cistern in litres.

286. A man sold a horse for 837 f. and lost 7 per cent; find the prime cost of the horse.

287. What sum lent at compound interest at 4 per cent. will amount to £2050. 10*s.* 8*d.* in 3 years?

---

288. If a cu. foot of a solid be cut into $\frac{1}{2}$-inch cubes, and if these cubes be piled up in a column, what will be the height of the column?

289. Simplify $\frac{2}{5}(\frac{3}{2}\times\frac{1}{6}+\frac{3}{8}\div\frac{3}{4})-\dfrac{1\frac{2}{5}}{6\frac{2}{3}}$ of $\frac{18}{20}+\frac{3}{7}$ of $\dfrac{6\frac{6}{11}}{3\frac{3}{3}}$.

290. A person settling his bills paid $\frac{1}{3}$ of his money to one man, $\frac{2}{5}$ of the remainder to another and $\frac{4}{7}$ of the rest to a third. If he had £3. 6*s.* remaining, what had he at first?

291. If £1 = 10 florins, 1 florin = 10 cents, 1 cent = 10 mils, find the number of cents and mils in 2*s.* 6*d.* and in 1*d.*

292. By selling at £12 per ton, 6 per cent. is gained on the outlay; what would be lost per cent. by selling at the rate of 7 lb. for 6*d.*?

293. Tithe Rent Charge is of such value that every nominal £100 is really worth only £66. 10*s.* $9\frac{1}{4}d.$ Find the amount of Tithe Rent Charge which is worth £1. 14*s.* 4*d.* less than its nominal value.

294. A tradesman, having promised me $12\frac{1}{2}\%$ reduction from a bill for £16. 16*s.* 8*d.*, takes off only 10%. By what sum am I entitled to further diminish his bill? and how much per cent. is this of the account rendered?

---

295. If when a number is divided continuously by 8, 9 and 11 the remainders are 5, 2 and 4 respectively, what would be the remainder if the same number were divided by 792?

296. In a certain town the average attendance in Board Schools was 40,514, and in Voluntary Schools 63,531. Express each of these numbers as a percentage of the whole, to two decimal places.

297. A man paid ·15 of a certain sum to one person and $\frac{15}{32}$ of the remainder to another. If he then had £38, what had he originally?

298. Find the circulating decimal which will become 2 when multiplied by $2\frac{2}{5} \div 4\frac{1}{2}$.

299. If a field of 5 ac. 1 ro. produce 28 qrs. of wheat, what is the area of a field which produces 100 qrs.?

300. A wine-merchant was selling wine so as to gain 20 %. He increased the price by 6 f. 25 c. a dozen, and his rate of profit was then $32\frac{1}{2}$%. At what rate per dozen was he selling the wine at first?

301. Find the compound interest on £2718. 15*s.* for 2 yrs. at 3 %.

---

302. A cistern full of water lost ·07 of its contents by leakage; 301 litres were then drawn off, which left the cistern half full. How many litres did it contain at first?

303. If a cu. yd. of clay make 460 bricks, each 9 in. by $4\frac{1}{2}$ in. by $2\frac{1}{2}$ in., how much does clay contract in the baking?

304. Find the sq. root of $7{\cdot}4538 - 6{\cdot}8 \div 8{\cdot}5 - 2{\cdot}03 \times 1{\cdot}17$.

305. If a man travel 108 miles in 6 days walking 6 hours a day, in how many days will he travel 273 miles walking 7 hours a day?

306. If £1 in France exchanges for 25·2 f., and 20 f. in England exchange for 15*s.* 9*d.*: what is lost in £100 by the two exchanges?

307. Divide 17871 into parts in the ratio of 13, 6, and 4.

308. What sum will amount to £1157. 12*s.* 6*d.* in 3 years at 5 per cent. compound interest?

---

309. How many yards of velvet trimming, $\frac{1}{16}$ yard wide, can be cut from $1\frac{1}{2}$ yards of velvet which is $\frac{3}{4}$ yard wide? What is the whole trimming worth at $2\frac{1}{2}$*d.* per yard?

310. The sum of 11 consecutive numbers is 2761. Find the middle number.

311. Two cubical tanks togetner contain 7666 Hl. 56 l. of water, and the edge of one is twice as long as the edge of the other. Find the edge of each.

312. Simplify $\frac{3}{5}$ of $\frac{\frac{1}{7} - \frac{1}{2} \text{ of } \frac{1}{11}}{\frac{1}{9} - \frac{1}{5} \text{ of } \frac{1}{7}}$ of $\frac{1}{2}$ of $\frac{\frac{7}{8} - \frac{2}{5} \text{ of } \frac{17}{28}}{\frac{2}{7} \text{ of } \frac{2}{7} - \frac{1}{14}}$.

313. Find the cost of 24 cwt. 3 qrs. 16 lb. 10 oz. at £2. 0*s.* 8*d.* a cwt.

314. By selling a carriage for £73. 3*s.* I should lose 5 per cent. At what price must I sell it to gain 15 per cent.?

315. Find the present value of £1680 due in $2\frac{2}{3}$ yrs. at $4\frac{1}{2}$ %.

---

316. The population of India was 190531440, of whom 139421250 were Hindoos. Find the ratio of the Hindoos to the whole.

317. Find the difference between the average length of a month in the first and second halves of 1904.

318. Simplify $\dfrac{7\frac{6}{11}}{8\frac{7}{22}}$ of $\dfrac{5\frac{1}{12}}{81\frac{4}{11}} \div \dfrac{2\frac{5}{6}}{7\frac{1}{2}}$.

319. Find the value of 128 tons 15 cwt. 3 qrs. 24 lb. of iron at £16. 6*s*. 8*d*. per ton.

320. Divide £125. 8*s*. $8\frac{1}{4}d$. into 3 parts in the ratio of 11, 13, 17.

321. Find the discount on £397. 14*s*. due in 6 mos. at 5 %.

322. A landlord's net rental, after paying 5 % for collection and 8*d*. in the £ income-tax on the remainder, is £551. Find his gross rental.

---

323. Reduce £2. 4*s*. $5\frac{1}{4}d. + \dot{3}\dot{6}$ of 12*s*. $1\frac{3}{4}d$. to the decimal of £5.

324. The sum of £17 is made up of the same number of sovereigns, half-sovereigns, half-crowns, shillings and sixpences: how many are there of each?

325. Simplify $\dfrac{4\frac{1}{7}-2\frac{1}{4}}{6\frac{1}{2}+2\frac{1}{7}} \div \dfrac{\frac{3}{7}+\frac{1}{2}}{\frac{5}{7}-\frac{1}{2}}$.

326. If income-tax is levied at the rate of 8*d*. in the £, what per cent. is this of a man's income?

327. By the reduction of income-tax from 6*d*. to 4*d*. in the £, a person saves £6. 9*s*. 2*d*. What is his income?

328. If mackerel be bought at the rate of 16 for a shilling, and sold so as to gain $33\frac{1}{3}$ %, what is the selling price of each?

329. The discount on a certain sum due in 3 yrs. is £30, and the interest on the same is £33. 12*s*. Find the sum and the rate per cent.

---

330. If the 3rd satellite of Jupiter be ·0000885 of the mass of the planet and ·02947 of the mass of the Earth, find the mass of the Earth in terms of the mass of Jupiter.

331. A person bought 374 eggs at 2 a penny and others at 3 a penny. He paid £1. 9*s*. 11*d*. altogether. How many did he buy at 3 a penny?

332. A receives $\frac{5}{12}$ of a sum of money, B receives $\frac{2}{3}$ and C the rest. C's share is 13*s*. $11\frac{1}{2}d$. Find A's share.

333. How often must I run round a square of 10 acres to run a mile?

334. Find the value of a nugget of gold weighing 47 oz. 18 dwt. 4 grs. at £3. 17*s*. 6*d*. per oz.

335. A grocer sells coffee containing 25 % of chicory at 3 f. 20 c. per Kg. What does he gain or lose per cent., if pure coffee cost 2 f. 40 c. per Kg., and chicory 80 c. per Kg.?

336. A bill for £152. 1*s*. 8*d*. is drawn on the 5th of April at 4 months, and is discounted by a banker on the 8th of May at $3\frac{1}{2}$ per cent. How much does the holder receive?

---

337. The remainder after a division is 97, the quotient is 665, and the divisor is 91 more than the sum of both; what is the dividend?

338. If it cost £9. 5*s*. 4*d*. to carpet a floor containing 278 sq. feet, how many sq. feet can be carpeted for £12. 4*s*. 8*d*.?

339. The external length, breadth and depth of an open box are 6 ft. 2 in., 3 ft. 8 in. and 2 ft. respectively, and it is made of wood 1 inch thick. Find the quantity used.

340. Two pipes can together fill a cistern in 8 min., and one of them alone in 24 min. How long would the other alone take?

341. If 12 men working 8 hrs. a day do $\frac{4}{5}$ of a piece of work in 20 days, in how many days will 15 men working 10 hrs. a day do $\frac{7}{8}$ of it?

342. Find the simple interest on £11000 for 4·4 yrs. at 3·15 %.

343. The sum of £116. 13*s*. 4*d*. is lent on March 11th, and on the 16th of October the simple interest is found to be a guinea and a half. What is the rate of interest?

---

344. On what days of the week will the 10th of February fall in the years 1960 and 2060? (In 1904 it fell on a Wednesday.)

345. If the interest on the National Debt is 18*s*. 6*d*. per second, how much is that for a year of 365 days?

346. If 25 men do a certain amount of work in 14 days of 8 hrs., in how many days will 7 men do $\frac{2}{5}$ of the work, working 10 hrs. a day?

347. Find the simple interest on £1373. 6*s*. 2½*d*. for 5 yrs. at 4 %.

348. If the true discount on £775 for one year were £25, find, to the next penny, the true discount on the same sum for half a year.

349. Find the cost of papering a room 37 ft. 3 in. long, 18 ft. 4 in. wide and 14 ft. high, with paper 2 ft. 4 in. wide, at 6*s*. per piece.

350. How much must be invested in a 3 per cent. stock at 87, to produce a net income of £295 after deducting income-tax at 4*d*. in the £?

---

351. Subtract $\frac{1}{16}\frac{5}{6}$ of £16. 2*s*. 4*d*. from ·0125 of £1626. 15*s*.; and find by what decimal the result must be multiplied to produce £1. 6*s*. 1½*d*.

352. If 56 Kg. of lead be worth 64 m. 40 pf., what would be the value of 7·6 Kg.?

353. Find the value of 1 qr. 2 bush. 2 pks. at £29. 16*s*. 6*d*. a bushel.

354. Find the compound interest on £1397. 11*s*. 3*d*. for 3 years at 4 per cent. Of what amount, due 3 years hence at 4 per cent. compound interest, is £1397. 11*s*. 3*d*. the present value?

355. A clock loses 8·5 sec. an hour when the fire is alight, and gains 5·1 sec. an hour when the fire is out, but on the whole it neither loses nor gains; how long in the 24 hours is the fire burning?

356. If £500 be spent in buying 2½ per cent. consols at 99⅜, and in paying the broker's commission at the rate of 2*s*. 6*d*. per cent., what amount of stock will be bought, and what income will be obtained?

357. If the cost of carriage be 1½*d*. per ton per mile, express, to the nearest centime, this rate in centimes per metric ton per kilometre. Given 1 m. = 39·37 in., 1 Kg. = 2·2046 lb., and £1 = 25·22 f.

---

358. How many revolutions will be made by a wheel, which revolves at the rate of 151 revolutions in 3 min., while another wheel revolving 241 times in 7 min. makes 723 revolutions?

359. Divide the product of 13·68 and ·038̇ by 1·33; and multiply 3273·5620976 by 5·3698527, correct to 5 decimal places.

360. Reduce $\frac{3\frac{1}{2}}{7}$ of $\frac{99}{100}$ of $\frac{50}{451}$ of ·040$\dot{9}$ of 1 cwt. 3 qrs. 7 lb. to the decimal of $2\frac{1}{2}$ tons.

361. A creditor received 16*s.* 3*d.* in the £, and thereby lost £135. 10*s.*: how much was due to him originally?

362. In what time will £1260 amount to £1496. 5*s.* at $3\frac{3}{4}$ per cent. simple interest?

363. What sum must be invested in a $3\frac{1}{2}$ per cent. stock at 104 to produce an income of £329?

364. How long will it take a man to walk round a square field whose area is $5\frac{5}{8}$ acres, at the rate of a mile in $10\frac{2}{3}$ minutes?

---

365. Find the total amount of money paid in Income-Tax in 9 years on an annual income of £960, if the Tax is at the rate of 8*d.* in the £ the first year, and rises 2*d.* in the £ each year up to 1*s.* 8*d.*, and then remains steady.

366. Simplify 5·1̇42857̇ − 5·142857; and divide 763·3205841 by 67·4369013, correct to 3 decimal places.

367. When a regiment of 962 men is drawn up in a solid square, one man is left out; find the number of men in the face of the square.

368. Find the cost of 227 tons 7 cwt. 84 lb. of coal at 18*s.* 4*d.* per ton.

369. Find the amount of £260. 10*s.* in 3 years at $5\frac{1}{2}$ per cent. compound interest, neglecting fractions of a penny.

370. The true discount on £147 is £7. Find the interest on £147 for the same time at the same rate per cent.

371. What income is derived from £1120 of $2\frac{1}{2}$ per cent. consols?

---

372. A German 20-mark piece is worth £·979: find to the nearest farthing the value of 3725·39 marks.

373. Find, correct to 5 places, the value of $\frac{14{\cdot}52}{83{\cdot}1}+\frac{1{\cdot}575}{13{\cdot}85}$.

374. If 1 ton 13 cwt. 1 qr. 9 lb. 5 oz. cost £186. 13*s.* $3\frac{3}{4}$*d.*, what will 17 cwt. 7 oz. cost?

375. If a train goes 45 miles an hour, how many revolutions does the driving-wheel, 22 ft. in circumference, make in a second?

376. Express the rate of 45 miles per hour in metres per second, to two decimal places. Given that 1 m. = 39·37 in.

377. A watch is offered for sale for 115 f. If that price be reduced

by 5 %, the dealer will still make $9\frac{1}{4}$ % profit on his outlay. How much did the watch cost him?

378. Which yields the larger income; a $3\frac{1}{4}$ per cent. stock at $98\frac{5}{8}$, or a $3\frac{1}{2}$ per cent. stock at par?

---

379. Find the value of ·07890625 of a ton.

380. Simplify $\dfrac{\frac{1}{2}+\frac{1}{4}+\frac{1}{7}+\frac{1}{14}+\frac{1}{28}}{\frac{1}{2}+\frac{3}{4}+\frac{6}{7}+\frac{13}{14}+\frac{27}{28}}$.

381. Find, by Practice, the value of 4112 things at £1. 1*s*. $7\frac{1}{4}d$. each.

382. Find the square root of $410\frac{1}{16}$ and of ·00138384.

383. Find the present value of £324. 6*s*. $1\frac{3}{4}d$. due in $2\frac{3}{4}$ years at $3\frac{1}{2}$ %.

384. A banker gains £1. 0*s*. 8*d*. by deducting interest instead of true discount from the value of a bill due 10 months hence at 5 %. What is the amount of the bill?

385. What income does a man obtain by investing £3220 in a $3\frac{1}{2}$ per cent. stock at $80\frac{1}{2}$?

---

386. If, when a number is divided continuously by 6, 5 and 7, the remainders are found to be 2, 4 and 3 respectively, what would be the remainder if the number were divided by 210?

387. In 1881 the population of a town was 5 per cent. less than in 1871. In 1881 the population was 11628; what was it in 1871?

388. If the manufacturer makes a profit of 25 %, the agent one of 8 %, and the shopkeeper one of 20 %, what is the cost to the manufacturer of an article which is sold in the shop for £32. 8*s*.?

389. If $1004\frac{7}{13}$ cu. yds. of earth are carted $3\frac{5}{7}$ miles for £70; how far should 181 cu. yds. 10 cu. ft. 216 cu. in. be carted for the same sum?

390. Multiply $\dfrac{3\frac{1}{15}-4\frac{3}{20}+2\frac{1}{12}}{3\frac{1}{25}-2\frac{1}{10}}$ by $\dfrac{3\frac{2}{15}}{\frac{2}{3}+\frac{2}{3}-\frac{4}{48}}$.

391. A merchant insures a ship for £15320 at a premium of $4\frac{1}{4}$ % to cover the value of the ship and the cost of insurance. Find the value of the ship.

392. A person investing in a 4 per cent. stock receives $4\frac{3}{8}$ per cent. interest for his money; what is the price of the stock?

---

393. Divide £243. 2*s*. 6*d*. into 3 parts in the ratio of 3, 5, 7.

394. A game licence costs £3, and a cartridge $1\frac{1}{2}d$. A sportsman kills his bird once in 3 shots. If birds are worth 4*s*. 6*d*. a brace, how many birds must be shot just to pay expenses?

395. A person paid away $\frac{2}{3}$ of the contents of his purse, and was robbed of $\frac{2}{7}$ of the remainder. If in the end he had £3. 15*s*. left, how much had he at first?

396. If the railway fare for 470 Km. be 31 f. 60 c., find, to the nearest farthing, the fare for 76 miles at the same rate. Given that 1 Km. = 5 fur. and £1 = 25·2 f.

397. Find, to the nearest pfennig, the value of $15\frac{1}{4}$ cwt., if 20·5 Kg. cost 38 m. 35 pf. and 1 ton = 1016 Kg.

398. What income is derived from the investment of £801. 2*s.* 6*d.* in 3 per cent. stock at $94\frac{1}{4}$?

399. A litre of a certain liquid weighs 1·340 Kg. When it is mixed with water, a litre of the mixture weighs 1·270 Kg. Find the volume of each in a litre of the mixture.

---

400. Simplify $4\cdot0\dot{3}\div\cdot1\dot{4}0\dot{7}$.

401. Find the least sum of money that can be paid with an exact number of either sixpences, half-pennies, or dollars (1 dollar = 4*s.* 2*d.*).

402. How long will it take to walk round a square field of 13 ac. 81 sq. yds. at the rate of $3\frac{3}{4}$ miles an hour?

403. A cube contains 9261 cu. inches; find the cost of gilding the outside at $1\frac{1}{2}d.$ per sq. inch.

404. If 2 ro. 15 per. cost £59. 7*s.* 6*d.*, what will 8 ac. 17 per. cost?

405. If 40 shillings weigh 7 oz. 5 dwt. and contain 10 dwt. 21 grs. of alloy, how much pure silver is there in a shilling?

406. A man buys an article and sells it at a profit of 10 %. If he had bought it at 10 % less and sold it for 6*d.* less, he would have made a profit of 20 %. Find the cost price.

---

407. Express $\cdot00\dot{4}\dot{7}\dot{2}$ of an acre as a decimal of a square yard.

408. Find the square root of $\cdot05\dot{4}$ as a circulating decimal.

409. Find, to the nearest centime, the amount of 72050 f. in 3 years at 6 per cent. compound interest.

410. If a person's estate be worth 3000 guineas a year, and the land tax is 2*s.* $9\frac{1}{2}d.$ in the £, what is his annual income?

411. Divide £82. 7*s.* 3*d.* into 3 parts in the ratio of 4, $5\frac{1}{4}$, $9\frac{1}{4}$.

412. A person has £26041 of a 4 per cent. stock. He saves each year $\frac{1}{4}$ of his income, which he invests at 4 per cent. What is his income in the 4th year?

413. A, B and C rent a field for £85. 10*s.* A puts in 15 sheep for 4 months, B 24 for 5 months and C 27 for 6 months; what share of the rent should each pay?

---

414. How many articles at £3. 4*s.* $5\frac{1}{4}d.$ each cost £5647. 18*s.* $11\frac{1}{4}d.$?

415. Two pieces of cloth of the same length cost £11. 3*s.* 6*d.* and £14. 8*s.* respectively; the price of the first was 6*s.* $2\frac{1}{2}d.$ per yd.: what was the price of the second?

416. A man owns $\frac{7}{25}$ of a ship and sells $\cdot3\dot{5}71428\dot{}$ of his share: what fraction of the ship does he still own?

417. Find the value of $\frac{\cdot 093\dot{1}\dot{8}}{\cdot 56\dot{8}\dot{1}}$ of $2\frac{1}{12}$ of 2·5 days.

418. The population of a country is 3,789,560, and it decreases at the rate of 6 per 1000 per annum ; what will be the population at the end of 3 years ?

419. If 24 bushels of wheat cost £4. 1*s.* 6*d.*, how many bushels can be bought for £55. 0*s.* 3*d.* ?

420. Find how much must be invested in 4 per cent. Debenture stock at $104\frac{3}{4}$ to produce an income of £156. 18*s.*

---

421. A sum of money is divided among 3 men. If the first has $\frac{1}{7}$, the second $\frac{5}{9}$, and the third 38*s.* ; what did the money amount to ?

422. A banker has in his drawer 16 piles of sovereigns, each containing 13 ; his clerk steals them, and puts in their place 16 piles, each consisting of 12 farthings, with a sovereign at the top. How much does the banker lose ?

423. Find the greatest and least numbers of 6 digits which are exactly divisible by 789.

424. A merchant spends £1636. 5*s.* on equal quantities of wheat at 42*s.* a qr., barley at 21*s.* and oats at 14*s.* How much will he have ?

425. A rectangular grass plot 13 m. 50 cm. long and 4 m. 50 cm. broad is surrounded by a path 150 cm. wide. Find the area of the path.

426. If £1100 be laid out in the purchase of $3\frac{1}{2}$ per cent. stock at $82\frac{1}{2}$, determine the income obtained, and the price at which the stock must be sold to gain £100.

427. A man pays a poor-rate which exceeds his income-tax at 7*d.* in the £ by £22. 10*s.*, and, after paying both, his net income is £486. Find his gross income.

---

428. Reduce £4. 17*s.* $3\frac{1}{2}$*d.* to the decimal of a guinea, to 5 places.

429. Simplify $5 \div \left(6 + \frac{7}{8 + \frac{10}{11}}\right)$.

430. Find the weight of water in a dock, 24 feet deep and covering $\frac{1}{10}$ of an acre, if a cubic fathom of water weigh 6 tons.

431. If on analysis 1·95 g. of a substance afford ·984 g. of A, ·345 g. of B and ·621 g. of C ; find, to 2 decimal places, its percentage composition.

432. The rental of a parish is £1850, and the rates are taken on 80 per cent. of the rental. What is the amount of a rate at 9*d.* in the £ ?

433. A man sells a certain quantity of tea at 2*s.* per lb., thereby gaining $33\frac{1}{3}$ %. If his total gain was £14, what weight did he sell ?

434. A man invests £19040 in 5 per cent. stock at $74\frac{1}{4}$ (brok. $\frac{1}{8}$ per cent.) ; the stock rising 2 per cent. he sells out, paying the same brokerage ; what does he gain ?

---

435. If, when a number is divided continuously by 6, 7, 8, 9, 10, the remainders are 4, 2, 5, 1, 8 respectively, find the remainder when the number is divided by 30240.

436. A square field contains 15 ac. 6·25 sq. ch.; find, to the nearest foot, the length of a path crossing it diagonally.

437. A man bought £10. 13*s.* worth of tea at 3*s.* a lb. and £1. 4*s.* 4*d.* worth of coffee at 4*d.* a lb.; which was the heavier parcel?

438. A gives $\frac{1}{8}$ of his income in charity, and pays $\frac{1}{10}$ in rates and taxes. If he had £473. 15*s.* left, what was his gross income?

439. If 7 bushels of corn at 32*s.* 6*d.* a quarter will serve 3 persons for 10 weeks, find the cost of providing for 2 persons for 6 weeks?

440. If £333. 6*s.* 8*d.* be spent in buying $2\frac{1}{2}$ per cent. consols at $94\frac{1}{2}$, and in paying the broker's commission ($\frac{1}{8}$ per cent.), how much stock will be obtained and what income will be derived from it?

441. If a sovereign weighs 123·27445 grains, the gold being $\frac{11}{12}$ fine, and a 20-franc piece contains 5·8064 g. of fine gold, find, to the nearest centime, the value of £1 in francs. Given 1 grain = ·064799 g.

---

442. Simplify (i) $\frac{\cdot0038425 - \cdot00183}{\cdot035}$; (ii) $\frac{\cdot203 \times \cdot0003 \times 16}{\cdot008 \times \cdot0029}$.

443. Divide £6. 1*s.* 11*d.* into 3 shares so that the second share may be $\frac{4}{9}$ of the first, and the third $\frac{2}{5}$ of the second.

444. Find the square root of 1383·0961 and of $2\frac{2}{4}\frac{3}{9}$.

445. A gramophone with 5 rolls of music costs £7; the same gramophone with 20 rolls of music costs £9; how much should be paid for the gramophone with 50 rolls of music?

446. Find the amount of £10 in $3\frac{1}{2}$ yrs. at 4 % compound interest.

447. The true discount on a certain sum of money due $2\frac{1}{2}$ years hence at 3 per cent. is £17. 14*s.* 6*d.* What is the sum?

448. A person invests £25935 in 3 per cent. stock at 90. What income will he derive from it?

---

449. If an 18-carat gold chain weigh 2 oz. avoir., how many grains of pure gold does it contain?

450. From $2\frac{3}{4}$ of 11*s.* 8*d.* take $1\frac{1}{3}$ of 17*s.* $2\frac{1}{2}d.$, and reduce the remainder to the fraction of 13*s.* $5\frac{1}{4}d.$

451. A man, walking at the rate of 5 Km. per hour, crosses a square field diagonally in 3 minutes. Find in ares the area of the field.

452. A, B and C enter into partnership. A advances £1080 for 5 months, B £960 for 3 months and C £1120 for 6 months. They gain £992 altogether. Find the share of each.

453. Express $\frac{\frac{1}{2} \text{ of } 2\cdot179 - \frac{5}{6} \text{ of } \cdot8684}{\frac{2\frac{1}{5}}{5} - \frac{2}{5\frac{5}{8}} + \frac{1\frac{5}{6}}{4\frac{2}{5}}}$ as a decimal.

454. If $5\frac{1}{2}$ per cent. would be gained by selling 121 yds. of silk for £26. 11*s.* $10\frac{3}{4}$*d.*, at how much per yd. must it be sold to gain 12 per cent.?

455. Determine the price of $2\frac{1}{2}$ per cent. consols when £5230 stock can be bought for £4975. 0*s.* 9*d.*: find also the income obtained.

---

456. Reduce 3 pks. 1 gall. 2 qts. 1 pt. to the decimal of 8 bushels.

457. A vulgar fraction has for its numerator 209, and its nearest approximate value in thousandths is ·511: what is its denominator?

458. If a railway train travel 42 ft. in a second, find, to the nearest integer, its rate in kilometres per hour. (1 ft. = 30·47945 cm.)

459. Extract the square root of 3108169, and of $2\frac{4}{49}$.

460. Find the cost of papering a room 25 ft. long, 18 ft. 6 in. wide, and 10 ft. high, with paper 2 ft. wide at 3*s.* per piece of 12 yards.

461. How many pounds of tea at 1*s.* 6*d.* per lb. should be added to 25 lb. at 2*s.* 6*d.* per lb., so that by selling the mixture at 2*s.* 1*d.* per lb a gain of 25 % may be made?

462. A man sold £19200 of three per cent. stock at 85, and invested the proceeds in 4 % stock at 96; find the increase in his income.

---

463. Simplify $3\frac{1}{7} \times 2\frac{1}{4} - 1\frac{1}{3} \div 3\frac{3}{5} + \frac{3}{16} + 3\frac{1}{112} \times \frac{1}{27}$.

464. On what day of the week was June 20th, 1837? (In 1904 it fell on a Monday.)

465. A sum of £50000 for drainage purposes has to be raised by rates from four townships A, B, C, D. A is rated at £175002, B is rated at £82434, C is rated at £22116, and D is rated at £20448. How much in the £ must the rate be, and how much must each contribute?

466. If coffee and chicory cost £8. 10*s.* and £2. 10*s.* per cwt. respectively, what is the proportion of coffee and chicory in a mixture of which 7 lb. are worth 7*s.* 6*d.*?

467. The area of a rectangular field contains 975744 sq. ft., and one side is $3\frac{1}{2}$ times as long as the other. What is the length of each side?

468. Find the true discount on £929. 10*s.* due in $2\frac{1}{4}$ yrs. at $2\frac{1}{2}$ %.

469. £3600 of $3\frac{1}{2}$ per cent. stock is bought for £3519. What is the price of the stock, and what income will be thence derived?

---

470. Find the square root of 549081, and of $5\frac{4}{9}$.

471. At what rate per cent. simple interest must £230,398. 6*s.* 8*d.* be invested in order that it may amount to £298,941. 16*s.* 9*d.* in 7 years?

472. Shew that at 5 per cent. the interest on £650 for 3 months is equal to the true discount on £495. 12*s.* 6*d.* due in 4 months' time.

473. What income would be obtained by investing £12002. 12*s.* $4\frac{1}{2}$*d.* in 3 per cent. stock at 93?

474. After a tour in Italy a man found that he had spent daily half as many shillings as the total number of days he had been from home. His tour cost £57. 12*s.* How many days did it occupy?

475. The basin of the Severn is 4350 sq. miles in area, and the yearly rainfall is 29 in. Half the rain that falls flows to the sea. Find, to the nearest million gallons, how much water flows to the sea per hour. (1 cu. ft. = 6·23 gall.)

476. Two settlers in New Zealand own adjoining farms of 3000 and 5000 acres respectively. They unite their farms, taking at the same time an additional partner who pays them £8000, on the understanding that $\frac{1}{3}$ of the land shall in future belong to each. How is the £8000 to be divided between the original owners?

---

477. A person, paid at the rate of 8*s.* 4*d.* in the £, received £140. 16*s.* 8*d.*; what was the whole debt due to him?

478. Express in grams per sq. cm., to the nearest integer, a pressure of 14 lb. to the sq. in. Given 1 Kg. = 2·2046 lb., 1 inch = 2·54 cm.

479. After paying income-tax at 5*d.* in the £, a man had £606. 11*s.* $10\frac{1}{2}$*d.* left. What was his gross income, and what will he have left if the tax is raised to 6*d.* in the £?

480. Find the cost of papering a room 30 ft. 4 in. long, 20 ft. 2 in. broad, and 10 ft. high, with paper 3 ft. 4 in. wide at 7*s.* 6*d.* per piece.

481. If sugar can be bought at £3. 14*s.* per cwt., how must it be sold to gain $12\frac{1}{2}$ per cent., after allowing a discount of $7\frac{1}{2}$ per cent.?

482. The width of a rectangular cistern is twice its height, and its length is five times its height. What are its dimensions in feet, if it contain $6\frac{1}{8}$ tons of water, a cubic foot of water weighing $62\frac{1}{2}$ lb.?

483. In the centre of a room 21 ft. square, there is a square carpet; the rest of the floor is covered with oil-cloth. The carpet and oil-cloth cost respectively 16*s.* 6*d.* and 8*s.* 6*d.* per sq. yd.; and the whole cost is £35. 4*s.* 6*d.* Find the width of the oil-cloth border.

---

484. Reduce £3. 15*s.* $6\frac{1}{4}$*d.* to the decimal of £3.

485. Simplify $\frac{11}{40}$ of $\sqrt{\frac{(1\cdot345)^2-(\cdot905)^2}{1\cdot\dot{2}+\cdot\dot{2}\dot{3}}} \times \sqrt[3]{\cdot\dot{0}3\dot{7}}$.

486. Find the value of ·6215 of £5 + 1·2586805 of £3.

487. A tradesman buys milk at 11*d.* per gallon, and, after diluting it with water, sells it at $3\frac{1}{2}$*d.* per quart, making a profit of 50%. What proportion of water did he add to the milk?

488. If a person transfers £1234 stock from a $3\frac{1}{4}$ per cent. stock at $88\frac{7}{8}$ to a $3\frac{1}{2}$ per cent. stock at $95\frac{3}{8}$, find the difference in his income.

489. A garden whose length is 67 ft. 9 in. has a path 4 ft. wide on two sides and at one end. If it cost £4. 10*s.* $3\frac{3}{4}$*d.* to turf the remainder at 6*d.* per sq. yard, what is the width of the garden?

490. Of a certain store of potatoes, 11 men would in 3 days consume all but 201 lb., and 21 men would in 4 days consume all but 48 lb. How many pounds of potatoes does the store contain?

PROBLEMS.

1. A pays £9. 3*s.* 4*d.* more rates than B, though their assessments are equal: living in different towns they are rated at 2*s.* and 1*s.* 4*d.* in the £ respectively; what is the amount of the assessment?

2. A seaman in the British Navy is allowed $33\frac{1}{3}$ oz. of food daily; supposing him to eat 30 oz., and to weigh $11\frac{1}{2}$ stone, how many times his own weight does he eat in an ordinary year?

3. In English gunpowder 75 parts by weight are saltpetre, 10 parts sulphur, and 15 parts charcoal. How many pounds weight of each ingredient are used in the manufacture of 16 cwt. of gunpowder?

4. The surface of the earth is about 200 million sq. miles, of which $\frac{1}{4}$ is habitable; if the population be now 1500 millions, and double itself every 25 years, how many persons will there be to a sq. mile of habitable land at the end of 125 years?

5. If a pint of pure water weighs a pound and a quarter, and a cu. foot weighs 1000 ounces, how many gallons will fill a cistern 5 feet long, $3\frac{1}{2}$ feet wide, and $2\frac{3}{4}$ feet deep?

6. A gallon of water weighs 10 lb., and a cu. foot of water weighs 1000 oz.; how many gallons are there in a cubic foot?

7. A gallon contains 277·274 cu. in.: a cu. foot of water weighs 1000 oz. How many gallons weigh a ton? what is the weight of a pint?

8. The imperial gallon contains 277·27 cubic inches, and a cubic foot of water at its maximum density weighs 62·42 lb. Find the weight of a pint of water correctly to two places of decimals.

9. If the weight of one cubic foot of water is 62·25 lb., find the error in calculating the weight of 1000 cu. ft. on the approximate assumption that one cubic foot weighs 1000 oz.

10. Two flag-staffs are 9 metres apart and are respectively 10 metres and 11·65 metres high; what is the distance in metres from the top of one to the top of the other?

11. How many bricks, each 9 in. long, $4\frac{1}{2}$ in. broad, and 4 in. thick, will be required for a wall 80 feet long, 12 feet high, and $2\frac{1}{2}$ feet thick, allowing $6\frac{1}{4}$ per cent. of the space for the mortar?

12. If 3 men, 5 women, or 8 children, could do a certain work in $26\frac{1}{2}$ hours, in what time will 2 men, 3 women and 4 children do it?

13. If 81 gallons fill a cistern 4 ft. 4 in. by 2 ft. 8 in. by 1 ft. $1\frac{1}{2}$ in., find the number of cubic inches in a pint.

14. A rectangular cistern, 2 yds. 2 ft. long, and 2 yds. wide, contains a depth of 4 ft. of water. If a cu. foot of water weigh 1000 oz., find the weight of the water in the cistern.

15. $I = M \cdot \frac{a^2+b^2}{12}$. If $M = 35$, $a = 10{\cdot}12$, $b = 3{\cdot}86$, find I correct to 2 decimal places.

16. What sum of money will amount to £699. 13*s*. 2·4*d*. in 2 years, reckoning compound interest for one year at $3\frac{1}{2}$ per cent. and for the other at 4 per cent. per annum?

17. The difference between the simple and the compound interest on a certain sum in 3 years at 5 % is £5. 10*s*. $6\frac{3}{4}d$.; find the sum.

18. A gentleman found that if the income-tax were 2 per cent. instead of 6*d*. in the £, it would make a difference to him of £11. 3*s*. $2\frac{3}{4}d$.: what is his annual income?

19. A bill bought in London at 25·6 francs per £ sterling is sold in Lisbon at 172 reis per franc. What is the exchange between London and Lisbon? (1000 reis = 1 milrei.)

20. A contractor contracts to finish a piece of work in 30 days, and employs 15 men upon it; at the end of 24 days the work is only half done. How many additional men must he employ to fulfil the contract?

21. If 13 solid inches of copper balance 17 of iron, and 15 of iron balance 16 of tin, and 19 of tin balance 12 of zinc, how many solid inches of zinc balance 1235 of copper?

22. If a litre be ·22 of a gallon, find the value of a pint of liquid which is worth 20 francs the litre: 1200 francs being equivalent to £49.

23. The surface of a sphere of radius r inches is $4\pi r^2$ sq. inches, where $\pi = 3{\cdot}14159$. If π is taken to be $\frac{22}{7}$, find, to the nearest square mile, the error made in calculating the earth's surface, the earth being a sphere of radius 4000 miles.

24. If steel weighs 488 lb. per cu. foot, find, to the nearest cg., the weight of a c.cm. of steel. (1 Kg. = 2·2046 lb., 1 m. = 39·37 in.)

25. If $v^2/r = g/289$, find v, to 2 decimal places; given that $r = 4000$, and $g = \frac{32{\cdot}2}{5280}$.

26. The volume of a tree trunk is taken to be equal to the length multiplied by the square of $\frac{1}{4}$ of the girth. How much per cent. would this be in error if the trunk were a circular cylinder? ($\pi = 3{\cdot}1416$.)

27. If $t = 2\pi\sqrt{\frac{l}{g}}$, find l, correct to 3 decimal places, if $t = 1$, $g = 32{\cdot}2$ and $\pi = 3{\cdot}1416$.

28. The following rules for conversion of units are sometimes employed: (*a*) to convert oz. to grams multiply by $\frac{8}{3}\frac{5}{5}$; (*b*) to convert lb. to Kg. multiply by $\frac{5}{11}$. Given that 1 Kg. = 2·2046 lb., find, to 2 decimal places, the percentage error in each case.

29. A donkey is tethered 5 metres from a long straight hedge, and it can graze at a distance of 13 metres from its tether. Find how much of the hedge it can nibble.

30. A certain length of pathway has to be constructed. It is found that 3 men can construct $\frac{1}{5}$ of it, all but 1 mile, in 2 days, whilst 18 men can construct 1 mile more than $\frac{2}{5}$ of it in 1 day. What is the length of the path?

31. Find a number, between 500 and 1000, which is a multiple of 7, and is such that whether divided by 4, 5 or 6 the remainder is 1.

32. A merchant buys 230 hhd. of wine, and sells 207 for the same amount that he gave for the whole. The rest he sells at a profit of 12 per cent. If the price of each hhd. is £30, and the expense of sale on each is 12*s.*, what is his whole gain, and also his gain per cent.?

33. A hundred gallons of liquid contain 70 per cent. of wine and the rest water. How much wine should be added to the mixture to raise the proportion of wine to 80 per cent.?

34. If £1 is worth 24·6 francs, and the French 3 per cent. Rentes are at 70·3 francs, how much of this stock will £351. 10*s.* buy?

35. What is the market value of 3 per cent. stock if, after deducting the income-tax at 1*s.* 4*d.* in the £, it yields $3\frac{1}{2}$ % interest?

36. If 3 per cent. stock be at such a price as to give $3\frac{1}{3}$ per cent. interest, what rate of interest will it pay when the price has risen £6?

37. If I invest £790. 10*s.* in a 4 per cent. stock at $87\frac{5}{6}$, and sell out when it rises to 90, what change would be caused in my income, if I were to invest the proceeds in shares at 150 yielding $7\frac{1}{2}$ per cent.; the shares when at par being at 100?

38. A ship's crew consists of 1 captain, 3 officers and 36 men; the captain gets 4 times as much prize money as each officer, and the officers together get 12 times as much as each man. What do they respectively get out of £1000?

39. A colliery employs 120 men working 8 hours a day for 5 days a week. How many men can it employ for 4 days a week working 6 hours a day, if only half as much coal is required to be raised?

40. A, B and C contribute sums of £250, £500 and £750 respectively towards a venture, on the understanding that the profits shall be divided in such a way that the *rate* of interest which each receives shall be in proportion to the amount of his contribution. If the profits for a year amount to £245, how much will each receive?

41. If you invest a sum of money in such ways that on $\frac{1}{3}$ of it you gain 3 %, on $\frac{1}{5}$ of it you lose 4 %, and on the remainder you gain 6%, what average rate per cent. do you make on the whole sum?

42. The incomes of two men would be equal if one were increased by 7 % and the other diminished by $7\frac{1}{2}$ %; and the sum of their incomes is £418. 19*s.* What is the income of each?

43. If it costs the same amount to keep 4 horses as 9 oxen, and 5 horses can do as much work as 8 oxen, which will it be more profitable to employ—20 horses and 32 oxen, or 12 horses and 48 oxen?

44. Two fields, one measuring 95 m. by 247 m. and the other 532 m. by 703 m., are to be divided into square plots of the same size: if the squares are as large as possible, find the area of one of the squares.

45. A person left London for Cambridge at 8 a.m., travelling the first 23 miles by rail at the rate of $23\frac{1}{3}$ miles per hour, and walking the other 29 miles at $\frac{1}{7}^{\text{th}}$ of that rate; at what o'clock did he arrive?

46. A cistern can be filled by two pipes A and B in 12 min. and 14 min. respectively, and can be emptied by a third C in 8 min. If all the taps are turned on at the same moment, what part of the cistern will remain unfilled at the end of 7 min.? If the cistern then contain 25 gallons, how many gallons have been carried off by the waste pipe C?

47. A detachment of cavalry consisted of 520 men after engaging in 2 battles: in the first it lost one man in every 25, and in the second $\frac{5}{18}$ of the remainder. Of how many men did the troop originally consist?

48. The large wheel of an engine is 20 ft.. and the wheel of a carriage is 12 ft., in circumference. If the large wheel slips 2 in. in every revolution, how many revolutions will the carriage wheel make more than the large one in going a distance of 12 mi. 1728 yds.?

49. A takes twice as long to do a piece of work as B, and C half as long again as B; the three together take $3\frac{3}{13}$ days; how long would each one take by himself?

50. Two men undertake to do a piece of work for £2. One could do it alone in 5 days, the other in 8 days. With the help of a boy they finish it in 3 days. How should the money be divided among them?

51. A tradesman bought some goods and sold them so as to gain 4%. If he had bought them at 4% less and had sold them for £4. 15*s.* less, he would have gained 2%. Find the original cost.

52. A tank contains 5600 Kg. of water, and another of the same horizontal section but 103 cm. deeper, has a capacity of 10,750 litres. Find the area of the horizontal section of the tank, and the depth.

53. The squares of the times of revolution of the planets are as the cubes of their distances from the Sun. Find, to the nearest day, the time Jupiter takes to go round the Sun, given that his distance from the Sun = 5·2 times the Earth's distance. (1 year = $365\frac{1}{4}$ days.)

54. $t = \pi\sqrt{\dfrac{\mathsf{I}}{\mathsf{MH}}}$. If $t = 7{\cdot}5$, $\mathsf{H} = {\cdot}18$, $\mathsf{I} = 789{\cdot}6$, and $\pi = 3{\cdot}1416$, find M to one decimal place.

55. A man walks a third as far again as a boy in a given time. His stride is 5 cm. longer than the boy's, and he takes 125 strides per min. while the boy takes only 100. At what rate in Km. per hour does each walk?

56. The tonnage of a vessel is $\dfrac{(\mathsf{L} - \frac{3}{5}\,\mathsf{B}) \times \mathsf{B} \times \frac{1}{2}\mathsf{B}}{94}$, where L and B are the length and breadth of the vessel measured in feet. Find, to the nearest integer, the tonnage when L = 280 and B = 32.

57. If the true length of an arc be 91·6 cm., find the error per cent. in the value given by the approximation $\dfrac{8\mathsf{B} - \mathsf{A}}{3}$, where A = 23·4 cm. and B = 35·1 cm.

58. The volume of the frustum of a cone is $\frac{1}{3}h(a+b+\sqrt{ab})$, h denoting the height of the frustum, a and b the areas of the two ends. Calculate the volume, to the nearest cubic inch, when $h=4{\cdot}5$ in., $a=28{\cdot}5$ sq. in., $b=78{\cdot}6$ sq. in.

59. The mass of a sphere varies directly as its density and the cube of its diameter conjointly. The density of the moon is to that of the earth as 4891 to 4000, and the diameter of the moon is to that of the earth as 100 to 365. Find, to three decimal places, the mass of the earth in terms of that of the moon.

60. A man buys goods and finds that the cost of carriage is 4 % on the cost of the goods. He is compelled to sell at a loss of 5 % on his total outlay ; if however he had received £3. 5*s*. more than he did, he would have gained $2\frac{1}{2}$ %. What was the original price of the goods?

61. $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$. If $u=36{\cdot}8$, $v=25{\cdot}4$, find f, to 2 places.

62. A, B and C divide 2 gross of pens in the following manner : as often as A takes 3, B takes 5 ; and as often as B takes 7, C takes 8. How many will each have?

63. A cube contains 1953125 cu. inches. Find, to 4 places of decimals, the difference between the length of its edge and of its diagonal.

64. If $a=12{\cdot}7564$, correct to 4 places, $b={\cdot}1946$, correct to 4 places, and $c=851{\cdot}27$, correct to 2 places, find as accurately as possible the value of $a\times b\times c$, and state the limits of error.

65. A rectangular vessel formed of material one inch thick, and of external dimensions 13 ft. 6 in., 6 ft. 8 in. and 6 ft. 11 in. respectively, weighs 1 cwt. 1 qr. 10 lb. Find the weight of a solid mass of the same material and of the same dimensions.

66. Two cogged wheels, of which one has 16 cogs and the other 27, work in each other ; if the latter turns 80 times in $\frac{3}{4}$ of a minute, how often does the former turn in 8 seconds?

67. In a large institution where there is always the same number of inmates to be fed, the contract price of meat rises 20 per cent. and the daily allowance of each person is at the same time reduced from 9 oz. to 8 oz. If the yearly charge for meat is thenceforward £597, what was it before the changes were made?

68. Taking the exact equivalents to be those given in Chap. XLV. find, to 2 decimal places, the error per cent. in the following approximate equivalents :—10 metres = 11 yds., 1 Km. = 5 fur. and 4 litres = 7 pints.

69. A pond whose area is 3 acres is frozen over with ice to the uniform thickness of 6 inches. If a cu. foot of ice weighs 896 oz., find the weight of the ice on the pond in tons.

70. A river 30 feet deep and 200 yds. wide flows 4 miles an hour ; find the number of cubic feet of water that falls into the sea in a minute, and its weight in tons (1 cu. ft. weighs 1000 oz.).

71. Divide £1690 among 4 persons so that the first may have $\frac{1}{11}$ as much again as the second, the second $2\frac{2}{3}$ times as much as the third, and the fourth as much as the second and third together.

72. 33 sq. feet of paper weigh 3500 grains; what is the weight of 24 sheets each 18 in. by 22 in.? If this paper be torn into pieces each an inch square, and a person running at the rate of 9 miles an hour drop a piece every yard, in what time will the whole be exhausted?

73. In a cricket match A got $\frac{6}{5}$ as many runs as B, B $\frac{11}{9}$ as many as C, C $\frac{5}{4}$ as many as D, D $\frac{9}{7}$ as many as E, E $\frac{4}{3}$ as many as F and F $\frac{3}{5}$ as many as the rest of the eleven. If F got 21, what was the total score, including 4 wides and 10 byes?

74. A person invests £6825 in a 3 per cent. stock at 91; he sells out £5000 stock when it has risen to $93\frac{1}{2}$, and the remainder when it has fallen to 85. How much does he gain or lose by the transaction? If he invest the proceeds in $4\frac{1}{2}$ per cent. stock at par, what is the difference in his income?

75. A founder is required to supply a ton of fusible metal consisting of 8 parts by weight of bismuth, 5 of lead and 3 of tin. The only bismuth he has in stock is in an alloy consisting of 9 parts bismuth, 4 lead and 3 tin. How much of the alloy must he take, and how much lead and tin must he add to make up the order?

76. If a ship cost £80000 building, $\frac{3}{5}$ as much equipping and $\frac{1}{8}$ as much storing, find the cost of 12 such vessels, supposing that 5 % is saved on the cost of building, $17\frac{1}{2}$ % on fitting and 15 % on storing.

77. A post, whose cross section is a square of the greatest possible area, is cut from a cylindrical tree trunk. How much per cent. of the wood is wasted? (Answer to nearest integer.)

78. A square courtyard consists of a lawn 44 yards square, with a path of uniform width all round the lawn. The area of the lawn is to that of the path as 16 is to 9. How wide is the path?

79. Six men agree with a seventh man to provide a certain sum of money among them. The first six are to subscribe £10 each, and the seventh is to pay £2 more than the average of the seven. What is the whole sum provided?

80. The sum which will pay A's wages for $61\frac{1}{4}$ days will pay B's wages for $81\frac{3}{5}$ days. For how many days will it pay both A and B?

81. A train starts with its full complement of passengers. At the first station it drops $\frac{1}{3}$ of these and takes in 96. At the next it drops $\frac{1}{2}$ of the new total and takes in 12. On reaching the third station there are 248 left. What number started?

82. A cask contains 3 parts ale and 1 part porter; how much of the mixture must be drawn off and porter substituted in order that the resulting mixture may be half-and-half?

83. A besieged garrison have sufficient provisions to last them for 23 weeks at the rate of 18 oz. per man per diem; but, receiving a reinforcement of 40 per cent. upon their original number, this allowance is reduced to 15 oz. per diem. How many days will they be able to hold out?

84. Three apples are worth as much as 4 plums, 4 pears as much as 9 apples, 13 apricots as much as 7 pears and 7 apples sell for 3*d.* I wish to buy an equal number of each of the four fruits and to spend an exact number of pence ; find the least sum I need spend.

85. A bankrupt has book debts equal in amount to his liabilities, but on £3000 of such debts he can recover only 9*s.* 8*d.* in the £, and on £1200 only 5*s.* 7*d.* in the £ ; the expenses of the bankruptcy amount to £1053. 15*s.*, and he finds he can pay 10*s.* 9*d.* in the £. Find the amount of his debts.

86. Two passengers are charged for excess of luggage 3*s.* 7*d.* and 8*s.* 5*d.* respectively. Had the luggage all belonged to one person he would have been charged 19*s.* for excess. How much is allowed free, the charge for excess being $\frac{1}{2}d.$ per lb. ?

87. The specific gravity of mercury is 13·596, correct to 3 places. Find, to the nearest cg., the weight of mercury in a bottle, whose capacity is 49·974 cu. cm., correct to 3 places. Shew that your result may be in error by 3 cg.

88. A runs a two-mile race with B, and loses. Had he run $\frac{1}{5}$ quicker he would have won by 22 yds. Compare the rates at which they ran.

89. A and B run a mile race, and A wins by 80 yds. When A and C run over the same course, A wins by 20 sec. B and C run, and B wins by 5 sec. In what time can A run the mile ?

90. A starts on a walking tour and walks 20 miles a day. After the lapse of a certain number of days B starts on the same tour, and walking 25 miles a day overtakes A at the end of 8 days. How many days' start had A ?

91. The present value of a sum of money due 4 years hence is £150. If it were due in 2 years, the present value would be £165. 7*s.* 6*d.* Find the rate of Compound Interest.

92. A shilling weighs 3 dwt. 15 grs. of which 3 parts in 40 are alloy, and the rest pure silver. If the value of pure silver rises 8 per cent., what must be the reduction of pure silver in a shilling ?

93. Standard gold is formed by mixing 11 parts of fine gold with one of copper. How many sovereigns can be coined from 4 oz. avoir. of fine gold, if 480 oz. troy of standard gold is coined into 1869 ?

94. Standard silver contains 37 parts in 40 of fine silver, and 12 oz. troy of standard silver is coined into 66 shillings. Calculate the value of the money which can be coined from 222 lb. avoir. of fine silver, it being given that 144 lb. avoir. are equal in weight to 2100 oz. troy.

95. If $1\frac{3}{4}$ oz. of gold, 18 carats fine, be worth £5. 11*s.* 5*d.*, of what fineness must gold be in order that $6\frac{3}{4}$ dwt. of it may be worth £1. 3*s.* $10\frac{1}{2}d.$? (Pure gold is 24 carats fine.)

96. 280 people dine together 5 days a week, but on the remaining 2 days some are absent ; the consumption of food is thus reduced (for the whole week) in the ratio of 69 to 70. Find the number of absentees.

97. If 35 cu. in. of brass and 16 cu. in. of iron together weigh 241 oz., and 11 cu. in. of brass and 25 cu. in. of iron together weigh 157 oz., find the weight of 1 cu. in. of each metal.

98. A person shooting at a target, distant 550 yards, hears the bullet strike the target 4 seconds after he fires. A spectator, equally distant from the target and the shooter, hears the shot strike the target $2\frac{1}{2}$ seconds after he heard the report; find the velocity of sound.

99. A ship 600 miles from shore springs a leak which admits $6\frac{1}{4}$ tons of water in 20 min.; 60 tons of water would suffice to sink her, but her pumps can throw out 70 tons in 4 hours. Find her average rate of sailing that she may reach the shore just as she begins to sink.

100. A man has part of his money invested at $3\frac{1}{2}\%$ and part at $2\frac{1}{2}\%$. He realizes half of each and reinvests the proceeds at 6 %. If his original and final incomes are £469 and £684. 10*s.* respectively, find his total capital and the amounts of his original investments.

101. If a cu. inch of water at a certain temperature weighs 252·458 grs., which is the more accurate of the following rough statements—a cu. ft. of water weighs 1000 oz., or a cu. yd. weighs $\frac{3}{4}$ of a ton?

102. A landlord has an estate worth £3900 a year, but has to pay 12 per cent. on the gross income for taxes, etc. He sells it at 24 years' purchase on the gross income, and invests the proceeds in a 3 per cent. stock at $97\frac{1}{2}$. What is the difference in his income?

103. A certain company agree that each member shall subscribe as many guineas as there are members in all; the whole subscription being £1047901. 1*s.*, how many members are there?

104. A person started at 2.30 p.m. and walked to a village, arriving there when the church clock indicated 3.15 p.m. After staying 25 min., he drove back by a road one-fourth as long again, at a rate twice as fast as he had walked, and reached home by 4.0 p.m. How far wrong was the church clock?

105. Two clocks strike 9 together on Monday morning. On Tuesday morning one wants 8 minutes to 11, when the other strikes 11. How much must the slower be put on, that they may strike 9 together in the evening?

106. If a watch showing correct time at noon on Tuesday gains 3 min. 40 sec. a day, what is the correct time on Saturday evening when it is 10 o'clock by the watch?

107. Two clocks point to 2 o'clock at the same instant on the afternoon of Christmas Day: one loses 7 sec. and the other gains 8 sec. in 24 hours. When will one be half an hour before the other, and what time will each clock then shew?

108. A's present age is to B's as 9 is to 5, and 23 years ago the ratio was 10 to 3. Find the present age of each.

109. A ball, of external diameter 6 cm. and of thickness 5 mm., weighs 162 g. when it is filled with liquid. If a cu. cm. of the substance of the ball weigh 2 g., find, to the nearest centigram, the weight of a cu. cm. of the liquid.

110. The military pace is 30 in., and the "quick" march is 3 mi. 120 yds. an hour. If the "double" increases the length of the pace in the proportion of 3 : 4, and the number of steps per min. in the proportion of 2 : 3, how many paces are taken in a march of 1 mile, half at the "quick" and half at the "double," and how long a time is occupied?

111. In firing at a mark A hits in 2 out of 3 shots, B in 3 out of 4 and C in 4 out of 5. The mark was hit 931 times. Supposing each to have fired the same number of shots, find how many hits each made and the total number of shots fired.

112. A man buys an article and sells it at a profit of 2*s.* 7*d.* On calculating his profit he finds that if he estimates it as a percentage on the cost price, it is one-fifth as great again as it would be if he estimated it as a percentage on the selling price. Find the cost price of the article.

113. Two contractors, A and B, employ the same number of workmen at the same daily wages. A employs his men for $6\frac{3}{4}$ weeks, B employs his for $12\frac{1}{2}$ weeks and pays £31. 1*s.* more in wages than A. Find what A and B paid their workmen.

114. A ship and cargo were worth £68541 ; the owner insured them at $17\frac{1}{2}\%$ so as to cover his venture, and also to recover his premium. For how much did he insure?

115. A sum of £630 is divided between 10 men, 32 women and 48 children. Each man gets twice as much as a woman, and the 32 women get as much as the 48 children. What is the share of each person?

116. A man buys eggs at a certain price per score, and sells them at half that price per dozen ; what is his gain or loss per cent.?

117. A monkey, climbing up a greased pole, ascends 3 feet and slips down 2 feet in alternate seconds until he reaches the top of the pole. If the pole be 60 feet high, how long will it take him to reach the top?

118. A drover buys 87 oxen for £1740. He sells 25 at 7 per cent. profit, and 40 at $12\frac{1}{2}$ per cent. profit ; he loses 3 by disease, and sells the rest at cost price. How much does he gain?

119. From a cask of wine $\frac{1}{4}$ is drawn off and replaced with water ; $\frac{1}{4}$ of the mixture is then drawn off and the cask is again filled with water ; after this process has been repeated 4 times altogether, what fraction of the original quantity of wine will be left in the cask?

120. One vessel contains 24 gallons of water ; another contains 24 gallons of wine. One gallon is taken from each, and is then poured into the other. This is done 3 times. How much wine and how much water will the vessels then respectively contain?

121. A merchant purchases wheat at 35*s.* 9*d.* a quarter ; at what price must he sell it to gain $12\frac{1}{2}$ per cent., and allow a purchaser $2\frac{1}{2}$ per cent. discount?

122. The price of a diamond per grain varies as its weight. If a diamond of 12 grains be worth £60, how much will a diamond of 16 grains be worth?

123. A cask weighing 2 cwt. 12 lb. 4 oz. floats in a square cistern of water whose side is 2 ft. 6 in.; find how much the water in the cistern will sink on the removal of the cask, supposing a cu. foot of water to weigh 63 lb.

124. If gold weighs 19·3 times as much as water, and copper 8·89 times as much, find how many times as heavy as water a coin is which contains 11 volumes of gold to 1 of copper.

125. Twenty-five years ago a man was four times as old as his son, whose present age is 33. What is the present age of the father?

126. The sum of the ages of A and B is now 60 years, and their ages 10 years ago were as 5 to 3. Find their present ages.

127. If 10 oz. 13 dwt. 16 grs. of gold, worth £3. 5*s.* per oz., be mixed with 1 oz. 2 dwt. 8 grs. of silver worth 5*s.* per oz., how much worthless alloy must be added to reduce the value of the compound metal to £2. 16*s.* 8*d.* per oz.?

128. Sound travels at the rate of 1140 feet a second. If a shot be fired from a ship moving at the rate of 10 miles an hour, how far will the ship have moved before the report is heard at a place $14\frac{1}{4}$ miles off?

129. If gold can be beaten out so thin that a grain will form a leaf of 56 sq. inches, how many of these leaves will be required to make up the thickness of a sheet of paper, the weight of a cu. foot of gold being 14580 oz. and 400 sheets of paper being 1 inch thick?

130. A bicycle was sold at a loss of 40 % on the cost price, and a second bicycle was bought with the proceeds when 5 guineas had been added. This second bicycle was sold at a loss of 35 %, and a third bicycle was bought with the proceeds after the addition of £5. 15*s.* 3*d.* This third bicycle cost £16: what was the price of the first?

131. If the price of bread be raised 5 %, while the price of meat remains the same, by how much per cent. must a person decrease his consumption of meat, in order to buy as much bread as before, and spend on bread and meat the same amount as before, assuming that the sums he spent on bread and meat respectively were as 5 to 2?

132. A sum of money is lent at compound interest. The increase in its amount for two years, which would result from an increase of 1 in the rate per cent., is 6*s.* 8*d.* more than this increase would have been if the initial rate had been 1 less. Find the sum of money.

133. A ditch is being dug at the rate of 81 ft. per day by 54 men; after 13 days' work 8 of them are replaced by boys, and the work goes on for 11 days more, at the end of which the whole length dug is 1889 feet. How much work per day do the boys do?

134. How many seconds will a train 184 feet long, travelling at 21 miles an hour, take to pass another train 223 feet long, proceeding at 16 miles an hour, (*a*) in the same, (*b*) in the opposite direction?

135. A person rows from A to B (a distance of $1\frac{1}{2}$ miles) and back again in an hour; how long would it have taken him if he had rowed equally hard, and there had been a stream of $1\frac{1}{2}$ miles an hour, running from A to B for the first 30 min., and then ceasing?

136. Two trains, 92 ft. and 84 ft. long respectively, move with uniform velocities on parallel rails in opposite directions, and pass each other in $1\frac{1}{2}$ sec.; when moving in the same direction, with the same velocities as before, the faster train passes the other in 6 sec.; find the rate at which each moves.

137. A cargo steamer leaves port at the rate of $8\frac{1}{2}$ miles an hour at 3 a.m. on Monday. At midnight a despatch boat starts in pursuit at the rate of 15 miles an hour. When will they be in sight of one another, supposing each visible from the other when 6 miles apart?

138. A four-wheeled carriage travels round on a circular railway. The circumferences of the two wheels of the carriage and of the two circles of rails are proportional to 6, 7, 7000, 7014. Find the number of revolutions made by each of the 4 wheels in a complete circuit.

139. Two boats start to row a race at 3 o'clock. The race is over at $6\frac{2}{3}$ minutes past 3, the losing boat being 40 yards behind at the finish. At 4 minutes past 3 this boat was 700 yards from the winning-post. Find the speed of each boat in miles per hour.

140. In a race B starts a certain distance ahead of A, and C an equal distance ahead of B. If A passes B in 12 min. and passes C 9 min. afterwards, in what time from the start will B pass C?

141. If I walk to the station at the rate of 4 Km. per hour I shall arrive one minute too late for a certain train; but if I run at the rate of 9 Km. per hour I shall arrive 3 minutes too early. Find in metres the distance of the station from my starting point.

142. On a stream, B is intermediate to and equidistant from A and C; a boat can go from A to B and back in 5 hrs. 15 min., from A to C in 7 hrs. How long would it take to go from C to A?

143. A car and a cyclist start together on a journey of 9 Km. 680 m., the car travelling uniformly. When the cyclist rides on the footpath his rate is to the car's as 38 to 33, but on the road as 10 to 11. If they finish together, how far did the cyclist ride on the footpath?

144. A messenger sets out at the rate of 30 miles a day, but falls off in his speed 4 miles daily. Four days afterwards another sets off from the same place on the same route, travelling 50 miles the first day, but falling off like the first 4 miles daily. After what time and at what distance will one overtake the other?

145. A man walking in a fog caught up a wagon, which was going in the same direction at the rate of 3 miles an hour. If the wagon was just visible to him at a distance of 55 yards, and he was able to see it for 5 minutes, how many miles an hour was he walking?

146. Apples are bought at the rate of 7 for 3*d.*; half of them are sold at 5 for 2*d.*, one-fifth at 7 for 4*d.*, and the rest at 7 for 5*d.* What is the smallest number of apples that can be so dealt in? What is the gain or loss per cent.?

147. Four apples are worth as much as 5 plums, 3 pears as much as 7 apples, 8 apricots as much as 15 pears, and 5 apples sell for 2*d.* I wish to buy an equal number of each of the four fruits, and to spend an exact number of pence; find the least sum I can spend.

148. The marching stride of an English soldier is 30 inches, and he takes 116 steps in a minute. The Italian soldier has a stride of only $29\frac{1}{2}$ inches, but he takes 120 steps in a minute. If a regiment of each start together for an eight hours' march, what will be the distance between them at the end of that time?

149. On a circular railway connecting three towns A, B, C, two trains are constantly running in opposite directions. Every four hours they leave A simultaneously, and, travelling uniformly at the same rate, cover the distances from A to B and from B to C in 65 min. and 75 min. respectively. If the trains pass at a point which divides the distance from B to C in the ratio of 7 to 5, and stop the same time at each station, find what this time must be, and also the time taken to go from A to C.

150. A man, his wife and boy worked in a garden: the man for 2 days, his wife for $3\frac{1}{2}$ days, the boy for 4 days. Their daily wages were—man's to wife's in the ratio of 7 to 4, and man's to boy's in the ratio of 7 to 3. Their total earnings amounted to £1. What were the daily wages of each?

151. If 75 men can do a piece of work in 12 days of 10 hours each, how many men will do a piece of work twice as large in $\frac{1}{10}$ of the time, supposing that they work the same number of hours in the day, and that 2 of the second set can do as much in an hour as 3 of the first?

152. A and B run a race of $\frac{2}{3}$ mile on a course $\frac{1}{4}$ of a mile round: they run in opposite directions and A wins by 40 yards; where was B when A passed the post the first time?

153. The external length, breadth and height of a rectangular wooden closed box are 18 inches, 10 inches and 6 inches respectively, and the thickness of the wood is half an inch. When the box is empty it weighs 15 lb., and when filled with sand 100 lb. Compare the weights of equal bulks of wood and sand.

154. A reservoir containing 1200 cu. feet of water is entered by means of a pipe whose cross section $= 7\frac{1}{4}$ sq. in. If the water flows out with a velocity of 2 miles an hour, how long will it take to empty it?

155. A railway proprietor receives one year a dividend of 3 per cent. on his stock, and pays an income-tax of 4*d.* in the £. The next year he receives a dividend of $3\frac{1}{2}$ per cent., pays an income-tax of 3*d.* in the £, and finds that his net income is £498 more. How much railway stock does he hold?

156. A person pays insurance on his life to the extent of 10 per cent. of his income: this he legally deducts from his income, and after paying 5*d.* in the £ income-tax on the remainder has £1057. 2*s.* 6*d.* left. What was his gross income?

157. The proprietor of a boarding school having already 30 pupils, finds that an addition of 5 increases his gross yearly expenditure by £300, but diminishes the average cost per head by £1. What did his annual expenses originally amount to?

158. A grocer buys 40, 50 and 63 Kg. of tea at 5*s.*, 4*s.* 6*d.* and 4*s.* 2*d.* per Kg. respectively. He sells half the first at a loss of 10 % and half

the second at a gain of 10 %. He mixes the rest. At how much per Kg. must he sell the remainder so as to gain 12 % on the whole?

159. A certain reef of quartz when crushed yields ·0011 per cent. of gold. If the working expenses amount to 62·5 per cent. of the gross receipts, and the net profit on each 100 tons is £52. 10*s.*, find the number of grains in a sovereign.

160. A sum of money is put out at compound interest for four years. During the first two years interest is paid at the rate of 4 per cent., but during the last two years at 5 per cent. The total amount at the end of that time is £77. 12*s.* 8¼*d.* What was the sum invested?

161. A milkman adulterates his milk as follows:—He takes a pint out of each gallon and replaces it with water. He then takes a pint out of the mixture and replaces that with water; and he repeats the operation a third time. If he then sells his adulterated milk at the price per gallon that pure milk cost him, what is his gain per cent.?

162. Two casks originally contain 60 gall. of wine and 30 gall. of water respectively. On three successive occasions 12 gall. of liquid are drawn from each cask and placed in the other. Express in decimals of a gallon the quantity of wine now in each cask.

163. If a coach travels at the rate of 9½ miles per hour, and a railway train runs 25 miles while the coach is accomplishing 8 miles how much time would a traveller save on a journey of 152 miles by adopting the train as his mode of conveyance?

164. A train, travelling at the rate of 18½ miles an hour, started at 6 o'clock on a journey of 148 miles. A second train started from the same station; its speed was to that of the former as 8 : 5, and it arrived 15 minutes after it. At what time did the second train start?

165. Soldiers take 108 paces of 30 in. per min., and sound travels 378 yds. per sec.; reckoning the breadth of a man to be 21 in., how many men were there marching in line when the extreme men were just completely out of step, the time being taken from a drum at one end?

166. There are in a manufactory a certain number of workmen who receive 50*s.* a week, twice as many who receive 31*s.* 6*d.* a week, and 11 times as many who receive 14*s.* a week; the total amount of the men's wages for one week is £93. 9*s.*; find the number of men.

167. A party of tourists set out for a station 3 miles distant and go at the rate of 3 miles per hour. After going half a mile one of them has to return to the starting point: at what uniform rate must he walk, if he at once increases his speed, in order to reach the station at the same time as the others?

168. The driving-wheel of a locomotive is 5 feet in diameter, and turned 5000 times in going 12 miles. Supposing the circumference of a circle to be 3·1416 times the diameter, find what distance in miles was lost owing to the slipping of the wheel on the rail.

169. If the manufacturer makes a profit of 10 %, the wholesale dealer one of 15 %, and the shopkeeper one of 20 %, what was the cost to the manufacturer of an article bought at a shop for £6. 6*s.* 6*d.*?

170. Assuming that an express train runs 40 miles an hour, and an ordinary train 30 miles an hour, and that the express fare is $\frac{1}{4}d.$ a mile more than the ordinary, find how much an hour a man's time is worth, if it costs him as much to travel by one as by the other.

171. A person bought $2\frac{3}{4}\%$ stock at 95, sold it, and with the proceeds bought $3\frac{1}{2}\%$ stock; he obtained £900 less stock than before, but the same income. How much money did he originally invest?

172. If a wholesale dealer sells to a retailer at 10 per cent. profit, and the retailer to the consumer at 50 per cent. profit, what proportion of the price paid by the consumer is profit? and in what proportion is this profit shared between the wholesale dealer and the retailer?

173. The debts of a bankrupt amount to £2152. 3*s.* 0*d.*, and his assets consist of property worth £916. 15*s.* 4*d.* and an undiscounted bill of £525 due four months hence, simple interest being reckoned at 4 per cent. How much in the £ can he pay his creditors?

174. A person had £6900 stock, paying 3 per cent., in a company, and was offered the choice of being paid off at par, or of receiving £110 of a new $2\frac{1}{2}$ per cent. stock for every £100 of the old stock. He chose the former alternative, and invested his money in a 3 per cent. stock at 92. Find the amount of his new stock and the excess of his new income above what it would have been if he had agreed to the proposed conversion.

175. If 15 men, 12 women and 9 boys can complete a piece of work in 50 days, what time would it take 9 men, 15 women and 18 boys to do 4 times as much, the parts done by each in the same time being as 3 : 2 : 1?

176. A and B enter into business together; and A provides $\frac{3}{5}$ of the capital. At the end of the 1st year each increases his capital employed by $3\frac{1}{2}\%$, and at the end of the 2nd by 5 %. The third year is a bad one, and at the end of it they dissolve partnership. The assets amount to £16663. 10*s.*, and each receives only 10*s.* in the £ on his share of the capital. How much did each originally provide?

177. A and B each invest a certain sum of money in a business. The sum which A invests is $\frac{2}{3}$ of that which B invests. At the end of 7 months A withdraws $\frac{1}{4}$ of his capital, and at the end of 9 months B withdraws $\frac{1}{4}$ of his. The profits at the end of the year are £132. 12*s.* How ought this sum to be divided?

178. If 5 persons contribute 60 dollars each, and employ 72 % of this joint capital in the purchase of meat at 18*s.* for 28 lb.; then, if dollars be exchanged at 4*s.* 2*d.* each, what fraction of a ton of meat will be obtained?

179. Find by how much the square root of $9+\cfrac{1}{1+\cfrac{1}{7+\frac{1}{6}}}$ differs from $\frac{355}{113}$. Which comes the nearer to $3+\frac{1}{10}\sqrt{2}$?

180. If 60 cu. in. of lead together with 54 cu. in. of cork are equal in weight to $1538\frac{2}{3}$ cu. in. of fir, and the sp. gravities of lead and fir are 11·324 and ·45, what is the sp. gravity of cork?

181. A sum of £3170. 7*s.* 6*d.*, not due for 3 years from the present time, is to be paid by three equal instalments at the end of one, two and three years from the present time. Money being worth 4 per cent. per annum, find what each instalment must be, (i) allowing simple interest, (ii) allowing compound interest.

182. If the duty on a certain commodity were reduced by 20 per cent. of the present amount, by how much per cent. must the consumption be increased that the same revenue may be derived from it? and by how much that the revenue may be increased 20 per cent.?

183. A and B were partners in business, and when the profits were divided, A received 5 per cent. on the capital he invested, and B 4 per cent. on his. A's share of the profits was the same as B's, and the whole capital was £22500. How much did each receive?

184. A person, set to calculate the decimal equivalent of $\frac{130909}{8371247}$, inadvertently transposes the 2 and 4 in the divisor, and in his third line of subtraction writes 20... where he should write 19.... If he makes no other error, show (by the simplest approximations which will give the result) that his answer will fall short of the true answer by more than ·00001 and less than ·00003.

185. A person, who has £9600 of a Railway stock on which 5 per cent. per annum is guaranteed, receives for his half-year's dividend the guaranteed interest and a further bonus of 5 per cent. for the half-year; find the amount of his dividend when an income-tax of 5*d.* in the £ has been deducted from it.

186. Supposing the cost of making a tunnel to vary jointly as the length of it and the quantity of earth excavated, and that the cost of making one 200 yards long was £20000, what will be the cost of making one a mile long, and half as wide again and half as high again as in the former case, the transverse sections being similar?

187. Supposing the Channel tunnel were constructed at a cost of 30 millions of money, that 100 trains per day passed each way, and that each train carried 200 tons of goods; what must be the freight per ton that the goods traffic alone should produce sufficient to pay 4 per cent. per annum on the outlay, it being assumed that there are annually 360 days on the average during which work might be carried on?

188. Two boats, A and B, row a race. A takes 4 strokes to B's 5, but 6 of B's are equal to 5 of A's. A starts in front of B at such a distance that B must take 10 strokes to row over it. How many strokes must B take before overtaking A?

189. A tradesman professes to retail his goods at 10 per cent. profit, but adulterates them with one-fourth of their weight of an inferior article which costs him two-thirds of the price of the better. How much per cent. profit does he really make? Also, in what proportion must he mix the two kinds so as to gain 50 per cent.?

190. A sum of £38. 16*s.* 8*d.* is to be divided among 25 men, 13 women, 7 boys and 11 girls. For every florin that a man gets, a boy gets 5*d.*, and for every half-crown that a woman gets, a girl gets 7*d.*

The whole money obtained by the boys is equal to that obtained by the girls. How much does each person get?

191. A person buys five shares in a concern at 40; next year the price is 45, and every succeeding year there is a fall of 4. Each year he sells out 1 share, and finds that at the end he has neither gained nor lost. What interest did the concern pay?

192. A creditor, holding a bill which falls due in 6 months, agrees to accept $\frac{1}{2}$ the amount at once, and the rest 1 year hence. Reckoning at the time when the bill would originally have fallen due, he finds that he is thus 9*s.* better off. Find the amount of the bill: interest at 6 % per annum.

193. A starts in trade with a capital of £624; after 4 months B joins him with a capital of £728; after 2 months more C joins the firm with £312 capital. At the end of 12 months from A's starting the profits are £316. 6*s.* 8*d.* What are their shares?

194. A rectangular fold is to be made with hurdles 6 feet long, to contain 1000 sheep, allowing at least 8 sq. ft. per sheep. Find the number of hurdles needed for the cases in which one side of the fold consists of 10, 11, ... 20 hurdles. Draw a graph shewing the relation between the total number of hurdles required and the number in one side. Hence find the smallest number of hurdles which will suffice.

195. A and B, starting at opposite corners of a square whose side is 100 yards long, walk round it in the same direction. A walks 19 yards while B walks 16. At what part of the square will they be together, and how many times will each have passed the corner from whence he started?

196. A tradesman has a certain article to sell. One intending purchaser offers him £22. 2*s.* at the end of one year from the purchase. Another offers him £11. 1*s.* at the end of six months, and £11. 1*s.* at the end of a year from the purchase. Which is the better offer, and by how much, money being worth 4 per cent. per annum?

197. A contractor agrees to execute a certain piece of work in a certain time; he employs 55 men who work 9 hours daily; when $\frac{3}{4}$ of the time is expired he finds that only $\frac{3}{7}$ of the work is done. How many additional men must he employ during the rest of the time, working 11 hours daily, in order that he may fulfil his contract?

198. A and B run a mile race. A goes 5 feet at each step, and takes 3·3 steps per second all through the race. B goes 6 feet at each step, and takes 3 steps per second for $\frac{3}{4}$ mile; but in the last $\frac{1}{4}$ mile he only goes 5·5 feet at each step, and takes 2·5 steps per second. Which won the race, and what time did each take?

199. Two men start with the same salary, £100 per annum. One gets a rise of 5 % on his salary for the previous year at the end of every year, and the other a rise of 10 % on the last year's salary at the end of every 2 years. Find, to the nearest penny, the difference in their salaries in the fifth year.

200. Two boats row a race over a straight course 1 mile 995 yards long, their rates of speed being 12 miles and $11\frac{23}{29}$ miles an hour respectively. Assuming that sound travels at the rate of 1140 feet in a second, find how much the faster boat will be ahead of the other when the sound of the gun fired at starting is heard at the winning-post.

201. A fast train leaves a place A for another place B at the same moment that a slow train leaves B for A. The fast train takes 2 hours on the journey, and if the slow train kept its time they would meet at a distance from B equal to $\frac{3}{5}$ of the whole distance from A to B. Instead of this, the place where they meet is at a distance from B equal to $\frac{1}{4}$ of the distance from A to B. How much behind time will the slow train be when it arrives at A?

202. A mass of lead ore weighed 7 tons; one portion of it yielded lead 78 per cent., and silver 8 oz. per ton: the remaining portion yielded lead 75 per cent., and silver $7\frac{1}{2}$ oz. per ton. The total yield of silver was 55 oz. What average per cent. of lead did the whole mass contain?

203. Teak is 45 lb. to the cubic foot, while water is $62\frac{1}{2}$ lb., and a certain kind of iron is 7·492 times as heavy as water. What thickness of iron will be of the same weight as a 4-inch plank of teak?

204. A postman has to walk 10 miles in 3 hours; he walks the first 4 miles at the rate of $3\frac{3}{4}$ miles an hour, the next 2 at the rate of $3\frac{1}{3}$ miles an hour and the next mile at 3 miles an hour. At what rate must he walk the rest to finish his journey at the proper time?

205. Two trains on the same railway are running past each other in opposite directions, one at 40 and the other at 30 miles per hour. Each has an engine and tender: the first has 12 carriages and the second 17. If the length of an engine and tender be 40 ft., and the length of a carriage 32 ft., and the coupling spaces be each 5 ft., how much time will elapse from the moment that the engines meet till the last carriages of each train have passed each other?

206. An up train 88 yds. long, travelling at the rate of 35 miles an hour, meets a down train 88 yds. long at 12 o'clock, and passes it in 6 seconds. At 15 min. 6 sec. past 12 o'clock the up train meets a second down train 132 yds. long, and passes it also in 6 seconds. At what time will the second down train run into the first?

207. During a distillation the fluid that comes over in 3 consecutive hours contains 53, 50 and 48 per cent. of alcohol respectively. The rates at which it comes over during these 3 hours are as 1, 2 and 3. What is the percentage of alcohol in the whole mixture?

208. I bought a parcel of nuts at 49 for 2*d*. I divided the parcel into two equal parts, one of which I sold at the rate of 24 and the other at the rate of 25 nuts for 1*d*. I spent and received an integral number of pence, but bought the least possible number of nuts. How many nuts did I buy, what did they cost me, and what did I gain?

209. In a distilling operation, during 5 hours the fluid contained 62·5 per cent. of alcohol, during $4\frac{1}{2}$ hours 31·4 per cent., and during the

remaining 3 hours 18·4 per cent. What is the average strength of the whole fluid distilled over, assuming that it came over at a uniform rate during the whole time?

210. During a distillation the fluid that comes over in three consecutive hours contains 51, 50 and 47·5 per cent. of alcohol respectively. But it comes over twice as fast in the last two hours as in the first hour. What is the percentage of alcohol in the whole when the three are mixed together?

211. A tradesman defrauds his customers (i) by an adulteration of the article to the extent of 7 per cent., (ii) by using a balance which indicates 1 lb. when the amount in the scale is really 15 oz. Which of these practices is the more fraudulent, and to what extent is the customer cheated in the first case when he orders 1 lb. of the commodity?

212. Two persons, A and B, being on opposite sides of a wood which is 476 yards about, begin to go round it both the same way at the same instant of time; A, who is 250 yards in advance of B at starting, saunters at the rate of 11 yards per minute, and B at 34 yards in 3 minutes; how many times will A have gone round the wood before the quicker overtakes the slower?

213. 20 English navvies, each earning 7*s.* a day, can do the same piece of work in 15 days that it takes 28 foreign workmen, each earning 6 francs a day, to complete in 20 days. If a piece of work done by navvies cost £3000, what would be the cost of the same work done by foreign workmen? [£1 = 25 f.]

214. Standard gold contains $\frac{1}{12}$ of alloy, and 480 oz. troy of standard gold is minted into 1869 sovereigns. How many grains of pure gold are in a sovereign? If there were one grain less of pure gold in a sovereign, what would be the loss sustained by the owner of a sovereign if the coin became defaced, and he received for it the value of the pure gold in it at £3. 17*s.* 9*d.* per oz.?

215. A tradesman's stock in trade is valued on January 1st, 1903, at £8000; he has also £350 in cash, and owes £1870; during the year his personal expenses, £300, are paid out of the proceeds of his business, and on January 1st, 1904, his stock is valued at £7950, he has £570 in cash, and he owes £1510. What is the whole profit on the year's transactions, after deducting 5 per cent. interest on the assets with which he began the year?

216. A contractor employs a fixed number of men to complete a work. He may employ either of two kinds of workmen: the first at 26*s.* 6*d.* per week each, the second at 18*s.* 6*d.* per week each; the work of one of the former is to that of one of the latter as 5 to 4. If he finishes it as quickly as possible, he spends £270 more than he would have done if he had finished it as cheaply as possible, but takes 4 weeks less time. What would it have cost if he had employed equal numbers of the two kinds of workmen?

217. Two monkeys, having stolen a pile of walnuts and filberts from a garden, are on the point of beginning their feast, when they see the injured owner of the nuts approaching with a stick. At once they see that he will take $2\frac{1}{2}$ minutes to reach them. There are twice as many filberts as walnuts, and one monkey finishes the latter at the rate of 15 a minute in $\frac{4}{5}$ of the time and runs away; the other manages to eat the filberts *just* in time. If the first monkey had stopped to help the other till all were finished, find when they would have got away, (i) if they eat filberts at equal rates, (ii) if the first monkey eats filberts at the same rate as he eats walnuts.

218. A coast-guardsman near the seashore sees the flash of a gun fired in a vessel steaming directly towards him, and hears the report in 15 sec. He then walks towards the ship at the rate of 3 miles an hour and sees a second flash 5 minutes after the first, and immediately stops; the report follows in $10\frac{1}{3}$ sec. Find the speed of the ship, taking the velocity of sound at 1200 feet per second.

219. The pressure of compressed air varies inversely as its volume. If the pressure on the inner surface of a cylinder, fitted with a piston, be 20 lb. on the sq. inch, and when the piston is forced in 2 in. the pressure becomes 30 lb. on the sq. in., what is the length of the cylinder?

220. A contractor agrees to sink an artesian well on the following terms: £5 per fathom for the first 30 fathoms consisting of gravel only, £35 per fathom through sand, £20 per fathom through clay and £25 through chalk. He found the thickness of the chalk to be equal to half the depth of the well, and that the sand beds were 3 times as thick as the clay. At the end of the work he got £2700 for his trouble. What was the depth of the well?

221. The height of a solid 6 in. cube is diminished by pressure to $5\frac{3}{5}$ in.; supposing the lateral expansion to be uniform throughout the mass, find, correct to 3 places of decimals, the length of an edge of a new square face, if the volume remain unaltered.

222. A prisoner of war, in attempting to escape to a friendly ship moored within sight of land, swims out at the rate of 3 miles per hour, and is assisted by the tide, which ebbs at the rate of 200 yards per hour. After he had been swimming for $\frac{3}{4}$ of an hour, the tide turns and impedes him as much as it had previously assisted him. He also slackens his speed from 3 to $2\frac{1}{2}$ miles per hour, and succeeds in reaching the vessel in another hour. How far was the vessel from the shore?

223. If a company of 160 men in 6 days of 11 hours each can dig a trench 230 yds. long, $5\frac{1}{2}$ wide and $1\frac{1}{2}$ deep, in how many days of 8 hours each would 96 men dig a trench 220 yds. long, $3\frac{1}{2}$ wide and 1 deep, supposing the hardness of the ground in the former case is to that in the latter as 5 : 7, and that 4 men of the latter company can do as much work as 5 of the former in the same time?

224. A vessel is sent to the Arctic ocean to catch whales, and the captain, who is entitled to $\frac{1}{18}$ of the entire produce of oil, receives at the end of a 4 years' voyage £735. Supposing the oil to be worth

60 guineas per tun (each whale on an average yielding 6 tuns), and the crew to have been $\frac{2}{3}$ of the entire period on the look out, what was the average number of days between the captures.

225. A company borrows £1,658,775 on the understanding that at the end of each year a portion of the principal is to be paid off with interest at 4% on the amount standing unpaid during that year. If the annual payments are equal, find the amount which must be paid each year in order that the debt may be cleared off in 4 years.

226. A person buys 6% bonds, the interest on which is payable yearly, and which are to be paid off at par 3 years after the time of purchase. If money be worth 5 per cent., what price should he give for the bond, reckoning Compound Interest?

227. Three boys begin to fill a 53-gallon cistern: one of them brings a pint at the end of every 3 min., another a quart every 5 min. and the third a gallon every 7 min. In what time will it be filled?

228. Three men are employed to dig an acre of land. A can dig 38 perches in 6 days, B can dig 60 perches in 8 days and A, B and C can do the whole in 7 days. What are the three lowest integral numbers expressing their relative working powers, and in what time would A and C do five-sixths of the work?

229. Four men and 10 boys together do one-half of a piece of work in 6 days. Two more men and 2 more boys having joined them, another one-third of the work is done in the next 3 days: how many more boys must then be put on if the rest of the work is to be finished in one day more?

230. Three men run a mile race: the first beats the last by $76\frac{2}{3}$ yds.: the second beats the last by 11 sec.; and the pace of the first is to that of the second as 45 : 44. In what time does each run the mile?

231. A and B row on a river, starting simultaneously from the same point, A down stream, B up stream. In $3\frac{3}{4}$ min. they are 5 furlongs apart; A then turns to follow B, and at the end of $12\frac{1}{2}$ min. from that time the boats have together rowed 1 mi. 7 fur. in all. If the speeds of A, B and the stream are constant, find the rate of the stream.

232. A tradesman spends £1500 in buying cloth in England at 3*s.* 6*d.* a yard, takes it to France at a cost of 2*d.* per yard and pays duty on it of 40 centimes a metre. Half of the goods, being damaged, he sells at a loss of 20% on their total cost to him and the rest at 7 francs a metre. Find his gain. [25 f. = £1, 1 metre = $39\frac{3}{8}$ in.]

233. The number of passengers in the 2nd class carriages of a train was three times the number in the first class, and the number in the 3rd class exceeded the other two together by 36: the fares for the several classes were at the rate of 3*d.*, 2*d.* and 1*d.* per mile: the average distance travelled by each of the 1st class passengers was 120 miles, by each of the 2nd class 50 miles and by each of the 3rd class 25 miles: and the whole of the fares amounted to £390. 1*s.* 8*d.* How many passengers were there of each class?

ANSWERS TO EXAMPLES.

ANSWERS TO EXAMPLES.

PART I.

EXAMPLES 3. PAGE 13.

1. 32700. **2.** 282008. **3.** 1163863. **4.** 3039679. **5.** 1885.
6. 8530586. **7.** 96124. **8.** 873503. **9.** 1069033. **10.** 3219254.

EXAMPLES 4. PAGE 21.

1. 110718, 1217898, 123007698, 12228188, 122921584, 1229953960, 61510, 307550, 1537750.

2. 605709, 6662799, 672942699, 66897194, 672471592, 6728753980, 336505, 1682525, 8412625.

3. 5059287, 55652157, 5620867857, 558770142, 5616932856, 56203057140, 2810715, 14053575, 70267875.

4. 4388589, 48274479, 4875722379, 484695274, 4872309032, 48752347580, 2438105, 12190525, 60952625.

5. 2946708, 32413788, 3273792588, 325447528, 3271500704, 32734651760, 1637060, 8185300, 40926500.

6. 68597865, 754576515, 76212228015, 7576253090, 76158874120, 762046060300, 38109925, 190549625, 952748125.

7. 13733174. **8.** 2295864. **9.** 2558192. **10.** 800290491.
11. 448602324. **12.** 67259907. **13.** 996275287620. **14.** 9254146475076.
15. 39768021981. **16.** 178501325263625. **17.** 97. **18.** 7. **19.** 118.
20. 217. **21.** 1042. **22.** 2197. **23.** 523920.
24. 556665. **25.** 589410. **26.** 622155. **27.** 1342545.
28. 2652345. **29.** 3307245. **30.** 3962145. **31.** 108.

EXAMPLES 5. PAGE 23.

1. 4, 13, 27, 98, 99. **2.** 159, 264, 338, 509, 590. **3.** 690, 628, 609.
4. 499, 721, 429. **5.** 1200, 1660, 1708. **6.** 999, 1990, 2858.
7. 11286, 13694. **8.** 61201, 60010, 82605. **9.** 59452, 99419.
10. 560749. **11.** 2651109.

EXAMPLES 6. PAGE 24.

1. IV, XIV, XXIV, XXXIV, XLIV, LIV, LXIV, LXXIV, LXXXIV, XCIV, CIV, CXIV.
2. IX, XIX, XXIX, XXXIX, XLIX, LIX, LXIX, LXXIX, LXXXIX, XCIX, CIX, CXIX.
3. CXLIX, CXLIV, CXCIV, CXCIX.
4. CD, CDXL, CDXLIV, CDXLIX, CDXC, CDXCIV, CDXCIX.
5. CM, CMXL, CMXLIV, CMXLIX, CMXC, CMXCIV, CMXCIX.
6. MDLXXXIV, MCDXCIX, MDCII, MCCCLI, MDCCCLXXXV.
7. MMML, MMMCXLIX, MMCXXXIX, MMMCCLXXIII, MMDCCLXVIII.
8. $\overline{\text{V}}$DCLXX, $\overline{\text{VIII}}$CMLIII, $\overline{\text{VII}}$CCCXX, $\overline{\text{VI}}$DCLXVI, $\overline{\text{VIII}}$DCCCLXXXVIII.
9. $\overline{\text{LXVII}}$CCXLI, $\overline{\text{XL}}$LXIX, $\overline{\text{XCVIII}}$DCCLXII, $\overline{\text{XLV}}$DCCXCVIII, $\overline{\text{XCIX}}$CMXCIX.
10. MDLXV$\overline{\text{II}}$DCCCXL, $\overline{\text{MMMCDLXXV}}$IDXXI, $\overline{\text{MMMV}}$DCCLXIII.

EXAMPLES 7. PAGE 27.

4. (i) 512, 11172, 14916, and 601830; (ii) 795, 11172, 2859, 14916, 38187 and 601830; (iii) 512, 11172, and 14916; (iv) 795, 1235, and 601830; (v) 11172, 14916, and 601830; (vi) 512; (vii) 38187 and 601830; (viii) 14916; (ix) 11172 and 14916.

EXAMPLES 8. PAGE 27.

1. $2^2.3.5$. 2. 3.5^2. 3. $2.3.13$. 4. 2.7^2. 5. $2.3.17$.
6. $3.5.7$. 7. $2.3.5.7$. 8. $3.7.11$. 9. $2^2.3^2.7$. 10. $2^3.3.7$.
11. $3^2.5.7$. 12. $3.11.13$. 13. $3.5^2.7$. 14. $2.3^3.11$. 15. $2^2.3^2.17$.
16. $3^2.7.11$. 17. $5.11.13$. 18. $2^3.3.5.7$. 19. $2.7.61$.
20. $3^2.11^2$. 21. 11^3. 22. $2.5.11.13$. 23. $2^4.7.13$. 24. $3^2.5.7.11$.
25. $3.5^2.7^2$. 26. $2^3.3^4.7$. 27. $2^2.3.5.7.11$. 28. $3^2.7^2.11$.
29. $2.3.5^3.7$. 30. $3^2.7.11^2$. 31. $3^2.5^2.7^2$. 32. $3^2.11^2.13$.
33. $3^2.7.11.41$. 34. $2.3^2.5.7^2.11$. 35. $5^3.11^2.17$.
36. $2^4.3^2.7.89$. 37. $7^4.13^2$. 38. $2^2.3.11^3.47$.
39. $2.3.5^3.7.11.13$. 40. $2^5.3^3.5^3.11^2.43$.

EXAMPLES 9. PAGE 29.

1. 384.	2. 445.	3. 7300.	4. 1575.	5. 39400.
6. 3215.	7. 22000.	8. 79152000.	9. 31960000.	10. 399.
11. 9975.	12. 8091.	13. 159879.	14. 359375.	15. 809919.
16. 1439991.	17. 1209375.	18. 1689856.	19. 11025.	20. 12544.
21. 14161.	22. 478864.	23. 672400.	24. 422500.	25. 992016.
26. 99960004.	27. 100060009.			

EXAMPLES 10. PAGE 32.

1. (i) 3447; (ii) 17567; (iii) 1590 or 1596; (iv) 701217; (v) 63525.
7. Four. **11.** When n is odd. **12.** When n is even.
13. When n is a multiple of 3; 9009009009.
14. Of three consecutive odd numbers, one is always a multiple of 3.
15. The error is a multiple of 99

EXAMPLES 12. PAGE 39.

1. 9. **2.** 9. **3.** 18. **4.** 33. **5.** 6. **6.** 24. **7.** 9. **8.** 6.
9. 38. **10.** 22. **11.** 240. **12** 117. **13.** 36. **14.** 39. **15.** 12. **16.** 672.
17. 87. **18** 39. **19.** 53 **20.** 5544. **21.** 649. **22.** 121. **23.** 3. **24.** 1155.
25. 3009. **26.** 9. **27.** 3. **28.** 81. **29.** 2. **30.** 2. **31.** 12. **32.** 75.
33. 442. **34.** 8. **35.** 10. **36** 23. **37.** 2. **38.** 2. **39.** 121. **40.** 117.
41. 422, 633, 844, 1055, 1266. **42.** 2, 6, 17. **43.** 5 Kilog. = 11 pounds.
44. 24 francs = 19 shillings.

EXAMPLES 14. PAGE 43.

1. 120. **2.** 294. **3.** 252. **4.** 180. **5.** 1260. **6.** 360.
7. 195. **8.** 1680. **9** 4200. **10.** 1620 **11.** 12600. **12.** 360.
13. 3822. **14.** 5040. **15.** 288. **16.** 1716. **17.** 13728. **18.** 3360.
19. 1320. **20.** 7056. **21.** 20475. **22.** 476. **23.** 5250. **24.** 240.
25. 240. **26.** 180. **27.** 1008. **28.** 63000. **29.** 1260. **30.** 1080.
31. 33696. **32.** 5775. **33.** 65520. **34.** 2520. **35.** 7920. **36.** 31920.
37. 4800. **38.** 3780. **39.** 3556. **40.** 23400. **41.** 778050. **42.** 77489220.
43. 39648. **44.** 7956. **45.** 16170. **46.** 940500. **47.** 432432. **48.** 453492.
49. 41580 pounds. **50.** 13. **51.** 327638. **52.** 3360 inches; 120, 112, 105.
53. 144, 120, 100, 75. **54.** 8226240 ft. = 1558 mi. = 1353 knots = 2508 Km.
55. 453. **56.** 9153. **57.** 15, 165, 75, 825.

EXAMPLES 15. PAGE 45.

1. Fifteen million, six thousand, and twenty-one; 1000.
2. 7013501. **3.** 71478. **4.** 118682878. **5.** 1864070025; 34522576863.
6. 2729. **7.** 87. **8.** 7920. **9.** 3468.
10. Nine million, ten thousand, two hundred and one; 104007082.
11. 28076. **12.** 8929248. **13.** 7484125; 293796811.
14. 10. **15.** $2^3.3^2.5.7.11$. **16.** 1848. **17.** 3360.
18. 120. **19.** 501000550050. **20.** 130130. **21.** 45110.
22. 2800, 2607, 2400. **23.** 2376. **24.** 459000.
25. 12. **26.** 1330 miles. **27.** (i) 7; 5, 4; 5, 0. (ii) 6, 8; 4; 5, 3.
28. 384, 521, 637. **30.** A, 115; B, 77; C, 88.
31. 31, 32, 33. **32.** 71280. **33.** 360360. **34.** 492.
35. 7, 140; or 35, 28. **36.** 188121. **37.** A, 45; B, 24.
38. 987. **39.** 2378. **40.** 13. **41.** 54528.
42. 95, 99. **43.** 28618. **44.** 27. **45.** 11.

46. 2886353. **47.** A to B, 25 mi. ; B to C, 15 mi. ; C to D, 35 mi.
48. 5. **49.** 1982. **50.** 4. **51.** 23.
52. 609840. **53.** 42. **54.** (i) 5, 6, 8 ; 1, 9 ; (ii) 7 ; 7, 8 ; 9, 3, 9.
55. 314 red, 291 white, 180 blue. **56.** 5 times. **57.** 317450.
58. 0, 0, 4, 0, 0. **59.** 21605. **60.** 23185. **61.** $2^6 . 3 . 7^4$.
62. 21 gallons. **63.** (i) 34578×9 ; (ii) 47568×7. **64.** 30244, 7561.
65. 537915, 536389, 409846976, 704. **66.** 17, 51.
67. quot. 9985 ; rem. 141. **68.** 56. **69.** 1009.
70. Composite, 67×109. **72.** 823×15 ; 975×18.
73. Multiply the tens' digit by itself increased by unity, and annex 25 to the right of the product.
74. 22265 ; 61. **75.** 8371. **76.** 999663 ; 100203.
77. $2^6 . 3 . 13 . 53$. **78.** 13, 14, 15, 16. **79.** 971. **80.** 324.
81. (i) 1524×32 ; (ii) 3413×97.
82. Four billion, one hundred and sixty-seven thousand three hundred and six million, two thousand, and fifteen.
83. A, 50 ; B, 20. **84.** quot. 60748 ; rem. 73. **85.** 882491355. **86.** 29
87. 3 ways. **88.** 997. **89.** 3780 miles.
90. (i) $3425 \div 8$; (ii) $34568 \div 7$.

EXAMPLES 17. PAGE 55.

1. 820, 1060, 2100, 7360, 145680. **2.** 1027, 2392, 4662, 112259.
3. 10800, 16080, 46800, 1470000, 11493600.
4. 1272, 30168, 191580, 823092. **5.** 800, 3650, 56370, 319180, 1151562.
6. 16536, 46404, 143840, 352452, 5396776.
7. 47337, 169695, 766686, 4332718, 94780773.
8. (i) 60, 512, 26900, 195744, 833884 ; (ii) 600, 5120, 269000, 1957440, 8338840.
9. (i) 35, 58, 1309, 20759, 68517 ; (ii) 525, 870, 19635, 311385, 1027755.
10. (i) 57, 698, 3323, 6535, 15366 ; (ii) 285, 3490, 16615, 32675, 76830.
11. (i) 85, 275, 4108, 25613, 383302 ; (ii) 850, 2750, 41080, 256130, 3833020.
12. 459120, 174888, 156000.

EXAMPLES 19. PAGE 57.

1. $167\frac{3}{4}$, $2107\frac{1}{4}$, $14030\frac{3}{4}$, $153086\frac{3}{4}$, $218240\frac{3}{4}$.
2. 39*s.* 4*d.*, 271*s.* 9*d.*, 4683*s.* 7*d.*, 11907*s.* 7*d.*, 285672*s.* 8*d.*
3. £17. 2*s.* ; £117. 9*s.* ; £4236. 5*s.* ; £21605. 9*s.* ; £428600. 18*s.*
4. £1. 19*s.* 4*d.* ; £37. 4*s.* 4*d.* ; £19. 19*s.* 1*d.* ; £258. 13*s.* 8*d.* ; £15503. 14*s.* 7*d.*
5. 15*s.* $5\frac{3}{4}$*d.* ; 43*s.* $6\frac{1}{4}$*d.* ; 754*s.* $2\frac{1}{4}$*d.* ; 17652*s.* $0\frac{1}{2}$*d.* ; 1322463*s.* $6\frac{3}{4}$*d.*
6. £11. 14*s.* $2\frac{1}{2}$*d.* ; £1754. 7*s.* $0\frac{1}{2}$*d.* ; £7559. 6*s.* $3\frac{1}{2}$*d.* ; £101172. 17*s.* $9\frac{1}{2}$*d.*
7. £6. 9*s.* $10\frac{1}{2}$*d.* ; £60. 10*s.* $6\frac{1}{2}$*d.* ; £71325. 2*s.* $0\frac{1}{4}$*d.* ; £585252. 3*s.* $6\frac{3}{4}$*d.*
8. 4034, 29689, 176766, 3115818 guineas.
9. 375 half-guineas. **10.** 4788. **11.** 82, 410, 820.
12. 144. **13.** 115, 5750. **14.** 7000. **15.** 400.

EXAMPLES 21. PAGE 60.

1. 2*s.* **2.** 1*s.* 8½*d.* **3.** 1*s.* 1*d.* **4.** 2*s.* 5¼*d.* **5.** 3*s.* 6*d.*
6. 18*s.* 10*d.* **7.** £1. 2*s.* 10*d.* **8.** £2. 3*s.* 9*d.* **9.** £1. 8*s.*
10. £1. 0*s.* 11⅜*d.* **11.** £1. 17*s.* 9⅜*d.* **12.** £2. 0*s.* 4⅜*d.* **13.** £2. 5*s.* 1*d.*
14. £6. 11*s.* 4*d.* **15.** £16. 16*s.* 11*d.* **16.** £21. 0*s.* 2*d.* **17.** £39. 9*s.* 0¼*d.*
18. £43. 14*s.* 5*d.* **19.** £69. 5*s.* 3⅝*d.* **20.** £72. 14*s.* 5½*d.*
21. £54. 11*s.* 3⅜*d.* **22.** £95. 17*s.* 10¼*d.* **23.** £109. 5*s.* 10½*d.*
24. £164. 5*s.* 0*d.* **25.** £239. 16*s.* 3½*d.* **26.** £255. 14*s.* 10⅜*d.*
27. £230. 1*s.* 5½*d.* **28.** £349. 10*s.* 5½*d.* **29.** £451. 7*s.* 8¼*d.*
30. £499. 2*s.* 0¼*d.* **31.** £429. 18*s.* 3¼*d.* **32.** £454. 8*s.* 10*d.*
33. £454. 11*s.* 10¾*d.* **34.** £1196. 1*s.* 4½*d.* **35.** £1353. 0*s.* 9*d.*
36. £1947. 0*s.* 5¼*d.* **37.** £2004. 5*s.* 1½*d.* **38.** £28681. 14*s.* 11⅝*d.*
39. £48185. 14*s.* 6*d.* **40.** £420186. 12*s.* 11½*d.*

EXAMPLES 23. PAGE 63.

1. 8*s.* 3*d.* **2.** £2. 3*s.* 3*d.* **3.** £12. 6*s.* 4*d.* **4.** £22. 3*s.* 11*d.*
5. £2. 2*s.* 8*d.* **6.** £20. 19*s.* 4*d.* **7.** £11. 15*s.* 9*d.* **8.** £16. 14*s.* 7¾*d.*
9. £85. 15*s.* 9¾*d.* **10.** £43. 15*s.* 8¾*d.* **11.** £108. 17*s.* 11¾*d.*
12. £107. 6*s.* 4¾*d.* **13.** £55. 16*s.* 9½*d.* **14.** £38. 17*s.* 8¾*d.*
15. £122. 12*s.* 11*d.* **16.** £175. 16*s.* 8½*d.* **17.** £2684. 15*s.* 6½*d.*
18. £1777. 14*s.* 6*d.* **19.** £2426. 13*s.* 2¾*d.* **20.** £20671. 19*s.* 8*d.*
21. £9954. 16*s.* 10*d.* **22.** £22706. 14*s.* 8*d.* **23.** £116590. 18*s.* 10*d.*
24. £23258. 16*s.* 10*d.* **25.** £233773. 12*s.* 11*d.* **26.** £20. 18*s.* 7*d.*
27. £27. 10*s.* 4¼*d.* **28.** £115. 1*s.* 10½*d.* **29.** £30. 18*s.* 9¼*d.* **30** £264. 17*s.* 6*d.*

EXAMPLES 24. PAGE 66.

1. £86. 13*s.* 4*d.*; £121. 6*s.* 8*d.*: £156; £190. 13*s.* 4*d.*
2. £207. 10*s.*; £311. 5*s.*; £415; £518. 15*s.*
3. £251. 0*s.* 4½*d.*; £585. 14*s.* 2½*d.*; £669. 7*s.* 8*d.*; £920. 8*s.* 0½*d.*
4. £299. 1*s.* 1½*d.*; £1046. 13*s.* 11¼*d.*; £1196. 4*s.* 6*d.*; £1794. 6*s.* 9*d.*
5. £2712. 6*s.* 4¼*d.*; £3254. 15*s.* 7½*d.*; £5424. 12*s.* 8½*d.*; £6509. 11*s.* 3*d.*
6. £96. 15*s.*; £129; £193. 10*s.*
7. £167. 14*s.* 8*d.*; £230. 12*s.* 8*d.*; £293. 10*s.* 8*d.*
8. £387. 19*s.* 4½*d.*; £496. 12*s.*; £543. 3*s.* 1½*d.*
9. £722. 15*s.* 7*d.*; £1445. 11*s.* 2*d.*; £1626. 5*s.* 0⅜*d.*
10. £1445. 6*s.* 4¼*d.*; £1858. 5*s.* 3¾*d.*; £2271. 4*s.* 3¼*d.*
11. £4173. 11*s.* 3*d.*; £4869. 3*s.* 1½*d.*; £6260. 6*s.* 10½*d.*
12. £18009. 14*s.* 2*d.*; £25213. 11*s.* 10*d.*; £49526. 13*s.* 11½*d.*
13. £78081. 16*s.* 1½*d.*; £116640. 14*s.* 5½*d.*; £72297. 19*s.* 4½*d.*
14. £481489. 2*s.* 6*d.*; £449389. 17*s.*; £513588. 8*s.*
15. £342274. 11*s.* 4*d.*; £825126. 3*s.* 9*d.*; £2292017. 3*s.* 9*d.*

EXAMPLES 25. PAGE 67.

1. £343. 19*s.* 7¾*d.*; £355. 12*s.* 10¼*d.*; £518. 17*s.* 9¼*d.*
2. £859. 8*s.* 0½*d.*; £2253. 0*s.* 6½*d.*; £1927. 16*s.* 11½*d.*
3. £4490. 8*s.* 2¼*d.*; £4035. 13*s.* 8¾*d.*; £4945. 2*s.* 8¼*d.*
4. £1581. 16*s.* 1¾*d.*; £1613. 2*s.* 7¼*d.*; £1675. 15*s.* 6¼*d.*
5. £1923. 3*s.* 4½*d.*; £1993. 14*s.* 10½*d.*; £2099. 12*s.* 1½*d.*
6. £2080. 1*s.* 11¼*d.*; £2547. 10*s.* 8¼*d.*; £2968. 4*s.* 6¾*d.*
7. £6159. 4*s.* 0¾*d.*; £7120. 4*s.* 3¼*d.*; £8561. 14*s.* 7*d.*
8. £30046. 11*s.* 4½*d.*; £34806. 0*s.* 2¾*d.*; £39855. 3*s.* 2¼*d.*
9. £6446. 18*s.* 10*d*; £31369. 14*s.* 5½*d.*; £23174. 2*s.* 10*d.*
10. £55290. 19*s.* 8*d.*; £28280. 6*s.* 2*d.*; £15506. 11*s.* 2*d.*
11. £46338. 5*s.* 1¼*d.*; £40638. 9*s.* 9½*d.*; £60522. 16*s.* 3*d.*
12. £18472. 5*s.* 6*d.*; £6737. 8*s.*; £16236. 9*s.*
13. £6602. 17*s.* 10*d.*; £3608. 19*s.*; £11489. 3*s.* 4*d.*
14. £45959. 6*s.* 8*d.*; £34736; £61230.
15. £20819. 10*s.* 11¼*d.*; £28688. 4*s.* 4½*d.*; £11450. 9*s.* 4½*d.*
16. £8975. 12*s.* 5*d.*; £15899. 16*s.* 5¾*d.*; £12734. 9*s.* 5¾*d.*
17. £17934. 2*s.* 11*d.*; £14719. 16*s.* 8*d.*; £13729. 15*s.* 5*d.*
18. £6122. 14*s.* 3½*d.*; £4225. 10*s.* 8½*d.*; £24788. 15*s.* 3*d.*
19. £17292. 5*s.* 6*d.*; £12962. 14*s.* 6¾*d.*; £11662. 15*s.* 8¼*d.*
20. £25525. 19*s.* 2*d.*; £41946. 13*s.* 4*d.*; £58291. 15*s.*

EXAMPLES 27. PAGE 70.

1. £14. 5*s.*; £11. 17*s.* 6*d.*
2. £78. 6*s.* 3d.; £59. 13*s.* 4*d.*
3. £50. 6*s.* 5½*d*; £40. 5*s.* 2*d.*
4. £65. 11*s.* 6¾*d.*; £21 17*s.* 2¼*d.*
5. £83. 14*s.* 5¾*d.*; £96. 9*s.*
6. £1192. 8*s.* 1½*d.*; £687. 1*s.* 0¾*d.*
7. £1. 17*s.* 7½*d.*; £2. 8*s.* 4½*a.*
8. £148. 11*s.* 2½*d.*; £424. 9*s.* 2*d.*
9. £12. 15*s.* 0¾*d.*; £8. 2*s.* 3¾*d.*
10. £30. 11*s.* 2¼*d.*; £25. 0*s.* 0¾*d.*
11. £10. 3*s.* 7½*d.*; £16. 19*s.* 4½*d.*
12. £191. 10*s.* 2½*d.*; £275. 15*s.* 6*d.*
13. £110. 3*s.* 1½*d.*; £282. 14*s.* 8¼*d.*

EXAMPLES 28. PAGE 71.

1. £1. 7*s.*
2. £1. 5*s.*
3. £1. 3*s.* 4*d.*
4. £1. 6*s.* 8*d.*
5. £1. 7*s.* 1*d.*
6. £1. 2*s.* 3½*d.*
7. £1. 6*s.* 7¼*d.*
8. 19*s.* 8¾*d.*
9. £1. 1*s.* 1¼*d.*
10. £1. 8*s.* 7¾*d.*
11. £2. 15*s.* 4½*d.*
12. £2. 7*s.* 5¾*d.*
13. £3. 18*s.* 6½*d.*
14. £12. 19*s.* 11¼*d.*
15. £23. 5*s.* 2½*d.*
16. £34. 16*s.* 3½*d.*
17. £23. 19*s.* 8¾*d.*
18. £14. 10*s.* 6¼*d.*
19. £33. 17*s.* 6¼*d.*
20. £64. 18*s.* 5¾*d.*
21. £148. 4*s.* 8½*d.*
22. £157. 3*s.* 4½*d.*
23. £281. 5*s.* 7½*d.*
24. £1473. 2*s.* 9¾*d.*

EXAMPLES 29. PAGE 73.

1. 29.	**2.** 53.	**3.** 87.	**4.** 43.	**5.** 47.
6. 57.	**7.** 111.	**8.** 52.	**9.** 29.	**10.** 29.
11. 42.	**12.** 73.	**13.** 44.	**14.** 148.	**15.** 251.
16. 541.	**17.** 1038.	**18.** 1683.	**19.** 1464.	**20.** 2709.

EXAMPLES 30. PAGE 74.

1. 452 f. 90 c.	**2.** 931 f. 93 c.	**3.** 464 m. 90 pf.	**4.** 244 dol. 30 c.
5. 122 f. 97 c.	**6.** 1216 f. 39 c.	**7.** 80 dol. 75 c.	**8.** 7719 m. 60 pf.
9. 1744 f. 32 c.	**10** 14530 f.	**11.** 97287 f. 75 c.	
12. 3183 dol. 45 c.	**13.** 22495 dol. 84 c.	**14.** 293940 m. 23 pf.	
15. 31 dol. 34 c.	**16.** 4 f. 56 c.	**17.** 14 f. 74 c.	
18. 11 m. 12 pf.	**19.** 90 f. 9 c.	**20.** 84 dol. 75 c.	

EXAMPLES 31. PAGE 75.

1. £1029. 15*s.* $2\frac{3}{4}d.$	**2.** £86055. 16*s.* $8\frac{1}{2}d.$	**3.** £11295949. 19*s.* 2*d.*		
4. B by 1*s.* 6*d.*	**5.** 13.	**6.** 5 seconds.	**7.** £1. 3*s.* 5*d.*	
8. £13692. 13*s.* $8\frac{3}{4}d.$	**9.** 96849.	**10.** 3720.	**11.** 15.	**12.** 2*s.* $10\frac{1}{2}d.$
13. £14.	**14.** 7*s.* $10\frac{1}{4}d.$	**15.** £17. 0*s.* $1\frac{3}{4}d.$; £5. 5*s.* $6\frac{3}{4}d.$		
16. 786926 farthings.	**17.** 1155.	**18.** 16.	**19.** £77. 7*s.* $7\frac{3}{4}d.$	
20. £11. 15*s.*	**21.** 15*s.*	**22.** 1716.	**23.** 1*s.* $4\frac{1}{2}d.$	
24. 13 times.	**25.** £1. 16*s.*	**26.** £600.	**27.** £87. 10*s.*, £62. 10*s*	
28. 42.	**29.** £2. 5*s.*		**30.** £157. 10*s.*	
31. 9600 f. 75 c.	**32.** 59.		**33.** 19 f. 25 c.	
34. 102 f.	**35.** 400.		**36.** 351 f.	
37. 329.	**38.** £134. 14*s.*		**39.** 201 yards.	
40. 13*s.* 6*d.*	**41.** 30; £1 11*s.* $10\frac{1}{2}d.$		**42.** £75. 12*s.*	
43. £3. 1*s.* 9*d.*	**44.** 100.		**45.** 178 litres.	

46. A, 25 f.; B, 28 f. 20 c.; C, 30 f. 30 c.; D, 31 f. 50 c.

47. 10 sovereigns, 40 shillings, 80 threepences.

48. 15 yds. **49.** 16, 32, 64, 208.

EXAMPLES 33. PAGE 80.

1. 3192, 4296.	**2.** 3576, 4728.	**3.** 3736, 4218.	**4.** 5606, 5517.
5. 36120, 71904.	**6.** 90143.	**7.** 344150.	**8.** 8870892.
9. 4308481.	**10.** 5777328.		

EXAMPLES 34. PAGE 81.

1. 21000.	**2.** 1897000.	**3.** 40075000.	**4.** 132.
5. 470.	**6.** 4656.	**7.** 2746.	**8.** 496.
9. 1104.	**10.** 10288.	**11.** 28087.	**12.** 476.
13. 12864.	**14.** 1763.	**15.** 11200.	**16.** 6769.
17. 253471.	**18.** 1706516.	**19.** 17700583.	**20.** 467344.

EXAMPLES 35. PAGE 82.

1. 7 dwt. 8 grs.; 19 dwt. 23 grs. 2. 60 dwt. 11 grs.; 99 dwt. 2 grs.
3. 49 oz. 7 dwt.; 38 oz. 4. 7 oz. 14 dwt. 19 grs.; 11 oz. 14 dwt. 5 grs
5. 18 oz. 3 dwt. 8 grs.; 48 oz. 18 dwt. 4 grs.
6. 20 oz. 15 dwt.; 236 oz. 1 dwt. 7. 288 oz. 3 dwt.; 315 oz. 7 dwt.
8. 30 oz. 15 dwt. 2 grs.; 98 oz. 10 dwt. 5 grs.
9. 745 oz. 1 gr. 10. 1322 oz. 9 dwt. 16 grs.
11. 3073 oz. 2 dwt. 15 grs. 12. 3255 oz. 3 dwt. 17 grs.
13. 175667 oz. 17 dwt. 3 grs. 14. 195302 oz. 13 dwt. 19 grs.

EXAMPLES 36. PAGE 82.

1. 9 lb. 6 oz. 2. 2 lb. 15 oz. 13 drs. 3. 297 lb. 11 drs.
4. 681 cwt. 1 qr. 20 lb. 5. 40858 cwt. 3 qrs. 21 lb.
6. 125268 cwt. 3 qrs. 1 lb. 7. 881665 cwt. 3 qrs. 16 lb.
8. 7 cwt. 10 lb. 9 oz. 9. 55 cwt. 3 qrs. 6 lb.
10. 209 cwt. 3 qrs. 21 lb. 2 oz. 11. 799 cwt. 12 lb. 14 oz.
12. 85 cwt. 2 qrs. 16 lb. 7 oz. 13. 3030 cwt. 22 lb. 2 oz.
14. 109504 tons 2 cwt. 15. 19 tons 10 cwt. 25 lb.
16. 24 tons 2 cwt. 1 qr. 19 lb. 17. 2438 tons 14 cwt. 3 qrs. 8 lb.
18. 6590 tons 12 cwt. 3 qrs. 23 lb.
19. 3839 tons 16 cwt. 3 qrs. 24 lb. 12 oz.
20. 212144 tons 7 cwt. 1 qr. 21 lb. 13 oz.

EXAMPLES 39. PAGE 88.

1. 120 in. 2. 26820 in. 3. 31853 in. 4. 7427 ft. 5. 9901 ft
6. 151146 in. 7. 166447 in. 8. 332722 in. 9. 871269 in.
10. 2906759 in. 11. 3231432 in. 12. 10961287 in. 13. 7279 ft. 6 in.
14. 2064 yds. 6 in. 15. 85 ch. 5 yds. 1 in. 16. 7 fur. 4 ch. 5 yds. 5 in.
17. 1 m. 4 fur. 2 ch. 21 yds. 2 ft. 8 in.
18. 4 mi. 5 fur. 5 ch. 16 yds. 2 ft. 10 in.

EXAMPLES 40. PAGE 90.

1. 14 dwt. 15 grs. 2. 9 oz. 18 dwt. 13 grs.
3. 24 oz. 17 dwt. 14 grs. 4. 49 oz. 12 grs.
5. 33 oz. 3 dwt. 20 grs. 6. 1 lb. 8 oz. 8 drs.
7. 1 lb. 8 oz. 6 drs. 8. 1 qr. 1 lb. 3 oz. 11 drs.
9. 1 qr. 5 lb. 10 oz. 2 drs. 10. 2 tons 8 cwt. 2 lb.
11. 2 tons 9 cwt. 2 qrs. 6 lb. 12. 77 tons 1 cwt. 1 qr. 16 lb. 9 oz.
13. 56 tons 5 cwt. 21 lb. 15 oz. 14. 146 g. 58 cg.
15. 1 g. 37 cg. 8 mg. 16. 28 g. 7 dg. 9 mg.
17. 16 Hg. 6 Dg. 18. 23 Kg. 424 g. 19. 14 Kg. 1 Hg. 4 Dg. 4 g
20. 17 t. 2 q. 42 Kg. 21. 10 t. 4 q. 29 Kg. 5 Hg.

EXAMPLES 41. PAGE 91.

1. 12 yds. 8 in.
2. 1 ch. 10 yds. 2 ft. 4 in.
3. 1 fur. 7 ch. 9 yds. 1 ft. 6 in.
4. 12 mi. 2 fur. 5 ch. 19 yds.
5. 22 mi. 5 fur. 170 yds.
6. 2 mi. 5 fur. 6 ch. 10 yds. 1 ft.
7. 23 m. 82 cm. 1 mm.
8. 14 m. 7 dm. 3 cm. 2 mm.
9. 25 Km. 378 m. 83 cm.
10. 31 Km. 1 Hm. 97 m.
11. 23 Km. 7 Hm. 33 m. 45 cm.
12. 2 Hm. 4 Dm. 2 m. 59 cm. 8 mm.

EXAMPLES 42. PAGE 92.

1. 6 dwt. 15 grs.
2. 4 oz. 17 dwt. 19 grs.
3. 2 oz. 14 dwt. 11 grs.
4. 5 oz. 4 dwt. 7 grs.
5. 202 oz. 11 dwt. 4 grs.
6. 14 drs.
7. 1 lb. 13 oz. 14 drs.
8. 4 lb. 14 oz. 10 drs.
9. 1 cwt. 3 qrs. 27 lb.
10. 13 cwt. 12 lb.
11. 2 cwt. 1 qr. 14 lb.
12. 4 tons 4 cwt. 1 qr. 7 lb.
13. 1 cg. 7 mg.
14. 24 cg. 3 mg.
15. 2 g. 9 cg. 8 mg.
16. 3 Kg. 9 Hg. 5 Dg. 2 g.
17. 2 Kg. 9 Hg. 80 g.
18. 3 Kg. 1 g. 76 cg. 5 mg.
19. 2 t. 1 q. 46 Kg.

EXAMPLES 43. PAGE 92.

1. 1 yd. 1 ft. 9 in.
2. 4 in.
3. 14 yds. 1 ft.
4. 6 mi. 3 fur. 6 ch.
5. 2 fur. 1 ch. 7 yds. 1 ft.
6. 4 fur. 71 yds.
7. 1 yd. 2 in.
8. 1 ch. 2 yds. 1 ft. 4 in.
9. 21 cm. 2 mm.
10. 56 cm. 8 mm.
11. 2 Km. 3 Hm. 32 m.
12. 2 Hm. 45 m.
13. 13 m. 75 cm.
14. 4 Hm. 47 m. 93 cm.
15. 66 Km. 295 m. 995 mm.

EXAMPLES 44. PAGE 94.

1. 1314 oz. 16 dwt.; 1479 oz. 3 dwt.; 1972 oz. 4 dwt.
2. 125 oz. 9 dwt. 12 grs.; 292 oz. 15 dwt. 12 grs.; 418 oz. 5 dwt.
3. 157 oz. 5 dwt. 8 grs.; 196 oz. 11 dwt. 16 grs.; 308 oz. 18 dwt. 8 grs.
4. 7981 oz. 1 dwt.; 18242 oz. 8 dwt.; 24323 oz. 4 dwt.
5. 16715 oz. 10 dwt.; 17046 oz. 10 dwt.; 17708 oz. 10 dwt.
6. 357733 oz. 19 dwt. 14 grs.; 547832 oz. 10 dwt. 20 grs.; 838344 oz. 14 dwt. 9 grs.
7. 1 cwt. 1 qr. 11 lb. 8 oz. 3 drs.; 1 cwt. 2 qrs. 26 lb. 12 oz. 13 drs.; 1 cwt. 3 qrs. 20 lb. 7 oz. 2 drs.
8. 5 tons 13 cwt. 1 qr.; 8 tons 9 cwt. 3 qrs. 14 lb.; 10 tons 7 cwt. 2 qrs. 14 lb.
9. 37 tons 7 cwt.; 32 tons 13 cwt. 2 qrs. 14 lb.; 114 tons 2 cwt. 2 qrs.
10. 1183 tons 2 cwt. 14 lb.; 7887 tons 7 cwt. 2 qrs.; 8179 tons 10 cwt.
11. 45 tons 14 cwt. 3 qrs. 23 lb. 10 oz.; 158 tons 3 cwt. 2 qrs. 10 lb. 8 oz.; 589 tons 5 cwt. 3 qrs. 21 lb.
12. 10275 tons 17 cwt. 2 qrs. 24 lb.; 11402 tons 8 cwt. 3 qrs. 12 lb.; 24331 tons 10 cwt. 2 qrs. 8 lb.
13. 46 g.; 63 g. 25 cg.; 86 g. 25 cg.

14. 495 g, 838 mg.; 1 Kg. 275 g. 12 mg.; 1 Kg. 487 g. 514 mg.
15. 13 t. 120 Kg. 350 g.; 28 t. 522 Kg 5 Hg.; 35 t. 938 Kg. 350 g.
16. 7020 t. 64 Kg.; 8283 t. 6 q. 75 Kg. 520 g.; 8634 t. 6 q. 78 Kg. 720 g.
17. 7 t. 8 q. 34 Kg. 50 g.; 9 t. 7 q. 92 Kg. 562 g. 5 dg.; 63 t. 7 q. 51 Kg 758 g.
18. 1 ch. 2 yds. 1 ft. 6 in.; 1 ch. 6 yds.; 1 ch. 9 yds. 1 ft. 6 in.
19. 5 ch. 9 yds. 2 ft. 4 in ; 6 ch. 9 yds 1 ft. 8 in.; 6 ch. 20 yds. 1 ft. 4 in.; 9 ch. 8 yds. 2 ft. 8 in.
20. 1 mi. 5 fur. 6 ch. 16 yds.; 2 mi. 1 fur. 20 yds.; 2 mi. 6 fur. 2 ch. 4 yds.
21. 26 mi. 7 fur. 7 ch. 12 yds.; 108 mi. 3 fur 3 yds.; 157 mi. 2 fur. 5 ch. 15 yds.
22. 34 mi. 6 fur. 7 ch. 13 yds.; 243 mi. 7 fur. 3 ch. 3 yds.; 1707 mi. 3 fur 1 ch. 21 yds.
23. 8469 mi. 4 fur. 7 ch.; 12443 mi. 4 fur. 7 ch.; 17932 mi. 5 fur. 4 ch.
24. 17 mi. 7 fur. 6 ch. 16 yds. 1 ft.; 661 mi. 4 fur. 9 ch. 13 yds.; 886 mi. 3 fur. 7 ch. 13 yds. 1 ft.
25. 63 cm.; 1 m. 575 mm.; 2 m. 52 cm.; 3 m. 675 mm.
26. 179 m. 34 cm.; 461 m. 16 cm.; 1 Km. 84 m. 58 cm.
27. 39 Km. 288 m. 60 cm.; 145 Km. 624 m. 5 cm; 2422 Km. 227 m. 60 cm.
28. 325 Km. 584 m.; 1864 Km. 356 m.; 7139 Km. 592 m.

EXAMPLES 45. PAGE 96.

1. 9 in.; 7 in.
2. 2 yds. 5 in.; 1 yd. 2 ft. 3 in.
3. 1 ch. 19 yds. 9 in.; 2 yds. 2 ft. 3 in.; 2 ft. 3 in.
4. 2 ch. 16 yds. 2 ft.; 1 ch. 8 yds. 1 ft.; 3 yds. 2 ft.
5. 4 mi. 2 fur. 5 ch.; 1 mi. 2 fur. 1 ch.
6. 14 cm.; 13 cm.
7. 18 cm. 2 mm.; 2 cm. 6 mm.
8. 3 m. 4 dm. 5 cm.; 2 m. 8 dm. 5 cm.; 6 dm. 9 cm.
9. 154 m.; 143 m.; 77 m.
10. 2 Hm. 3 Dm. 2 m. 3 dm.; 4 Dm. 3 m. 7 dm.; 1 Dm. 1 m. 5 dm.
11. 3 Hm. 4 Dm. 5 m.; 2 Hm. 6 Dm. 4 m.; 8 m.
12. 7 Dm. 8 m. 1 dm.; 1 Dm. 4 m. 3 dm.; 4 m. 1 dm.
13. 12 dwt. 5 grs.
14. 1 oz. 3 dwt. 8 grs.
15. 3 oz. 4 dwt. 8 grs
16. 10 oz. 2 dwt. 23 grs.
17. 308 oz. 18 dwt. 16 grs.
18. 8 oz. 7 drs.
19. 1 qr. 5 lb. 8 oz.
20. 1 cwt. 2 qrs. 7 lb.
21. 3 cwt. 1 qr. 14 lb.
22. 2 cwt. 3 qrs. 7 lb. 8 oz.
23. 7 cwt. 2 qrs. 10 lb
24. 18 g. 31 cg. 5 mg.; 15 g. 87 cg. 3 mg.; 2 g. 14 cg. 5 mg.
25. 18 Dg. 3 g. 15 cg.; 2 Dg. 1 g. 45 cg.; 8 g. 25 cg.
26. 83 Kg. 25 g.; 22 Kg. 7 Hg. 25 g.; 2 Kg. 25 g.
27. 1 t. 3 q. 56 Kg. 3 Hg.; 1 q. 68 Kg. 3 Hg.; 9 Kg. 9 Hg.

EXAMPLES 46. PAGE 97.

1. 56. 2. 18. 3. 105. 4. 37. 5. 62. 6. 143. 7. 223.
8. 1392. 9. 3325. 10. 931. 11. 3010. 12. 4635. 13. 17. 14. 1067

EXAMPLES 48. PAGE 99.

1. 13; 26. 2. 187; 374. 3. 6109; 12218. 4. 5*s.* 10*d.*; 11*s.* 8*d.*
5. £4. 1*s.* 2*d.*; £8. 2*s.* 4*d.* 6. £5. 15*s.* 2*d.*; £11. 10*s.* 4*d.*
7. £621. 4*s.* 10½*d.*; £1242. 9*s.* 9*d.* 8. £1333. 6*s.* 8*d.*; £2666. 13*s.* 4*d.*
9. £1948. 12*s.* 7*d.*; £3897. 5*s.* 2*d.* 10. £8099. 18*s.* 3*d.*; £16199. 16*s.* 6*d.*
11. 2 cwt.; 4 cwt. 12. 325 tons; 650 tons.
13. 1 cwt. 26 lb.; 2 cwt. 1 qr. 24 lb.
14. 6 cwt. 3 qrs. 16 lb.; 13 cwt. 3 qrs. 4 lb.
15. 15 cwt. 24 lb.; 1 ton 10 cwt. 1 qr. 20 lb.
16. 6 tons 9 cwt. 12 lb.; 12 tons 18 cwt. 24 lb.
17. 11 cwt. 1 qr. 22 lb. 10 oz.; 1 ton 2 cwt. 3 qrs. 17 lb. 4 oz.
18. 1 oz. 9 dwt. 4 grs.; 2 oz. 18 dwt. 8 grs.
19. 3 oz. 11 dwt. 21 grs.; 7 oz. 3 dwt. 18 grs.
20. 28 oz. 15 dwt. 6 grs.; 57 oz. 10 dwt. 12 grs.

EXAMPLES 50. PAGE 101.

1. £3. 4*s.* 2. £58. 10*s.* 3. £23. 2*s.* 4. £39. 18*s.* 5¼*d.* 5. 18*s.* 4½*d.*
6. £17. 5*s.* 10*d.* 7. £33. 16*s.* 4*d.* 8. £3. 6*s.* 2*d.* 9. £1. 14*s.* 4*d.*
10. 3 cwt. 14 lb. 11. 21 lb. 12. 2 tons 10 cwt. 2 qrs. 14 lb.
13. 10 lb. 8 oz. 14. 25 oz. 10 dwt. 5 grs. 15. 110 oz. 8 grs.
16. £405. 5*s.* 2*d.* 17. 12 oz. 3 dwt. 18. 4 cwt. 1 qr. 24 lb.
19. £2988. 2*s.* 6*d.* 20. £1364. 0*s.* 6*d.* 21. 4 Km. 590 m.
22. 1 Kg. 360 *g.* 23. 48 f. 50 c. 24. 416 dol.

EXAMPLES 52. PAGE 104.

1. $\frac{17}{76}$. 2. $\frac{188}{729}$. 3. $\frac{15}{44}$. 4. $\frac{8}{27}$. 5. $\frac{1}{4}$. 6. $\frac{140}{183}$. 7. $\frac{16}{25}$.
8. $\frac{5}{6}$. 9. $\frac{1}{11}$. 10. $\frac{8}{27}$. 11. $\frac{12}{19}$. 12. $\frac{41}{49}$. 13. $\frac{103}{397}$. 14. $\frac{19}{67}$.
15. $\frac{15}{17}$. 16. $\frac{65}{243}$. 17. $\frac{153}{440}$. 18. $\frac{71}{111}$. 19. $\frac{51}{107}$. 20. $\frac{40}{223}$. 21. $\frac{497}{631}$.
22. $\frac{40}{1113}$. 23. $\frac{19}{34}$. 24. $\frac{901}{1900}$. 25. $\frac{284}{2997}$. 26. $\frac{288}{28163}$. 27. $\frac{658}{897}$.

EXAMPLES 55. PAGE 107.

1. $\frac{16}{20}, \frac{6}{20}, \frac{7}{20}; \frac{3}{10}, \frac{7}{20}, \frac{4}{5}$.
2. $\frac{6}{12}, \frac{8}{12}, \frac{10}{12}, \frac{7}{12}; \frac{1}{2}, \frac{7}{12}, \frac{2}{3}, \frac{5}{6}$.
3. $\frac{30}{36}, \frac{33}{36}, \frac{2}{36}; \frac{1}{18}, \frac{5}{6}, \frac{11}{12}$.
4. $\frac{20}{45}, \frac{27}{45}, \frac{24}{45}; \frac{4}{9}, \frac{8}{15}, \frac{3}{5}$.
5. $\frac{30}{42}, \frac{17}{42}, \frac{27}{42}, \frac{32}{42}; \frac{17}{42}, \frac{9}{14}, \frac{5}{7}, \frac{16}{21}$.
6. $\frac{16}{56}, \frac{44}{56}, \frac{21}{56}, \frac{22}{56}; \frac{2}{7}, \frac{3}{8}, \frac{11}{28}, \frac{11}{14}$
7. $\frac{48}{80}, \frac{56}{80}, \frac{45}{80}, \frac{50}{80}; \frac{9}{16}, \frac{3}{5}, \frac{5}{8}, \frac{7}{10}$.
8. $\frac{16}{24}, \frac{20}{24}, \frac{17}{24}, \frac{18}{24}; \frac{2}{3}, \frac{17}{24}, \frac{3}{4}, \frac{5}{6}$.
9. $\frac{20}{100}, \frac{75}{100}, \frac{70}{100}, \frac{52}{100}; \frac{1}{5}, \frac{13}{25}, \frac{7}{10}, \frac{3}{4}$.
10. $\frac{80}{120}, \frac{105}{120}, \frac{110}{120}, \frac{114}{120}; \frac{2}{3}, \frac{7}{8}, \frac{11}{12}, \frac{19}{20}$
11. $\frac{60}{72}, \frac{45}{72}, \frac{16}{72}, \frac{39}{72}; \frac{2}{9}, \frac{13}{24}, \frac{5}{8}, \frac{5}{6}$.
12. $\frac{50}{120}, \frac{80}{120}, \frac{95}{120}, \frac{96}{120}; \frac{5}{12}, \frac{2}{3}, \frac{19}{24}, \frac{4}{5}$.
13. $\frac{300}{480}, \frac{408}{480}, \frac{304}{480}, \frac{435}{480}; \frac{5}{8}, \frac{19}{30}, \frac{17}{20}, \frac{29}{32}$.
14. $\frac{300}{420}, \frac{330}{420}, \frac{228}{420}, \frac{225}{420}, \frac{260}{420}; \frac{15}{28}, \frac{19}{35}, \frac{13}{21}, \frac{5}{7}, \frac{11}{14}$.

15. $\frac{78}{650}, \frac{455}{650}, \frac{39}{650}, \frac{70}{650}$; $\frac{3}{50}, \frac{7}{65}, \frac{3}{25}, \frac{7}{10}$.
16. $\frac{162}{180}, \frac{99}{180}, \frac{156}{180}, \frac{42}{180}, \frac{25}{180}$; $\frac{5}{36}, \frac{7}{30}, \frac{11}{20}, \frac{13}{15}, \frac{9}{10}$.
17. $\frac{108}{144}, \frac{132}{144}, \frac{60}{144}, \frac{111}{144}$; $\frac{15}{36}, \frac{3}{4}, \frac{37}{48}, \frac{11}{12}$.
18. $\frac{99}{198}, \frac{44}{198}, \frac{54}{198}, \frac{77}{198}, \frac{102}{198}$; $\frac{2}{9}, \frac{3}{11}, \frac{7}{18}, \frac{1}{2}, \frac{17}{33}$.
19. $\frac{210}{480}, \frac{380}{480}, \frac{384}{480}, \frac{225}{480}$; $\frac{7}{16}, \frac{15}{32}, \frac{19}{24}, \frac{4}{5}$.
20. $\frac{162}{432}, \frac{243}{432}, \frac{208}{432}, \frac{228}{432}$; $\frac{3}{8}, \frac{13}{27}, \frac{19}{36}, \frac{9}{16}$.

EXAMPLES 57. PAGE 108.

1. $18\frac{27}{29}$. 2. $16\frac{1}{2}$. 3. $15\frac{1}{6}$. 4. $12\frac{11}{21}$. 5. $5\frac{2}{91}$. 6. $7\frac{75}{103}$.
7. $18\frac{52}{105}$. 8. $18\frac{185}{241}$. 9. $14\frac{250}{481}$. 10. $15\frac{32}{373}$. 11. $34\frac{220}{283}$. 12. $32\frac{2}{3}$.
13. $32\frac{7}{120}$. 14. $91\frac{113}{143}$. 15. $339\frac{109}{152}$. 16. $73\frac{27}{545}$. 17. $47\frac{1159}{1251}$.
18. $22\frac{143}{1174}$. 19. $200\frac{27}{50}$. 20. $28\frac{283}{347}$. 21. $60\frac{513}{709}$. 22. $99\frac{2118}{2909}$.

EXAMPLES 59. PAGE 109.

1. $\frac{314}{25}$. 2. $\frac{960}{41}$. 3. $\frac{691}{51}$. 4. $\frac{1487}{33}$. 5. $\frac{633}{43}$.
6. $\frac{1243}{112}$. 7. $\frac{1338}{95}$. 8. $\frac{237}{17}$. 9. $\frac{1383}{38}$. 10. $\frac{3084}{43}$.
11. $\frac{2299}{18}$. 12. $\frac{3851}{21}$. 13. $\frac{10329}{50}$. 14. $\frac{4766}{15}$. 15. $\frac{8693}{14}$.
16. $\frac{75609}{29}$. 17. $\frac{119}{7}$. 18. $\frac{256}{16}$. 19. $\frac{589}{19}$. 20. $\frac{10521}{21}$.

EXAMPLES 61. PAGE 112.

1. $1\frac{19}{21}$. 2. $2\frac{4}{5}$. 3. $2\frac{31}{66}$. 4. $4\frac{7}{100}$. 5. $1\frac{9}{32}$.
6. $1\frac{47}{144}$. 7. $3\frac{49}{120}$. 8. $3\frac{2}{45}$. 9. $14\frac{43}{90}$. 10. $3\frac{47}{90}$.
11. $2\frac{131}{180}$. 12. $2\frac{17}{180}$. 13. $1\frac{121}{300}$. 14. $2\frac{5}{18}$. 15. $1\frac{119}{198}$.
16. $2\frac{22}{135}$. 17. $5\frac{15}{154}$. 18. $1\frac{171}{580}$. 19. $1\frac{193}{252}$. 20. $1\frac{47}{132}$.
21. $1\frac{31}{84}$. 22. $\frac{217}{324}$. 23. $1\frac{105}{136}$. 24. $3\frac{83}{198}$. 25. $1\frac{269}{528}$.
26. $1\frac{23}{39}$. 27. $3\frac{253}{390}$. 28. $3\frac{499}{660}$. 29. $1\frac{347}{1700}$. 30. $5\frac{439}{1820}$.
31. $1\frac{26505}{30003}$. 32. $8\frac{3}{4}$. 33. $9\frac{5}{9}$. 34. $14\frac{1}{4}$. 35. $47\frac{55}{72}$.
36. $10\frac{44}{45}$. 37. $12\frac{31}{120}$. 38. $21\frac{17}{24}$. 39. $68\frac{17}{18}$. 40. $7\frac{47}{70}$.

EXAMPLES 63. PAGE 113.

1. $\frac{41}{100}$. 2. $\frac{7}{54}$. 3. $\frac{11}{105}$. 4. $\frac{17}{21}$. 5. $\frac{85}{108}$.
6. $\frac{13}{30}$. 7. $\frac{32}{117}$. 8. $\frac{73}{112}$. 9. $\frac{241}{450}$. 10. $\frac{1}{60}$.
11. $\frac{8}{35}$. 12. $12\frac{11}{100}$. 13. $33\frac{41}{72}$. 14. $5\frac{5}{7}$. 15. $17\frac{7}{8}$.
16. $\frac{7}{8}$. 17. $4\frac{15}{16}$. 18. $3\frac{5}{8}$. 19. $6\frac{17}{18}$. 20. $5\frac{2}{3}$.
21. $4\frac{2}{5}$. 22. $4\frac{43}{72}$. 23. $330\frac{11}{20}$. 24. $43\frac{23}{24}$. 25. $1\frac{22}{35}$.
26. $2\frac{41}{80}$. 27. $6\frac{47}{80}$. 28. $\frac{17}{42}$. 29. $2\frac{29}{48}$. 30. $2\frac{55}{56}$.
31. $2\frac{51}{143}$. 32. $6\frac{349}{432}$. 33. $3\frac{61}{110}$. 34. $1\frac{279}{280}$. 35. $4\frac{343}{380}$.
36. $\frac{152}{385}$. 37. $7\frac{331}{390}$. 38. $45\frac{251}{260}$. 39. $5\frac{697}{770}$.

EXAMPLES 64. PAGE 114.

1. $2\frac{11}{12}$. 2. $4\frac{2}{5}$. 3. $3\frac{13}{45}$. 4. $7\frac{5}{12}$. 5. $2\frac{7}{30}$.
6. $13\frac{35}{36}$. 7. $6\frac{49}{60}$. 8. $11\frac{47}{72}$. 9. $39\frac{21}{40}$. 10. $2\frac{8}{23}$.
11. $9\frac{529}{624}$. 12. $3\frac{1}{504}$. 13. $5\frac{509}{1300}$. 14. $13\frac{913}{1120}$. 15. $9\frac{173}{216}$.
16. $9\frac{1}{500}$. 17. $4\frac{848}{1305}$. 18. $9\frac{1363}{2240}$. 19. $16\frac{109}{1008}$. 20. $18\frac{157}{432}$.

EXAMPLES 66. PAGE 116.

1. $2\frac{4}{5}$, $3\frac{1}{3}$. 2. $11\frac{3}{7}$, $11\frac{2}{3}$. 3. $37\frac{1}{3}$, $38\frac{2}{11}$. 4. $19\frac{1}{2}$, $15\frac{5}{17}$.
5. $14\frac{7}{10}$, 49. 6. 143, $14\frac{6}{7}$. 7. $46\frac{2}{13}$, $50\frac{10}{13}$. 8. $46\frac{6}{7}$, $61\frac{1}{2}$.
9. $103\frac{1}{5}$, $146\frac{1}{5}$. 10. $1192\frac{1}{4}$, $1362\frac{4}{7}$. 11. 10, $12\frac{1}{2}$. 12. $32\frac{2}{3}$, $51\frac{1}{3}$.
13. $20\frac{2}{5}$, $81\frac{3}{5}$. 14. $29\frac{1}{7}$, $109\frac{2}{7}$. 15. $73\frac{1}{7}$, $146\frac{2}{7}$. 16. $108\frac{9}{14}$, $158\frac{11}{14}$.
17. $56\frac{13}{22}$, $79\frac{5}{22}$. 18. $33\frac{9}{35}$, $60\frac{34}{35}$. 19. $16\frac{5}{7}$, $58\frac{1}{2}$. 20. $13\frac{6}{7}$, $19\frac{2}{5}$.
21. $7\frac{6}{11}$, $41\frac{1}{2}$. 22. $20\frac{13}{17}$, $176\frac{1}{2}$.

EXAMPLES 68. PAGE 118.

1. $\frac{7}{45}$, $\frac{7}{60}$, $\frac{7}{75}$. 2. $\frac{5}{134}$, $\frac{5}{201}$, $\frac{10}{603}$. 3. $\frac{5}{72}$, $\frac{5}{108}$, $\frac{5}{144}$.
4. $\frac{3}{100}$, $\frac{3}{200}$, $\frac{11}{800}$. 5. $\frac{9}{484}$, $\frac{9}{605}$, $\frac{7}{726}$. 6. $\frac{29}{1710}$, $\frac{29}{5130}$, $\frac{5}{1026}$.
7. $\frac{19}{840}$, $\frac{19}{1120}$, $\frac{3}{280}$. 8. $\frac{27}{520}$, $\frac{3}{65}$, $\frac{2}{325}$. 9. $\frac{25}{1386}$, $\frac{5}{378}$, $\frac{25}{2394}$.
10. $\frac{4}{213}$, $\frac{2}{213}$, $\frac{2}{355}$. 11. $1\frac{1}{4}$, $\frac{5}{12}$, $\frac{5}{28}$. 12. $\frac{15}{16}$, $\frac{5}{12}$, $\frac{15}{44}$.
13. $1\frac{7}{15}$, $1\frac{1}{21}$, $\frac{22}{27}$. 14. $3\frac{1}{8}$, $2\frac{8}{21}$, $1\frac{1}{9}$. 15. $\frac{2}{9}$, $\frac{2}{27}$, $\frac{1}{18}$. 16. $\frac{1}{6}$, $\frac{1}{30}$, $\frac{1}{54}$.
17. $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{25}$. 18. $\frac{2}{9}$, $\frac{1}{9}$, $\frac{2}{45}$. 19. $\frac{23}{30}$, $\frac{23}{66}$, $\frac{1}{12}$. 20. $\frac{6}{7}$, $\frac{114}{175}$, $\frac{57}{112}$.

EXAMPLES 70. PAGE 121.

1. $\frac{352}{665}$, $\frac{44}{175}$, $\frac{286}{875}$. 2. $\frac{34}{95}$, $\frac{4591}{1691}$, $\frac{5227}{8555}$. 3. $\frac{3}{4}$, $\frac{1}{2}$, $4\frac{1}{2}$. 4. $\frac{8}{45}$, $\frac{5}{54}$, $\frac{8}{15}$.
5. $1\frac{1}{8}$, $1\frac{9}{56}$, $\frac{60}{77}$. 6. $\frac{7}{45}$, $\frac{13}{55}$, $\frac{9}{46}$. 7. $\frac{10}{39}$, $\frac{32}{117}$, $\frac{27}{200}$. 8. $\frac{21}{143}$, $\frac{1}{5}$, $\frac{15}{52}$.
9. $\frac{5}{6}$, $1\frac{1}{4}$, $1\frac{7}{8}$. 10. $1\frac{7}{15}$, $\frac{11}{21}$, $3\frac{1}{18}$. 11. $3\frac{29}{44}$, $1\frac{7}{16}$, 4. 12. $10\frac{2}{3}$, $3\frac{31}{63}$, 4.
13. $2\frac{53}{56}$, $4\frac{19}{64}$, $1\frac{7}{48}$. 14. $9\frac{2}{8}$, $19\frac{11}{16}$, $28\frac{3}{4}$. 15. $13\frac{3}{4}$, $33\frac{3}{4}$, $28\frac{1}{2}$.
16. $1\frac{1}{2}$, $\frac{15}{28}$, 2. 17. $19\frac{1}{2}$, $14\frac{5}{8}$, $28\frac{7}{16}$. 18. $55\frac{1}{4}$, $27\frac{29}{36}$, $23\frac{5}{6}$.
19. $5\frac{22}{532}$, $16\frac{5}{16}$, $25\frac{7}{8}$. 20. $2\frac{11}{35}$, $19\frac{47}{55}$, $7\frac{1}{5}$. 21. $\frac{1}{30}$. 22. $\frac{3}{70}$.
23. $\frac{70}{297}$. 24. $\frac{6}{7}$. 25. $\frac{2}{7}$. 26. 2. 27. 3. 28. $4\frac{2}{7}$. 29. $\frac{60}{61}$. 30. $32\frac{5}{8}$.
31. $3\frac{1}{5}$. 32. $82\frac{2}{15}$. 33. $1\frac{67}{77}$. 34. $33\frac{1}{5}$. 35. 1. 36. 2*s.* 3*d.*
37. 6*s.* 38. $3\frac{3}{4}d.$ 39. £2. 40. 3*s.* 9*d.* 41. 10*s.* 3*d.* 42. $1\frac{11}{15}d$
43. £3. 6*s.* $1\frac{4}{5}d.$ 44. 2*s.* $7\frac{1}{2}d.$ 45. £37. 10*s.* 46. 3 lb. 9 oz.
47. 2 cwt. 22 lb. 48. £1. 12*s.* 1*d.* 49. £41. 2*s.* 3*d.* 50. £17. 15*s.* 3*d.*

EXAMPLES 72. PAGE 124.

1. $\frac{3}{4}$, $\frac{27}{32}$, $1\frac{1}{8}$, $\frac{81}{112}$. 2. $38\frac{2}{3}$, $43\frac{1}{2}$, $50\frac{3}{4}$, $31\frac{5}{12}$. 3. $162\frac{1}{2}$, 120, $212\frac{1}{2}$, $95\frac{5}{11}$.
4. $1\frac{1}{14}$, $4\frac{1}{8}$, $1\frac{3}{10}$, $1\frac{17}{35}$. 5. 10, $\frac{7}{16}$, $1\frac{2}{3}$, $\frac{5}{8}$. 6. $1\frac{5}{27}$, $\frac{3}{5}$, 2, $1\frac{7}{45}$.

7. 120, $300\frac{2}{3}$, $116\frac{3}{5}$, $247\frac{1}{2}$. 8. $1\frac{3}{16}$, $2\frac{1}{3}$, $1\frac{17}{2}$, $2\frac{4}{5}$. 9. $\frac{9}{10}$, $1\frac{4}{17}$, $1\frac{1}{2}$, $1\frac{1999}{000}$.
10. $1\frac{23}{52}$, $1\frac{1}{132}$, $\frac{9}{44}$, $1\frac{1}{6}$. 11. $1\frac{8}{23}$, $\frac{31}{105}$, $1\frac{23}{70}$, $3\frac{4}{9}$. 12. $\frac{1}{200}$, $\frac{1}{10}$, $\frac{3}{26}$, $\frac{3}{16000}$.
13. $\frac{2}{5}$, $\frac{5}{57}$, $\frac{2}{19}$, $\frac{1}{3}$. 14. $\frac{19}{22}$, $1\frac{1}{4}$, $\frac{19}{37}$, $1\frac{2}{17}$. 15. 9. 16. $\frac{3}{4}$. 17. $\frac{3}{4}$.
18. $\frac{51}{52}$. 19. $3\frac{5}{24}$. 20. $25\frac{1}{2}$. 21. 4. 22. $\frac{189}{323}$. 23. $22\frac{22}{25}$.
24. $1\frac{1}{4}$. 25. $\frac{8751}{8112}$. 26. $2\frac{2}{9}$. 27. 48. 28. $1\frac{1}{2}$. 29. $103\frac{1}{8}$.
30. $\frac{1}{11}$. 31. $2\frac{139}{18}$. 32. $103\frac{1}{8}$. 33. $1\frac{1}{4}$. 34. $\frac{105}{1024}$. 35. $7\frac{1}{5}$.
36. $\frac{189}{320}$. 37. $1\frac{1}{4}$. 38. $\frac{105}{1024}$. 39. $1\frac{7}{10}$.

EXAMPLES 73. PAGE 126.

1. $\frac{1}{60}$, 36. 2. $\frac{1}{42}$, $2\frac{2}{3}$. 3. $\frac{1}{4}\frac{1}{20}$, 2. 4. $\frac{1}{36}$, $25\frac{2}{3}$.
5. $\frac{1}{144}$, 1. 6. $\frac{1}{630}$, $4\frac{2}{7}$. 7. $\frac{2}{45}$, $13\frac{1}{3}$. 8. $\frac{3}{140}$, 1890.
9. $\frac{2}{315}$, 24. 10. $\frac{17}{60}$, $40\frac{4}{5}$. 11. $\frac{1}{5}\frac{1}{04}$, 5060. 12. $\frac{1}{36}$, $2426\frac{2}{3}$.
13. $\frac{11}{84}$, $9\frac{3}{7}$. 14. $\frac{17}{24}$, $2677\frac{1}{2}$. 15. $\frac{1}{900}$, 627264. 16. $\frac{13}{2310}$, $70\frac{10}{11}$.

EXAMPLES 74. PAGE 126.

1. $\frac{1}{7}$. 2. $\frac{5}{6}$. 3. $7\frac{1}{57}-4\frac{1}{19}=2\frac{55}{57}$. 4. $8\frac{2}{51}$. 5. $4\frac{1}{5}\times 2\frac{1}{7}$.
6. $5\frac{8}{27}\times 3\frac{6}{13}=18\frac{1}{3}$. 7. $4\frac{4}{27}\times 8\frac{7}{16}$ 8. $30\frac{2}{17}\div 4\frac{20}{51}=6\frac{6}{7}$.
9. $10\frac{4}{17}\div 1\frac{2}{27}$. 10. $\frac{719}{1247}$. 11. $1\frac{883}{150}$. 12. £5. 0*s*. 6*d*.
13. 1 ton 4 cwt. 3 qrs. 14. 17*s*. $7\frac{7}{10}d$. 15. £300. 16. $1\frac{113}{34}$. 17. $\frac{9}{40}$.
18. $3\frac{3}{5}$. 19. $\frac{31}{301}$. 20. $\frac{3}{4}$. 21. $3\frac{2}{15}$. 22. $\frac{6}{49}$. 23. 5*d*.; 1*s*. 8*d*.
24. $\frac{5}{48}$. 25. 2*s*. 4*d*.; 2*s*.; 4*d*. 26. $\frac{1}{3}$. 27. $\frac{19}{63}$; 18*s*. 28. 8*s*.
29. $\frac{8}{21}$; $\frac{4}{21}$. 30. $1\frac{1}{3}$ ft.; $\frac{2}{27}$ of pole. 31. $62\frac{2}{9}$. 32. £7. 15*s*. 33. 448.
34. $29\frac{17}{27}$ miles. 35. $\frac{9}{50}$. 36. $\frac{13}{80}$. 37. $\frac{4}{21}$. 38. $\frac{5}{36}$. 39. 10*s*. 5*d*. 40. $\frac{4}{35}$.

EXAMPLES 75. PAGE 130.

1. $29\frac{2}{3}$. 2. $14\frac{1}{8}$. 3. $16\frac{1}{3}$. 4. $\frac{5}{6}$. 5. $14\frac{1}{8}$. 6. $20\frac{5}{12}$. 7. $123\frac{17}{48}$.
8. $13\frac{3}{4}$. 9. $116\frac{11}{16}$. 10. $15\frac{35}{36}$. 11. $17\frac{35}{144}$. 12. $14\frac{19}{36}$. 13. $15\frac{115}{144}$.
14. $15\frac{1}{3}\frac{2}{7}\frac{5}{2}$. 15. $1\frac{8}{9}\frac{2}{3}$. 16. $15\frac{61}{372}$. 17. $2\frac{5}{93}$. 18. $8\frac{1}{3}\frac{8}{7}\frac{5}{2}$. 19. $15\frac{45}{52}$.
20. $14\frac{9}{13}$. 21. $14\frac{3}{52}$. 22. $13\frac{6}{13}$. 23. $18\frac{1}{4}$. 24. $15\frac{3}{4}$. 25. $6\frac{3}{4}$.
26. $9\frac{1}{4}$. 27. $1\frac{3}{4}$. 28. $4\frac{1}{4}$. 29. $13\frac{1}{4}$. 30. $10\frac{3}{4}$. 31. $7\frac{19}{36}$.
32. $1\frac{119}{180}$. 33. $5\frac{4}{15}$. 34. $4\frac{22}{336}$. 35. $25\frac{17}{72}$. 36. $\frac{3}{4}$ 37. $1\frac{1}{4}$. 38. $\frac{91}{124}$

EXAMPLES 76. PAGE 132.

1. $\frac{2}{3}$. 2. 1. 3. $6\frac{6}{37}$. 4. $\frac{7}{8}$. 5. $3\frac{117}{144}$. 6. $1\frac{3}{16}$.
7. $\frac{9}{16}$. 8. 6. 9. $1\frac{1}{2}$. 10. $\frac{2}{15}$. 11. $17\frac{13}{19}$. 12. $\frac{28}{29}\frac{7}{91}$.
13. $\frac{10}{25}\frac{9}{80}\frac{6}{08}\frac{5}{8}$. 14. $\frac{58}{429}$. 15. 2. 16. $\frac{61}{640}$. 17. $\frac{113}{355}$.
18. $13\frac{59}{60}$. 19. $\frac{172}{271}$. 20. $1\frac{108}{129}$. 21. 1. 22. $\frac{2}{3}$.

23. 1. **24.** $\frac{1}{360}$. **25.** $1\frac{1}{2}$. **26.** $\frac{173}{384}$. **27.** $\frac{22499}{31926}$. **28.** $8\frac{69}{260}$.
29. $1\frac{1}{4}$. **30.** $1\frac{3}{127}$. **31.** $5\frac{7}{36}$. **32.** $\frac{30}{37}$. **33.** $2\frac{1}{465}$. **34.** $\frac{25}{144}$.
35. 2904 sq. yds. **36.** $\frac{7}{4}\frac{12}{63}\frac{2}{05}$. **37** £27.
38. 2 cwt. 1 qr. 12 lb. **39** 15*s.* 6*d.*

EXAMPLES 81. Page 140.

1. 86·3. **2.** 6·6. **3.** 7·68. **4.** 33·01. **5.** 61·159.
6. 62·381. **7.** 67·434 **8.** 226·825. **9.** ·461. **10.** 67·381.
11. 173 8299. **12.** 379·4778. **13.** 10·39933. **14.** ·0113. **15.** 1·1111
16. 20·11201. **17.** 16·83115. **18.** 5·0232. **19.** 748·195.
20. 365·055291. **21.** 1988·61269201. **22.** 891 f. 12 c. **23.** 73 f. 43 c.
24. 104 f. 7 c. **25.** 160 f. 60 c. **26.** 39·24735 m. **27.** 8·980785 Km.
28. 4·18226 Kg. **29.** 15·3039 g. **30.** ·1302731 Km. **31.** 200·322 m.
32. 278·477 Kg. **33.** 58·68 m.; 130·29 f. **34.** 1·672 m.
35. 96·481 Km. **36.** 49·2 cm. **37.** 122·418 Kg.
38. 10 m.; 10·3575 m.; 13·1075 m.; 33·465 m.

EXAMPLES 82. Page 141.

1. 2·075. **2.** ·071. **3.** 999. **4.** ·99. **5.** 6·491.
6. 9·325. **7.** 10·886. **8.** 2·5239. **9.** 1·097. **10.** 86·273
11. ·0966. **12.** 6·124. **13.** 3·425. **14.** 1·523468.
15. 11·27654. **16.** 1·13995. **17.** 1·618996. **18.** 55·1985.
19. 863·71. **20.** ·0075. **21.** 66 c. **22.** 98 f. 97 c.
23. 2 f. 96 c. **24.** 2 f. 35 c. **25.** 88 c. **26.** 1 dol. 98 c.
27. 3 dol. 47 c. **28.** 11 m. 88 pf. **29.** 1 m. 88 pf. **30.** 1 m. 89 pf.
31. 9·966 m. **32.** 16 214 Km. **33.** 14·9446 Dg. **34.** 9·2315 Kg.
35. ·5116 m. **36.** 1 Km. 311 m. **37.** 3·805 m. **38.** 101·60 dol.
39. 15 mm. **40** 4·49 Km. **41.** ·0046213 lb. **42.** ·275 Kg.

EXAMPLES 83. Page 144.

1. 1; 1·5; 2·5; 4·5; 5. **2.** ·25, ·375, ·5, 1, ·625, 3·125, 15·625.
3. ·45, ·6, ·75, ·9, 1·2, 1·35, 1·65. **4.** ·4, ·7, ·9, 1·05, 1·25.
5. 12·96, 22·68, 181·44, 155·52, 194·4.
6. 9·5034, 142·551, 199·5714, 354·7936, 342·1224.
7. 129366·7125, 6390548·1325.
8. ·0000432412, ·0001297236, ·0001246364.
9. 21 f.; 278 f. 25 c.; 336 f.; 446 f. 25 c.; 504 f.
10. 45 dol. 15 c.; 90 dol. 30 c.; 722 dol. 40 c.; 1354 dol. 50 c.; 1625 dol. 40 c.
11. 40 m. 292 mm.; 362 m. 628 mm.; 2 Km. 901 m. 24 mm.
12. 63 Km. 765 m. 27 cm.; 765 Km. 183 m. 24 cm.; 4591 Km. 99 m. 44 cm.
13. 306 g. 625 mg.; 529 g. 625 mg.; 23 Kg. 303 g. 50 cg.
14. 325 Kg. 351 g.; 3904 Kg. 212 g.; 17443 Kg. 819 g.
15. 34 Kg. 605 g. 75 cg.; 519 Kg. 86 g. 25 cg.; 3056 Kg. 841 g. 25 cg.

EXAMPLES 84. PAGE 146.

1. 142·2. **2.** 146·6621. **3.** 211·06. **4.** 23·408. **5.** ·380716.
6. 13·110653369. **7.** ·0001292. **8.** ·00011355. **9.** ·000038665.
10. ·0000000036. **11.** 1293·9375, 1·3609375, 63825·875.
12. ·025, ·1, ·04, ·0625, ·01875, ·03125.
13. ·095034, ·142551, 1·995714, ·03547936. ·3421224.
14. ·00000004352, ·0000013056, ·0000039168.
15. 15·46875, 1·546875. ·1546875, ·01546875.
16. 97·565, 9·7565, 975·65, ·0097565.
17. ·01092347, 1·092347, ·0000001092347. **18.** ·0000505, ·000000505.
19. ·000001. **20.** ·00001111. **21.** ·0306. **22.** ·103125.
23. 40·092128184096. **24.** 2261 f. 76 c. **25.** 4 m. 68 cm.
26. 5618 Kg. 970 g. **27.** 1539 f. **28.** 207 f. 88 c.
29. 424 g. 41 cg. **30.** 6407 m. 39 pf. **31.** 8603·694 f.
32. Gain = 10·214 f. **33.** 4 m. 640·625 mm.
34. 8 Km. 468m. 941·25 mm. **35.** £43·732; £80; £58; £42·05.

EXAMPLES 85. PAGE 149.

1. 12·5, 6·25, 3·125, 1·5625, ·78125. **2.** 1·8, 1·6, 1·5, 1·44, ·5.
3. 8·5, 3·4, 1·7, ·425. **4.** 156·7, 31·34, 6·268, 1·2536, ·12536.
5. ·002002, ·0004004, 00008008, ·000016016. **6.** 1·7, ·17, ·15, ·05.
7. ·085, ·0075, ·0025, ·001. **8.** 1·2096, ·0576, ·0288, ·0192.
9. 2·49, ·075, ·0375, ·03. **10.** 59·514, 4·578, ·763.
11. ·0017, ·00085, ·0000425. **12.** 13·2545, ·2705, ·13525.
13. 3·249, 1·083, 1·6245. **14.** ·00000554, ·00000277. **15.** ·00038, ·000076.

EXAMPLES 86. PAGE 151.

1. 3122. **2.** 61. **3.** 941. **4.** 4000000. **5.** 7416400.
6. 300300. **7.** 85. **8.** 81, 8·1. **9.** 60240. **10.** 18820.
11. 18474. **12.** 602400. **13.** 23780000. **14.** 51, 85, 1700, 15.
15. 4000, 40, 40000. **16.** 9·236. **17.** 13·756. **18.** 1604·01.
19. ·06624. **20.** 6·056. **21.** 116·214, 116214000. **22.** 2537·9.
23. ·6268, 1·2536, 12·536. **24.** 47·93. **25.** 120. **26.** 530·6.
27. ·718. **28.** 1·092, ·1092, 1092. **29.** ·112, ·0112, ·00112, 11200.
30. ·02132. **31.** ·00102. **32.** ·00716. **33.** ·00501.
34. ·00756, 1·512, ·00216. **35.** ·00053. **36.** ·0000152.
37. 250 g. **38.** 420000. **39.** 84. **40.** 15 cm.
41. 5. **42.** 630. **43.** 144. **44.** 15, 105, 180, 375.
45. 74; ·001 in. **46.** 37; ·05 cm.

EXAMPLES 87. PAGE 152.

1. $\frac{1}{5}$. **2.** $\frac{2}{5}$. **3.** $\frac{1}{2}$. **4.** $\frac{3}{5}$. **5.** $\frac{4}{5}$. **6.** $\frac{7}{50}$.
7. $\frac{3}{25}$. **8.** $\frac{3}{20}$. **9.** $\frac{4}{25}$. **10.** $\frac{1}{4}$. **11.** $\frac{7}{20}$. **12.** $\frac{9}{20}$.

13. $\frac{11}{20}$. 14. $\frac{16}{25}$. 15. $\frac{13}{20}$. 16. $\frac{23}{40}$. 17. $\frac{1}{100}$. 18. $\frac{1}{25}$.
19. $\frac{3}{50}$. 20. $\frac{1}{20}$. 21. $\frac{2}{25}$. 22. $\frac{1}{8}$. 23. $\frac{31}{200}$. 24. $\frac{16}{125}$.
25. $\frac{32}{125}$. 26. $\frac{61}{200}$. 27. $\frac{69}{200}$. 28. $\frac{93}{250}$. 29. $\frac{157}{250}$. 30. $\frac{5}{8}$.
31. $\frac{1}{40}$. 32. $\frac{3}{200}$. 33. $\frac{3}{40}$. 34. $\frac{2}{125}$. 35. $\frac{8}{125}$. 36. $\frac{1}{500}$.
37. $\frac{1}{200}$. 38. $\frac{3}{500}$. 39. $\frac{49}{80}$. 40. $\frac{1}{80}$. 41. $\frac{1}{16}$. 42. $\frac{9}{400}$.
43. $\frac{13}{2500}$. 44. $6\frac{1}{2000}$. 45. $13\frac{9}{8000}$. 46. $29\frac{641}{20000}$.
47. $71\frac{1}{1600}$. 48. $93\frac{5001}{160000}$. 49. $108\frac{5001}{80000}$.

EXAMPLES 90. PAGE 156.

1. ·15. 2. ·35. 3. ·45. 4. ·72. 5. ·076. 6. ·888.
7. 4·48. 8. ·425. 9. ·3125. 10. ·5625. 11. ·6608. 12. ·1136.
13. ·6512. 14. ·078125. 15. ·46484375. 16. ·0005859375.
17. 9·1875. 18. 2·171875. 19. 3·59375. 20. 105·071875.
21. ·333333. 22. ·166667. 23. ·428571. 24. ·555556.
25. ·272727. 26. ·681818. 27. ·269230. 28. ·309523.
29. 2·916667. 30. 24·846154. 31. 8·642857. 32. 1·066667.
33. 24·352941. 34. 4·523810. 35. 2·037037. 36. 8·441176.
37. 3·088889. 38. 7·326087. 39. 3·385417. 40. 9·046296.

EXAMPLES 91. PAGE 160.

1. 29·96. 2. 44·12. 3. 18·87. 4. 5·37. 5. 10·46
6. 17·43. 7. 13·27. 8. 2·31. 9. 3·10. 10. 1·13.
11. 3·12. 12. 3·47. 13. 7·32. 14. 1·95.

EXAMPLES 92. PAGE 164.

1. 1·03. 2. ·91. 3. 1·72. 4. 4·25. 5. 14·08.
6. 221·26. 7. 369·95. 8. 5989·57. 9. 4090·65. 10. 4528·02.

EXAMPLES 93. PAGE 165.

1. 1·09. 2. 2·22. 3. ·37. 4. 1·10. 5. ·63.
6. 2·89. 7. ·41. 8. 24·87. 9. 19·92. 10. 1·91.

EXAMPLES 94. PAGE 165.

1. 18·07667. 2. ·00222. 3. ·01849. 4. ·52806. 5. 10·75188.
6. 27·02898. 7. 24·32769. 8. ·00008. 9. 2·7846. 10. 1·1902.
11. 254·4285. 12. 2115·1922. 13. ·0733. 14. 129·7322. 15. 726·3613.
16. 846. 17. 103 g. 18. 331 f. 19. 670942000. 20. 4 f. 7 c.
21. ·629 in.; ·037 in.; ·229 in. 22. 123·27 grs. 23. 3 g. 966 mg.
24. 2mm. 25. 1·680; ·280; 16·360; 2·372; 26·057.
26. 4810 m.; 3304 m.; 6281 m.; 8840 m.; 5171 m.
27. 15334·014988. 28. ·945775.

EXAMPLES 97. PAGE 171.

1. £7·20417. 2. £5·30521. 3. £2·40729. 4. £8·709375.
5. £9·81042. 6. £1·91146. 7. £9 31354. 8. £4·51875.
9. £3·62396. 10. £2·18125. 11. £6·284375. 12. £5·39375.
13. £4·89063. 14. £7·79792. 15. £3·68229. 16. £·0125.
17. £·021875. 18. £·065625. 19. £·36875. 20. £·809375.
21. £3·4375. 22. £5·70625. 23. £2·615625. 24. £5·003125.
25. £10·034375. 26. £14·93125. 27. £1·05625. 28. £2·99375.
29. £7·696875. 30. £8·390625. 31. £10 54375. 32. 8*s.* 5*d.*
33. £6. 9*s.* 5*d.* 34. £3. 2*s.* $8\frac{3}{4}$*d.* 35. £57. 18*s.* 3*d.* 36. £60. 11*s.* 7*d.*
37. £193. 15*s.* 11*d.* 38. £8. 16*s.* 8*d.* 39. £10. 10*s.* 8*d.* 40. £572886. 15*s.* 11*d.*

EXAMPLES 98. PAGE 172.

1. $\cdot 2\dot{6}$. 2. $\cdot 0\dot{6}$. 3. $\cdot 8\dot{6}$. 4. $10\cdot 3\dot{8}$. 5. $5\cdot 5\dot{6}$. 6. $\cdot 58\dot{3}$.
7. $\cdot 10\dot{7}$. 8. $3\cdot 1\dot{3}\dot{5}$. 9. $\cdot 4\dot{2}$. 10. $\cdot 0\dot{5}$. 11. $\cdot 09\dot{3}$. 12. $\cdot 5\dot{9}\dot{0}$.
13. $\cdot 1\dot{0}$. 14. $\cdot 1111\dot{5}$. 15. $\cdot 0\dot{2}43\dot{9}$. 16. $\cdot 31707\dot{5}$. 17. $\cdot 153846\dot{5}$.
18. $\cdot 8461\dot{5}\dot{3}$. 19. $1\cdot 2\dot{3}0769\dot{3}$. 20. $3\cdot 1\dot{9}047\dot{6}$. 21. $\cdot 0769\dot{2}\dot{3}$.
22. $3\cdot 1\dot{7}14285$. 23. $\cdot 02\dot{0}\dot{2}\dot{7}$. 24. $\cdot 089\dot{2}85714$. 25. $\cdot 16883\dot{1}$.

EXAMPLES 99. PAGE 175.

1. $\frac{1}{3}$. 2. $\frac{2}{3}$. 3. $\frac{4}{11}$. 4. $\frac{7}{11}$. 5. $\frac{1}{8}$. 6. $\frac{14}{33}$. 7. $\frac{24}{37}$.
8. $\frac{4}{37}$. 9. $\frac{23}{37}$. 10. $\frac{8}{37}$. 11. $\frac{255573}{333}$. 12. $\frac{19}{101}$. 13. $5\frac{2}{7}$. 14. $10\frac{4}{7}$.
15. $91\frac{1235}{111111}$. 16. $\frac{1}{41}$. 17. $\frac{11}{15}$. 18. $\frac{28}{495}$. 19. $\frac{31}{198}$. 20. $\frac{1}{300}$.
21. $15\frac{13}{15}$. 22. $5\frac{1}{888}$. 23. $3\frac{2557}{333330}$. 24. $2\frac{253}{675}$. 25. $9\frac{246645}{249975}$.
26. $2\frac{12888819}{33000000}$. 27. $15\frac{4855510}{2497750}$. 28. $3\frac{25339}{1332000}$.
29. $1\frac{26819}{133200}$. 30. $6\frac{47}{468}$ 31. $5\frac{327}{2020}$.

EXAMPLES 100. PAGE 176.

1. $1\cdot 12$. 2. $1\cdot 0\dot{1}8\dot{9}$. 3. $\cdot 61\dot{1}\dot{6}$. 4. $8\cdot \dot{7}19\dot{4}$.
5. $5\cdot 4\dot{3}86\dot{2}$. 6. $94\cdot 21\dot{2}\dot{4}$. 7. $47\cdot 76783$. 8. $5\cdot 110\dot{9}\dot{5}$.
9. $1\cdot 676\dot{7}\dot{0}$. 10. $305\cdot 18\dot{2}7\dot{5}$. 11. $4\cdot 23047$. 12. $8\cdot 14740$.
13. $38\cdot 25534$. 14. $212\cdot 10247$. 15. $4\cdot 08535$.

EXAMPLES 101. PAGE 177.

1. $34\cdot 01$. 2. $3\cdot 0\dot{6}$. 3. $\cdot \dot{8}$. 4. $31\cdot 0\dot{8}\dot{1}$.
5. $5\cdot 625\dot{7}\dot{6}$. 6. $6\cdot 9\dot{7}2\dot{5}$. 7. $70\cdot 51\dot{9}2\dot{2}$. 8. $24\cdot 22635\dot{1}$.
9. $\cdot 23679$. 10. $12\cdot 96522$. 11. $1\ 55104$. 12. $2\ 00731$.
13. $\cdot 52513$. 14. $9\cdot 75049$. 15. $1\cdot 71395$. 16. $\cdot 19048$.
17. $7\cdot 72137$. 18. $2\cdot 48098$. 19. $1\cdot 59044$. 20. $\cdot 00126$.

EXAMPLES 102. PAGE 177.

1. 23, $38\cdot \dot{3}$, $53\cdot \dot{6}$. 2. $\cdot 3\dot{0}$, $\cdot 9\dot{0}$, $1\cdot 2\dot{1}$. 3. $395\cdot 0\dot{1}$. 4. $4125\cdot 7\dot{8}\dot{5}$.
5. $5\cdot 877$. 6. $6\cdot 383$. 7. $3\cdot 991$. 8. $4\cdot 299$.

9. ·002 **10.** 31·824. **11.** ·585. **12.** 1291·507.
13. 27·702. **14.** 1059·636 **15.** 160289·966. **16.** 30319·381.

EXAMPLES 103 PAGE 178.

1. ·$\dot{1}$, ·0$\dot{6}$, ·048. **2.** ·067, ·013, ·003. **3.** 7·4$\dot{1}\dot{7}$, 4·121, ·824.
4. 12689·823, 1522·$\dot{7}\dot{7}\dot{8}$. **5.** ·006, 280·584. **6.** ·020, ·200, 19·990.
7. ·478. **8.** 45·620. **9.** 317·144.

EXAMPLES 104. PAGE 178.

1. ·$\dot{8}57142\dot{8}$. **2.** 2·$\dot{1}\dot{4}\dot{5}$. **3.** ·00$\dot{0}\dot{6}$. **4.** ·0026. **5.** ·$\dot{2}$46913580$\dot{8}$.
6. ·0$\dot{0}$99$\dot{9}$. **7.** ·0012$\dot{5}\dot{1}\dot{8}$. **8.** 5·3$\dot{1}\dot{6}\dot{2}$. **9.** 2·1$\dot{6}\dot{4}\dot{8}$. **10.** ·$\dot{4}12\dot{5}$.

EXAMPLES 106. PAGE 179.

1. $\frac{31}{120}$. **2.** $\frac{33}{376}$. **3.** $\frac{1}{480}$. **4.** $\frac{33}{82}$. **5.** $\frac{11}{592}$. **6.** $\frac{3}{310}$. **7.** $\frac{43}{54}$.
8. $\frac{173}{427}$. **9.** $\frac{57}{298}$. **10.** $\frac{153}{319}$. **11.** $\frac{27}{343}$. **12.** $\frac{219}{502}$. **13.** $\frac{3}{5}$ **14.** $\frac{9}{86}$.
15. $\frac{3}{5}$. **16.** $\frac{29}{107}$ **17.** $\frac{11}{26}$. **18.** $4\frac{9}{82}$. **19.** $\frac{26}{31}$. **20.** $\frac{3}{13}$. **21.** $\frac{3}{29}$.
22. $\frac{4}{25}$. **23.** $\frac{19}{67}$ **24.** $3\frac{1}{5}$. **25.** $2\frac{11}{12}$. **26.** $\frac{30}{127}$. **27.** $\frac{5}{9}$. **28.** $\frac{5}{88}$ **29.** $\frac{7}{10}$.

EXAMPLES 107. PAGE 181.

1. ·025. **2** ·225. **3.** ·1875. **4.** 5·6.
5. 3·23125. **6.** ·778125. **7.** ·1795138$\dot{8}$. **8.** 1·35859375.
9. ·002583$\dot{8}$. **10** ·839583$\dot{8}$. **11.** 7·3772916$\dot{8}$. **12.** ·048828125.
13. ·75. **14.** ·625. **15.** ·375. **16.** ·625.
17. ·3125. **18.** ·0$\dot{9}$523$\dot{8}$. **19.** ·02$\dot{7}$. **20.** ·347$\dot{2}$.
21. ·$\dot{2}$85714$\dot{1}$. **22.** ·15. **23.** ·062109375. **24.** ·30$\dot{5}$.
25. ·95. **26.** ·1015625. **27.** ·01171875. **28.** ·1390625.
29. ·24140625. **30.** ·053$\dot{5}$71428$\dot{5}$. **31.** ·18984375. **32.** ·02156.

EXAMPLES 109. PAGE 182.

1. £48. 8*s.* 1½*d.* **2.** £1. 10*s.* 5½*d.* **3.** £2. 17*s.* 3¾*d.*
4. £1. 11*s.* 6¾*d.* **5.** £3. 6*s.* 6·2*d.* **6.** £18. 14*s.* 7¾*d.*
7. £57. 1*s.* 0½*d.* **8.** £15. 12*s.* 4½*d.* **9.** 2 cwt. 3 qrs. 7 lb.
10. 6 lb. 8 oz. 7·5776 drs. **11.** 3 cwt. **12.** 1 lb. 9 oz. 4·2752 drs.
13. 18 tons 4 cwt. 2 qrs. **14.** 1 cwt. 1 qr. 20 lb.
15. 2 cwt. 2 qrs 19·475 lb. **16.** 1 cwt. 3 qrs. 18 lb.
17. 2 cwt. 3 qrs 6·9999916 lb. **18.** 5 mi. 5 fur. 2·2 yds.
19. 1 fur. 1 ch 14 yds. 2 ft. 9·966 in. **20.** 8 yds. 2 ft. 1·3 in.

EXAMPLES 110. PAGE 184.

1. 6*s.* 6*d.* **2.** 9¾*d.* **3.** 9*s.* 9*d.* **4.** £4. 15*s.* 10·74*d.*
5. 16*s.* **6.** 3*s.* 7½*d.* **7.** 6*s.* **8.** £19. 9*s.* 2*d.*
9. £2. 11*s.* 0¾*d.* **10.** £3. 11*s.* 10⅝*d.* **11.** 12*s.* 7⅜⅜*d.* **12.** £1
13. £30. 19*s.* 6*d.* **14.** £3. 11*s.* 4·65*d.* **15.** 18*s.* 2¼*d.* **16.** 12*s.* 1¾*d.*
17. 3 tons 7 cwt. 3 qrs. 22 lb.

EXAMPLES 112. Page 187.

1. 4827 sq. in. 2. 37330 sq. ft. 3. 46105 sq. yds.
4. 32569 sq. ch. 5. 598764 sq. yds. 6. 238427 sq. ft.
7. 305385 sq. in. 8. 5 sq. yds. 5 sq. ft. 121 sq. in.
9. 8 sq. yds. 118 sq. in. 10. 5 sq. ch. 15 sq. yds. 6 sq. ft. 80 sq. in.
11. 1747 ac. 2 sq. ch. 341 sq. yds. 12. 78 ac. 4 sq. ch. 5687 sq. links.
13. 5 sq. mi. 256 ac. 8 sq. ch. 14. 123 ac. 4 sq. ch. 5678 sq. links.
15. 50012 sq. cm. 16. 50705 ares. 17. 30508 sq. cm.
18. 5001509 sq. mm. 19. 17 sq. dm. 68 sq. cm. 49 sq. mm.
20. 56 sq. m. 84 sq. dm. 59 sq. cm. 21. 78 sq. Km. 68 Ha. 45 a.
22. 123 Ha. 45 a. 87 ca.

EXAMPLES 113. Page 189.

1. 8. 2. 15. 3. 28. 4. 30. 5. 72. 6. 91.
7. 120. 8. 170. 9. 143. 10. 100. 11. 144. 12. 567.

EXAMPLES 114. Page 190.

1. (i) 150 sq. ft., (ii) 168 sq. ft., (iii) $190\frac{1}{1\frac{1}{2}}$ sq. ft., (iv) $208\frac{1}{3}$ sq. ft., (v) $293\frac{5}{7\frac{5}{2}}$ sq. ft., (vi) $98\frac{7}{6\frac{4}{4}}$ sq. ft.
2. (i) 15 ft., (ii) 21 ft., (iii) $27\ \frac{3\frac{1}{5}}{4}$ ft., (iv) $31\frac{1\frac{1}{6\frac{1}{1}}}{1}$ ft.
3. (i) 178 sq. m. 50 sq. dm., (ii) 119 sq. m. 70 sq. dm., (iii) 214 sq. m. 71 sq. dm., (iv) 210 sq. m. 31 sq.dm. 1 sq. cm.
4. (i) 15·2 m., (ii) 15 m. 20 cm., (iii) 13 m. 2 dm. 5 cm.
5. (i) 150 ac. 2 ro., or 150 ac. 5 sq. ch., (ii) 338 ac. 3 ro. 36 per., or 338 ac. 9 sq. ch. 363 sq. yds., (iii) 131 ac. 1 ro. 25 per., or 131 ac. 4 sq. ch. 30 sq. yds. 2 sq ft. 36 sq. in.
6. (i) 30 ac. 5 sq. ch. 2500 sq. lks., (ii) 25 ac. 4 sq. ch. 9780 sq. lks., (iii) 999 ac. 7 sq. ch. 5000 sq. lks., (iv) 157 ac. 1 sq. ch. 6500 sq. lks.
7. (i) 47 a. 86·7 ca., (ii) 7 Ha. 3 a. 54·5 ca., (iii) 132 Ha. 83 a. 92 ca., (iv) 295 Ha. 40 a. 94 ca. 8. 105. 9. £47. 16*s.* 3*d.*
10. 15 ac. 3 r. 30 p. 22 sq. yds. 8 sq. ft. 136 sq. in. 11. 140 sq. yds.
12. £13. 10*s.* 13. 10000; £4166. 13*s.* 4*d.* 14. 16.
15. 12*s.* $4\frac{2}{9}$*d.* 16. £18. 15*s.* 17. £72. 6*s.* 3*d.*
18. £67. 12*s.* $0\frac{3\ 9}{8\ 4}$*d.* 19. 981 m. 20. 999 m. 21. 9962 links.
22. 2 mi. 4 fur. 6 ch. 23. 26 m. 12 cm. 24. 536 Ha. 25. 199 ac.
26. 2 ac. 27. 36. 28. 1200. 29. 12615. 30. 144.
31. 17337 f. 50 c. 32. £6667. 10*s.* 33. 2820 f. 6 c.
34. 152524 f. 72 c. 35. 12 Ha. 97 a. 32 ca. 36. 112 ac. 4 sq. ch.
37. 10·7643 sq. ft., 1·1960 sq. yds. 38. 10·7646.

EXAMPLES 116. Page 196.

1. 5. 2. 26. 3. 216. 4. 164. 5. 506. 6. 75. 7. 34.
8. 156. 9. 250. 10. 306. 11. 2016. 12. 1400. 13. 2280. 14. 10240.
15. 10256. 16. 480 qts. 2 gills. 17. 175 gal. 3 qts. 1 pt. 1 gill.

18. 232 gal. 1 pt. **19.** 1960 gal. 2 qts. 1 pt. 1 gill.
20. 1 cwt. 1 qr. 3 lb. 12 oz. **21.** 25. **22.** 13 tons 7 cwt. 3 qrs. 12 lb.
23. 2565·3. **24.** 187. **25.** 497 ; 75 litres. **26.** 195.
27. 12 steres $2\frac{1}{2}$ decisteres ; 2 steres $4\frac{1}{2}$ decisteres. **28.** 1960.
29. 1573. **30.** 112 l. 50 cl.

EXAMPLES 118. Page 201.

1. 162000. **2.** 216000. **3.** 1755. **4.** 87660 **5.** 139405.
6. 206190. **7.** 18220. **8.** 7862400. **9.** 13° 13′ 52″. **10.** 156° 31′ 12″.
11. 10 d. 8 h. 30 min. 31 sec. **12.** 12 d. 13 h. 10 min. 3 sec.
13. 5 w. 4 d. 19 h. 45 min. 7 sec. **14.** 97 w. 4 d. 2 hr. 5 min. 32 sec.
15. 26880. **16.** 81000000. **17.** 660. **18.** 36524.
19. 42003. **20.** 876576. **21.** Sunday. **22.** Monday.
23. Monday. **24.** Thursday. **25.** Friday. **26.** Monday.

EXAMPLES 119. Page 202.

1. 78 lb., 78 lb., 68 lb. **2.** 90 cwt. **3.** 1895. **4.** 555.
5. 104 ; 1 cwt. 3 qrs. 7 lb. **6.** 282, 355, 365. **7.** 121 ; 120. **8.** 1 inch.
9. 443520. **10.** 8. **11.** 504. **12.** 137896.
13. 704004. **14.** 14192640. **15.** 1000. **16.** 8000.
17. 275792. **18.** 2160. **19.** 14080. **20.** 2415.
21. 1 ft. 6 in. **22.** $36\frac{3}{8}$. **23.** 192 times ; No. **24.** 50 lb.
25. 2592000. **26.** 41760 ; 40320. **27.** 28 yrs. 41 wks. 4 dys.
28. 28. **29.** 432000. **30.** 113 dys. $2\frac{1}{2}$ hrs. **31** $1\frac{7}{15}$.
32. 288. **33.** 3 mi. 845 yds. **34.** 876576 ; £913. 2*s*.
35. 26 Mar. 1795. **36.** 3. **37.** $15\frac{3}{4}$ min. to 12. **38.** $\frac{4}{5}$ oz.
39. 12, 60, 84. **40.** 290. **41** 231 ; 24389. **42.** 999.
43. 1·68 oz. **44.** 128. **45.** 224 times ; 32 cl. **46.** 238 feet.
47. 14 ac. 2 sq. ch. 441 sq. yds. **48.** ·12 of an inch.
49. 7509 sq. mm. **50.** 1·853. **51.** ·018125.
52. 8*s*. **53.** £1. **54.** $1\cdot\dot{8}5\dot{1}$ or $1\frac{23}{27}$.
55. 46·55 yds. **56.** $\frac{19}{51}$. **57.** £126583. 6*s*. 8*d*.
59. (i) 7749×53, (ii) $56849 \div 7 \times 9$. **60.** Loses £12. **61.** £8. 18*s*. 9*d*.
62. £147. 15*s*. 0*d*. **63.** £25. **64.** £4. 7*s*. 11*d*. **65.** 1*d*.
66. 5*d*. **67.** £12. 17*s*. 10*d*. **68.** 666.
69. Loss, £5. 1*s*. 3*d*. **70.** 4*s*. 3*d*. per lb. **71.** 34*s*. per qr.

ANSWERS TO EXAMPLES.

PART II.

EXAMPLES 120.* Page 210.

1. $102\frac{2}{7}\frac{3}{2}$; £23. 0*s.* $5\frac{1}{4}d.$ **2.** 27 yds. 1 ft. $3\frac{4}{5}$ in. **3.** 30.
4. 74 feet. **5.** 88. **6.** 864; £233. 17*s.* 9*d.* **7.** £13. 7*s.* $2\frac{1}{4}d.$
8. 27 sq. yds. 4 sq. ft.; £13. 10*s.* 10*d.* **9.** £9. 15*s.* 0*d.*
10. 140 ft.; £3. 15*s.* **11.** £9. 12*s.* $7\frac{1}{9}d.$
12. 133 ft.; £9. 19*s.* 6*d.*; $191\frac{1}{4}$ sq. ft. **13.** 27648. **14.** 52080.
15. 17 ft. by 17 ft. **16.** 474 sq. yds. too large. **17.** 810 acres.
18. 330 sq. yds. $1\frac{1}{4}$ sq. ft. **19.** $24\frac{3}{4}$ miles + 220 yds.; 25 miles + 44 yds.

EXAMPLES 121. Page 211.

1. 140 sq. yds. **2.** $107\frac{1}{3}$ sq. yds.; £20. 2*s.* 6*d.* **3.** $115\frac{1}{9}$; £7. 8*s.* $8\frac{2}{9}d.$
4. 121695. **5.** 9. **6.** 15. **7.** $93\frac{1}{3}$; 11; £4. 2*s.* 6*d.*
8. 14 pieces; £3. 3*s.* **9.** 18 pieces; £2. 5*s.*
10. 800 sq. ft.; 11 pieces; £1. 7*s.* 6*d.* **11.** £1. 6*s.* 8*d.*
12. £6. 10*s.* 8*d.* **13.** £1. 19*s.* **14.** 10 pieces; £2. 5*s.* **15.** £2. 5*s.*
16. $378\frac{3}{16}$ sq. ft.; 1 ton 7 cwt. $1\frac{1}{2}$ lb.; £37. 16*s.* $4\frac{1}{2}d.$
17. 19*s.* $9\frac{1}{2}d.$ **18.** 12 feet. **19.** $2\frac{3}{4}\frac{4}{9}d.$; £3. 14*s.* 3*d.*
20. Carpet, £30. 16*s.* $1\frac{3}{4}d.$; paper, £14. **21.** $255\frac{1}{2}\frac{3}{4}$ sq. ft.; $242\frac{3}{7}\frac{1}{2}$ sq. ft.
22. $55\frac{5}{9}$ sq. ft.; 18*s.* $6\frac{2}{9}d.$ **23.** $34\frac{3}{4}.$

EXAMPLES 122. Page 214.

1. 6. **2.** 30. **3.** 42. **4.** 84. **5.** 105. **6.** 132.
7. 231. **8.** 252. **9.** 315. **10.** 350. **11.** 385. **12.** 495.
13. 693. **14.** 756. **15.** 825. **16.** 1575. **17.** 3528. **18.** 4312.
19. 4725. **20.** 17325. **21.** 72765. **22.** 232848. **23.** 848925.
24. 1097712. **25.** 2223375. **26.** $\frac{11}{13}$. **27.** $\frac{16}{29}$. **28.** $\frac{7}{8}$. **29.** $\frac{15}{32}$.
30. $1\frac{1}{14}$. **31.** $1\frac{1}{3}$. **32.** $6\frac{1}{4}$. **33.** $14\frac{1}{2}$. **34.** $6\frac{1}{7}$. **35.** $52\frac{1}{2}$.
36. $8\frac{1}{9}$. **37.** $16\frac{3}{7}$. **38.** 7. **39.** 13. **40.** 21. **41.** 22. **42.** 91.

* The printed answers to Examples 120 and 121 have been obtained by the method explained in Arts. 274, 275, and 278.

EXAMPLES 123. PAGE 218.

1. 19. **2.** 23. **3.** 41. **4.** 59. **5.** 83. **6.** 93. **7.** 119. **8.** 149.
9. 151. **10.** 253. **11.** 383. **12.** 476. **13.** 499. **14.** 503. **15.** 613.
16. 739. **17.** 1123. **18.** 1059. **19.** 3852. **20.** 6304. **21.** 4701. **22.** 8027.
23. 8513. **24.** 13509. **25.** 30507. **26.** 19321. **27.** 29897. **28.** 62573.
29. 62569. **30.** 37196. **31.** 322176. **32.** 100372. **33.** 131025.

EXAMPLES 124. PAGE 220.

1. 1·4. **2.** 1·8. **3.** 9·3. **4.** 1·19. **5.** 5·37. **6.** 18·2. **7.** 3·47.
8. 31·87. **9.** 13·05. **10.** 11·01. **11.** 2·946. **12.** 10·043.
13. 12·025. **14.** 90·909. **15.** 26·004. **16.** 5·1034. **17.** 1·7156.
18. 2·0162. **19.** 8·48071. **20.** 14·37121. **21.** 5·03121. **22.** ·06561.
23. ·0031. **24.** ·0074. **25.** ·0101. **26.** ·0219. **27.** ·01809.
28. ·01849. **29.** ·00043. **30.** ·00072.

EXAMPLES 125. PAGE 221.

1. 19. **2.** 144; 144. **3.** 150 m. **4.** 440 yds. **5.** 770 yds.
6. 306 m. **7.** 9·24 m. **8.** 1001 feet. **9.** 3 m. **10.** 8030.
11. 914 yds. 1 ft. 7 in. **12.** 4 minutes. **13.** 246·9 ft.
14. 176 yds. and 616 yds. **15.** 74 ft. 7½ in long, 49 ft. 9 in. wide.
16. 12·9505 ft.; 5·1802 ft. **17.** 4050 sq. ft. **18.** 40·8 cm.
19. 25 ft. **20.** 14·64 m. **21.** 578 m. **22.** 24 m. 64 cm.
23. Length 10½ yds., breadth 3½ yds., height 4 yds.

EXAMPLES 126. PAGE 225.

1. (i) 30 cu. ft., (ii) 84 cu. ft., (iii) 63000 c.cm., (iv) $58\frac{3}{8\,5\,2}$ cu. ft., (v) 3·26893 c.m., (vi) $12\frac{1\,6\,3}{2\,1\,6}$ cu. ft., (vii) 10·640565 c.m., (viii) $283\frac{4\,9}{7\,2}$ cu. ft.
2. 12½ cu. ft. **3.** 12 ft. **4.** 316224 c.cm. **5.** 230·656 c.m.
6. 3229 cu. ft. 1044 cu. in. **7.** $2037\frac{7}{6\,4}$ cu. ft. **8.** 1386 cu. ft.
9. 55296. **10.** 30000. **11.** $10\frac{5}{1\,2}$ ft.
12. 192 cu. ft.; 5 tons 7 cwt. 16 lb. **13.** 101372·5 Kg.
14. £60. 0*s.* 6*d.* **15.** £63. 7*s.* $11\frac{3\,5}{4\,8}d.$ **16.** 1 ton 3 cwt. 1 qr. $2\frac{2}{5}$ oz.
17. 3 tons 4 cwt. 3 qrs. 4 lb. 13 oz. **18.** 1¼ in. **19.** 3⅜ ft.
20. 125 gallons. **21.** $239\frac{1\,7\,3}{5\,7\,6}$ cu. ft. **22.** 87509 tons.
23. 345 metric tons. **24.** 26·4 cm. **25.** 1451 c.m.
26. 616 Hl. 14 l. **27.** 21 ft. 3·2 in. **28.** 69 cm.
29. $183264\frac{3}{8}$ c.cm. **30.** 64; 125 Kg. **31.** 100 nearly.

EXAMPLES 127. PAGE 229.

1. 6. **2.** 12. **3.** 14. **4.** 15. **5.** 21. **6.** 22. **7.** 24.
8. 33. **9.** 45. **10.** 55. **11.** 60. **12.** 77. **13.** 105. **14.** 154.

15. 165. **16.** 175. **17.** 225. **18.** 1512. **19.** 4851. **20** 47432.
21. $1\frac{1}{2}$. **22.** $\frac{3}{5}$. **23.** $3\frac{1}{2}$. **24.** $3\frac{2}{3}$. **25.** $1\frac{8}{9}$. **26.** $3\frac{1}{8}$. **27.** $1\frac{9}{14}$.
28. $1\frac{5}{14}$. **29.** $\frac{2}{9}$. **30.** $1\frac{2}{39}$. **31.** $7\frac{2}{5}$. **32.** $13\frac{1}{4}$. **33.** $\frac{5}{7}\frac{7}{9}$. **34.** $15\frac{1}{8}$.
35. $\frac{3}{4}\frac{0}{4}\frac{5}{4}$. **36.** $89\frac{2}{5}$. **37.** $75\frac{1}{8}$. **38.** $108\frac{1}{2}$. **39.** 11. **40.** 3. **41.** 12.
42. 4. **43.** 11 yds. **44.** $13\frac{1}{4}$ in.; $175\frac{9}{16}$ sq. in. **45.** 13254 sq. in.
46. 49 sq. m. **47.** Edge = 17·8 m. **48.** 12 cm. **49.** 2·5 c.m.

EXAMPLES 128. Page 232.

1. £27. **2.** £30. **3.** £24. 15*s*. **4.** £18. 13*s*. 4*d*. **5.** £36.
6. £83. 2*s*. 6*d*. **7.** £110. 5*s*. **8.** £326. 6*s*. 8*d*. **9.** £147. 5*s*.
10. £150. 3*s*. 4*d*. **11.** £20. 1*s*. 6*d*. **12.** £4. 0*s*. $2\frac{1}{2}d$. **13.** £105.
14. £1377. 12*s*. 6*d*. **15.** £176. 10*s*. 4*d*. **16.** £52. 12*s*. $7\frac{1}{2}d$.
17. £121. 12*s*. $0\frac{3}{4}d$. **18.** £201. 14*s*. $9\frac{1}{2}d$. **19.** £361591. 15*s*. 9*d*.
20. £28. 4*s*. 7*d*. **21.** £128. 17*s*. 2*d*. **22.** £2096. 4*s*.
23. £6840. 11*s*. 8*d*. **24.** £4307. 11*s*. $10\frac{1}{2}d$. **25.** £1199. 13*s*. $4\frac{1}{2}d$.
26. £1600. 5*s*. $1\frac{1}{2}d$. **27.** £1040. 2*s*. 6*d*. **28.** £429. 14*s*. $3\frac{1}{4}d$.
29. £511. 14*s*. $10\frac{1}{2}d$. **30.** £2088. 10*s*. $3\frac{3}{4}d$. **31.** £1330. 6*s*. 3*d*.
32. £4011. 6*s*. $6\frac{3}{4}d$. **33.** £6928. 1*s*. $5\frac{1}{4}d$. **34.** £2229. 1*s*. 4*d*.
35. £109482. 7*s*. $7\frac{1}{2}d$. **36.** £6843. 7*s*. 1*d*.
37. 31302 tons 17 cwt. 21 lb. **38.** 38893 mi. 4 fur. 7 ch. 11 yds.
39. 1948 hrs. 11 min. 15 sec. **40.** 3743 sq. yds. 1 sq. ft. 63 sq. in.

EXAMPLES 129. Page 235.

1. £40. 9*s*. $4\frac{1}{2}d$. **2.** £312. 13*s*. 9*d*. **3.** £33. 16*s*. 7*d*.
4. £2. 5*s*. 10*d*. **5.** £13. 17*s*. 5*d*. **6.** £1. 17*s*. 0*d*.
7. £310. 0*s*. 5*d*. **8.** £13. 9*s*. 1*d*. **9.** £40. 4*s*. 2*d*.
10. £80. 19*s*. 9*d*. **11.** £16. 5*s*. 7*d*. **12.** £412. 10*s*. 1*d*.
13. £2734. 15*s*. 3*d*. **14.** £279. 16*s*. 6*d*. **15.** £85. 10*s*. $11\frac{1}{2}d$.
16. £36. 3*s*. 2*d*. **17.** £155. 15*s*. 0*d*. **18.** 3*s*. 3*d*. **19.** £4. 9*s*. 1*d*.
20. 4*s*. 11*d*. **21.** £300. 15*s*. 10*d*. **22.** £75. 10*s*. $4\frac{1}{2}d$.
23. £169. 7*s*. $8\frac{1}{2}d$. **24.** £3093. 15*s*. **25.** £5118. 2*s*. 5*d*.
26. £10387. 17*s*. 10*d*. **27.** £62. 12*s*. $1\frac{1}{2}d$. **28.** £1492. 0*s*. 9*d*.
29. £4412. 12*s*. 4*d*. **30.** £191. 14*s*. 8*d*. **31.** £9. 7*s*. 5*d*.
32. £339. 12*s*. **33.** £734. 10*s*. 9*d*. **34.** £16. 18*s*. 6*d*.
35. £36. 4*s*. 6*d*. **36.** £424. 10*s*. 5*d*. **37.** £56. 0*s*. 6*d*.
38. £4567. 14*s*. 10*d*. **39.** £2045. 4*s*. $7\frac{1}{2}d$. **40.** £577. 12*s*. 7*d*.
41. £4147. 19*s*. 1*d*. **42.** £22. 3*s*. 3*d*. **43.** £564. 13*s*. 2*d*.
44. £369. 18*s*. 8*d*. **45.** £2. 4*s*. 8*d*. **46.** £3895. 19*s*. 1*d*.
47. £45. 5*s*. 5*d*. **48.** £385. 16*s*. 6*d*. **49.** 3517 lb. $9\frac{1}{4}$ oz.
50. 78 lb. 12 oz. **51.** 5 tons 6 cwt. 3 qrs. 14 lb. 5 oz.

EXAMPLES 130. PAGE 237.

1. £48. 1*s.* 11*d.* **2.** £3. 14*s.* 2*d.* **3.** £2. 16*s.* $0\frac{3}{4}d.$ **4.** £1. 4*s.* $9\frac{1}{2}d.$
5. 13*s.* $7\frac{1}{4}d.$ **6.** £1. 3*s.* $7\frac{1}{2}d.$ **7.** £4. 16*s.* 8*d.* **8.** £10. 12*s.* $3\frac{1}{4}d.$
9. £4. 0*s.* $3\frac{3}{4}d.$ **10.** £26. 14*s.* $10\frac{1}{2}d.$ **11.** £9. 10*s.* $1\frac{1}{2}d.$ **12.** £6. 2*s.* 9*d.*
13. £41. 5*s.* 9*d.* **14.** £2. 2*s.* $2\frac{1}{4}d.$ **15.** £4. 9*s.* 11*d.* **16.** £4. 19*s.* 10*d.*

EXAMPLES 132. PAGE 241.

1. $\frac{1}{18}$. **2.** $\frac{1}{12}$. **3.** $\frac{1}{4}$. **4.** $\frac{20}{3}$. **5.** $\frac{3}{1}$. **6.** $\frac{1}{44}$. **7.** $\frac{2}{65}$.
8. $\frac{6}{17}$. **9.** $\frac{26}{31}$. **10.** $\frac{35}{24}$. **11.** $\frac{114}{185}$. **12.** $\frac{195}{77}$. **13.** $\frac{105}{52}$.
14. $\frac{383}{256}$. **15.** $\frac{2}{3}$. **16.** $\frac{151}{210}$. **17.** $\frac{19}{6}$. **18.** $\frac{42}{79}$. **19.** $\frac{19}{10}$.
20. $\frac{101}{9801}$. **21.** $\frac{7}{9}$. **22.** $\frac{11}{5}$. **23.** $\frac{3}{20}$. **24.** $\frac{15}{16}$. **25.** $\frac{24}{1}$.
26. $\frac{8}{15}$. **27.** $\frac{8}{1}$. **28.** $\frac{8}{5}$. **29.** $\frac{10}{21}$. **30.** $\frac{3}{13}$; $\frac{1}{9}$. **31.** $10\frac{2}{5}$.
32. $3\frac{3}{8}$. **33.** $\frac{1}{9}$. **34.** £10. 0*s.* 10·6*d.* **35.** 1 cwt. 2 qrs. 4 lb. 4 oz.

EXAMPLES 133. PAGE 244.

1. 21. **2.** 4. **3.** 105. **4.** 12. **5.** 4. **6.** $2\frac{2}{5}$. **7.** $6\frac{2}{9}$.
8. 9. **9.** 850. **10.** 6. **11.** 11. **12.** 1750. **13.** $\frac{1}{3}$. **14.** 12.
15. 9. **16.** $736\frac{2}{3}$. **17.** 3. **18.** 8. **19.** $1\frac{1}{3}$. **20.** $\frac{3}{5}$.
21. 35. **22.** 30. **23.** $\frac{10}{13}$. **24.** 6. **25.** 8. **26.** 12.
27. 15. **28.** 25. **29.** 24. **30.** 3. **31.** $2\frac{2}{3}$. **32.** $1\frac{3}{5}$.

EXAMPLES 134. PAGE 246.

1. $4\frac{4}{9}$, $22\frac{7}{9}$, $29\frac{4}{9}$, $-11\frac{1}{9}$, $-33\frac{1}{3}$, degrees C.
$3\frac{5}{9}$, $18\frac{2}{9}$, $23\frac{5}{9}$, $-8\frac{8}{9}$, $-26\frac{2}{3}$, degrees R.

2. 68, 95, $197\frac{3}{5}$, 14, $-32\frac{4}{5}$, degrees F.
16, 28, $73\frac{3}{5}$, -8, $-28\frac{4}{5}$, degrees R.

3. $97\frac{1}{4}$, $110\frac{3}{4}$, $225\frac{1}{2}$, 14, $-1\frac{3}{4}$, degrees F.
$36\frac{1}{4}$, $43\frac{3}{4}$, $107\frac{1}{2}$, -10, $-18\frac{3}{4}$, degrees C.

EXAMPLES 135. PAGE 248.

1. $1\frac{1}{2}d.$ **2.** £37. 7*s.* 9*d.* **3.** 5 cwt. **4.** £3. 15*s.* **5.** £10. 8*s.* $1\frac{1}{2}d.$
6. £23. 6*s.* 8*d.* **7.** £11. 1*s.* 8*d.* **8.** £1. 10*s.* **9.** £960. 6*s.* **10.** £8. 19*s.*
11. £27. 10*s.* **12.** £115. 4*s.* **13.** £93. 8*s.* **14.** 123 yds.
15. 732 f. **16.** 108 miles. **17.** 48. **18.** 108.
19. 98. **20.** 15. **21.** 6. **22.** £43. 2*s.* $7\frac{1}{4}d.$
23. £46. 17*s.* 6*d.* **24.** 170 Kg. **25.** $10\frac{1}{2}$ m. **26.** 2 f. 8 c.
27. 43 dol. 70 c. **28.** £169. 5*s.* 9*d.* **29.** £6. 9*s.* 6*d.* **30.** £29. 8*s.*
31. £2340. **32.** 81 l. 90 cl. **33.** $\frac{15}{34}$. **34.** 152.
35. 864. **36.** £7. 11*s.* 8*d.* **37.** 2*s.* $8\frac{1}{2}d.$ **38.** 4*s.* $4\frac{1}{2}d.$

39. £156. 16*s*. **40.** 9*d*. **41.** £1. 14*s*. 1½*d*. **42.** £8. 1*s*.
43. 45 Kg. 5 Hg. **44.** ½ yd. **45.** 12 mi. 2 fur. 14 po. **46.** £6. 5*s*.
47. 35280 f. **48.** 10 f. 71 c. **49.** 26 c. cm. **50.** 136 Km. 9 Hm.
51. 261 f. 8 c. **52.** 1*s*. 10*d*. **53.** £187. 2*s*. 1½*d*. **54.** £1. 19*s*. 4½*d*.

EXAMPLES 136. PAGE 252.

1. £1. 11*s*. 8*d*. **2.** £60. **3.** £46. 7*s*. **4.** 630 Km. **5.** £150.
6. £37. 2*s*. 6*d*. **7.** 183⅓ Km. **8.** 56¼ miles. **9.** £6. **10.** 12 f.
11. 434 f. 70 c. **12.** 48 days. **13.** $28\frac{4}{5}$ min. **14.** 3 Kg. 640 g.
15. 1584 m. **16.** 94 ac. 2 ro. 16 per. **17.** £1. 7*s*. 1½*d*. **18.** £37. 10*s*.
19. 55 Kg. **20.** 403 Km. **21.** 30 cm. by 24 cm. **22.** 40 ares.
23. £9. 18*s*. **24.** 1616 m. **25.** £88. 15*s*. 1¾*d*. **26.** £5. 7*s*. 2*d*.
27. 88. **28.** 16 oz. 18 grs. **29.** 999 oz. **30.** 65.

EXAMPLES 137. PAGE 255.

1. 295·8 Km., 382·8 Km. **2.** 13 metres. **3.** 2257 f. 50 c. **4.** 15.
5. 704. **6.** 10. **7.** 192 f. **8.** 21 weeks.
9. 21. **10.** 660. **11.** 8. **12.** 5.
13. £50. 12*s*. 6*d*. **14.** 54. **15.** 198 Km. **16.** £40.
17. 4. **18.** 180 Ha. 48 a. **19.** 38. **20.** 165.
21. 270. **22.** 5. **23.** 81. **24.** $8\frac{1}{3}$.
25. 10. **26.** £4. 8*s*. **27.** 84. **28.** 18.
29. £51. **30.** 10*s*. **31.** 40 Kg. **32.** £11. 5*s*.
33. £10. 12*s*. 8*d*. **34.** English : French = 25 : 24. **35.** 80. **36.** 20.
37. 18*s*. 1·6*d*. **38.** 7½*d*. **39.** £51. 4*s*. 3½*d*. **40.** 18 days.
41. £1. 3*s*. 10*d*. **42.** £50. 10*s*. **43.** 6. **44.** ½ hour per day.
45. £21. 7*s*. 6*d*. **46.** 11. **47.** 30. **48.** 7. **49.** 19.
50. 10. **51.** £189. **52.** 75 days. **53.** 5. **54.** £12. 9*s*.
55. $26\frac{7}{8}$ miles. **56.** 50·5848 g. **57.** £105. 12*s*. **58.** $12\frac{2}{3}\frac{1}{2}$.

EXAMPLES 141. PAGE 261.

1. 18 miles. **2.** 2 in. **3.** 8 per. **4.** $345\frac{3}{5}$ cu. in.
5. 96 sq. in. **6.** £3. 14*s*. **7.** $16\frac{4}{5}$ lb. **8.** 487 tons 10 cwt.
9. 2429 men. **10.** 45·22 mi. **11.** 94 f. 15 c. **12.** 7 Km. 465 m.
13. 8 Kg. 456 g. **14.** 2 gall. 2½ gills. **15.** 494 ac. 30 per.
16. £2399. 11*s*. 3·6*d*. **17.** 3 qrs. 19⅕ lb. **18.** 2 fur. 1 ch.
19. £1260. **20.** £1425. **21.** 1940 dol.
22. 7374. **23.** 7210 f. **24.** £11223. 17*s*. 4·8*d*. **25.** $2395\frac{4}{5}$ sq. yds

EXAMPLES 142. PAGE 262.

1. 50. **2.** 20. **3.** 60. **4.** $66\frac{2}{3}$. **5.** $12\frac{1}{2}$. **6.** 20. **7.** 45. **8.** 4.
9. 50. **10.** $33\frac{1}{3}$. **11.** $7\frac{1}{2}$. **12.** $99\frac{1}{2}$. **13.** $14\frac{1}{4}$. **14.** 85. **15.** $53\frac{9}{47}$.

EXAMPLES 143. PAGE 263.

1. 1600. **2.** 2880. **3.** 2400. **4.** 26. **5.** 1040. **6.** 1500.
7. £90. **8.** 63 gall. **9.** £3000. **10.** £4000. **11.** 12400 f. **12.** 9740.

EXAMPLES 144. PAGE 265.

1. £340. **2.** £2. **3.** 16. **4.** $10\frac{1}{4}$ %. **5.** $\frac{4}{5}$ % excess. **6.** 650.
7. 145600. **8.** 75 Ha. **9.** 3700. **10.** 3. **11.** $\frac{3}{4}$.
12. 1 % decrease. **13.** 12. **14.** 32·825. **15.** $33\frac{1}{3}$. **16.** 20.
17. 2*d*. **18.** 4·03 %. **19.** 66·0 %. **20.** 44·53 %. **21.** 45·24 %.

EXAMPLES 145. PAGE 268.

1. £50. **2.** £12. 0*s*. 9*d*. **3.** £96. 5*s*. **4.** £202. 10*s*. **5.** £23.
6. £10526. 5*s*. **7.** £15. 0*s*. 8*d*. **8.** £85. 7*s*. 9·9*d*.; £5607. 7*s*. 2·1*d*.
9. £7. 13*s*. 7·2*d*. **10.** £8650. **11.** £78. 12*s*. $9\frac{3}{8}d$. **12.** £765.
13. £1600. **14.** £66. 5*s*. 3*d*. **15.** £879. 14*s*. $8\frac{1}{4}d$. **16.** £113. 4*s*. $7\frac{1}{2}d$.
17. £12. 8*s*. 4*d*. **18.** £1599. **19.** £20. 8*s*. $6\frac{1}{2}d$. **20.** £631. 10*s*.
21. £533. 6*s*. 8*d*. **22.** £945. **23.** £1449. **24.** 8*s*. 5*d*.
25. £2933. 6*s*. 8*d*. **26.** 3*s*. 4*d*. **27.** 1*s*. 2*d*. **28.** 3*d*.
29. £70. **30.** £360. **31.** £1020. 10*s*.

EXAMPLES 146. PAGE 271.

1. 25 % gain. **2.** 20 % gain. **3.** 50 % gain. **4.** $44\frac{4}{9}$ % gain.
5. 25 % loss. **6.** $37\frac{1}{2}$ % loss. **7.** 25 % gain. **8.** 20 % gain.

EXAMPLES 147. PAGE 271.

1. £700; £9450. **2.** 21*s*. per lb. **3.** £41. 17*s*. **4.** £31. 6*s*. 9*d*.
5. £38. 7*s*. 8*d*. **6.** $3\frac{3}{4}d$. **7.** 26*s*. 6*d*. **8.** £109. 2*s*. 6*d*. **9.** 95316 f.

EXAMPLES 148. PAGE 272.

1. £30. **2.** £125. **3.** £24. **4.** £58. 6*s*. 8*d*.
5. £533. 6*s*. 8*d*. **6.** £1. 13*s*. 4*d*. **7.** 52 f. 50 c. **8.** £290.
9. £1. 5*s*. $6\frac{1}{2}d$. **10.** 642 f. 50 c. **11.** 3*s*. $2\frac{1}{2}d$.

EXAMPLES 149. PAGE 273.

1. 20 %. **2.** 5 % loss. **3.** 35 % gain. **4.** $5\frac{1}{2}$ % loss.
5. 8 %. **6.** 40 % gain. **7.** $3\frac{3}{4}$ %. **8.** $15\frac{5}{8}$ % loss.
9. $3\frac{1}{3}$. **10.** Neither gain nor loss.

EXAMPLES 150. PAGE 274.

1. £2. 4*s*. **2.** 5 f. 80 c. **3.** 9*s*. 3*d*. **4.** 10*s*. $11\frac{1}{4}d$.
5. £8. 12*s*. 6*d*. **6.** 9*s*. $2\frac{1}{2}d$. **7.** 8*d*. **8.** 310 f.

EXAMPLES 151. PAGE 276.

1. $33\frac{1}{3}$. 2. 2 f. 25 c. 3. 1 f. 96 c. 4. 15 for 7*d*.
5. £58. 10*s*. 6. £2400. 7. £240. 8. 600 f.
9. £446. 10*s*. 10. $4\frac{1}{8}d$. 11. 12*s*. 6*d*. 12. $63\frac{7}{11}$.
13. 12*s*. 6*d*.; $19\frac{3}{13}$. 14. $33\frac{1}{3}$. 15. 400. 16. $34\frac{8}{13}$.
17. 23%. 18. $29\frac{3}{8}$. 19. $9\frac{1}{11}$. 20. $\frac{3}{4}d$.; 20.
21. $11\frac{1}{9}$. 22. £10 lost. 23. 25. 24. 576 Kg.
25. 16*s*. $10\frac{1}{2}d$. 26. 20. 27. $47\frac{11}{12}$. 28. 19*s*. $4\frac{8}{31}d$.
29. 35 for 2 f. 30. $3\frac{3}{4}d$. 31. Lose $5\frac{1}{2}$%. 32. £16. 16*s*.
33. 3 f. 75 c. 34. £44. 0*s*. 10*d*. 35. $5\frac{1}{8}$ gain. 36. £3. 16*s*. 8*d*.
37. £13. 6*s*. 8*d*. 38. 15*s*. 39. £1. 9*s*. $6\frac{6}{7}d$. 40. $30\frac{5}{64}$.
41. Loses 80%. 42. 8. 43. $14\frac{7}{12}$. 44. 25. 45. $2\frac{2}{5}$.
46. A, £105. 8*s*. 4*d*.; B, £104. 7*s*. 3*d*. 47. £75. 48. £10. 49. Loss $=2\frac{2}{49}$%.

EXAMPLES 153. PAGE 281.

1. £78. 2. £126. 15*s*. 3. £66. 11*s*. 2·4*d*.
4. £18. 7*s*. 6*d*. 5. 32 dol. 97 c. 6. 25 m. 74 pf.
7. 89 dol. 79 c. 8. 118 f. 16 c. 9. 13 f. 44 c.
10. 14 m. 70 pf. 11. £26. 14*s*. 12. £91. 17*s*. 6*d*.
13. £221. 14. £259. 4*s*. 15. £3141. 12*s*.
16. £339. 18*s*. 17. £621. 13*s*. 6*d*. 18. 838 dol. 8 c.
19. 897 f. 92 c. 20. £1658. 15*s*. $3\frac{1}{4}d$. 21. £32. 11*s*.
22. £250. 2*s*. 6*d*. 23. £109. 15*s*. $6\frac{3}{4}d$. 24. £154. 18*s*. 3*d*.
25. £257. 5*s*. 3*d*. 26. 105 f. 2 c. 27. 506 f. $37\frac{1}{2}$ c.
28. £897. 7*s*. $1\frac{1}{2}d$. 29. £35. 18*s*. 8*d*. 30. £563. 1*s*. $1\frac{1}{2}d$.
31. £702. 13*s*. $9\frac{3}{4}d$. 32. 1665 m. 3 pf. 33. 650 dol. 65 c.
34. £3963. 0*s*. $2\frac{3}{4}d$. 35. £946. 16*s*. $3\frac{1}{2}d$. 36. £887. 4*s*. $7\frac{1}{4}d$.
37. £7501. 11*s*. $10\frac{1}{2}d$. 38. 8327 dol. 45 c. 39. £42. 18*s*. 4*d*.
40. 693 m. 11 pf. 41. £3540. 14*s*. 8*d*. 42. £974. 6*s*. 3*d*.
43. 8528 m. 40 pf. 44. £1362. 11*s*. $9\frac{3}{4}d$. 45. £1529. 19*s*. $4\frac{3}{4}d$.
46. £376. 9*s*. $3\frac{1}{2}d$. 47. £6918. 1*s*. $9\frac{1}{2}d$. 48. £2290. 5*s*. 3*d*.
49. £539. 14*s*. 8*d*. 50. £194. 4*s*. 10*d*.

EXAMPLES 154. PAGE 284.

1. £117. 10*s*. 2. £385. 3. £1594. 2*s*. $9\frac{3}{4}d$.
4. £2311. 7*s*. 6*d*. 5. £8776. 6*s*. $10\frac{3}{4}d$. 6. £4763. 14*s*. $4\frac{1}{2}d$.
7. 3425 f. $62\frac{1}{2}$ c. 8. 242 dol. 34 c.

EXAMPLES 155. PAGE 285.

1. £750. 2. £350. 3. £370. 4. £175.
5. £4328. 2*s*. 6*d*. 6. £906. 13*s*. 4*d*. 7. £720. 8. £258. 17*s*. 2*d*.
9. £6307. 15*s*. 10. £37. 10*s*. 11. £275. 12. £555. 11*s*. $1\frac{1}{5}d$.

13. £40. 16*s*. 8*d*. 14. £1733. 6*s*. 8*d*. 15. £7144. 3*s*. 8·65*d*.
16. £8. 8*s*. $2\frac{3}{4}d$. 17. £356. 5*s*. ; £21. 7*s*. 6*d*. 18. £502. 13*s*. 4*d* ; £75. 8*s*.
19. £7877. 10*s*. ; £1516. 8*s*. $4\frac{1}{2}d$. 20. £200. 15*s*. ; 9*s*. $7\frac{1}{2}d$.

EXAMPLES 156. PAGE 287.

1. 25 yrs. 2. $3\frac{1}{2}$ yrs. 3. 7 yrs. 4. 8 yrs. 5. 7 yrs.
6. $3\frac{1}{4}$ yrs. 7. 3 yrs. 8. $4\frac{1}{4}$ yrs. 9. 5 yrs. 10. 8 yrs.
11. 4 yrs. 12. 5 yrs. 13. $4\frac{3}{4}$ yrs. 14. $4\frac{1}{2}$ yrs. 15. $3\frac{3}{4}$ yrs.
16. $2\frac{1}{2}$ yrs. 17. 4 yrs. 18. 3 yrs. 19. 3 yrs. 20. $5\frac{1}{8}$ yrs.
21. $3\frac{3}{4}$ yrs. 22. $\frac{1}{8}$ yr. 23. $3\frac{1}{4}$ yrs.

EXAMPLES 157. PAGE 288.

1. 9. 2. $8\frac{1}{3}$. 3. $3\frac{1}{3}$. 4. $3\frac{1}{2}$. 5. $10\frac{1}{2}$. 6. $3\frac{1}{4}$. 7. $9\frac{1}{2}$.
8. 10. 9. $2\frac{1}{3}$. 10. 5. 11. $2\frac{1}{2}$. 12. $5\frac{1}{4}$. 13. 1. 14. $3\frac{1}{3}$.
15. $37\frac{1}{2}$. 16. 6. 17. 5. 18. $5\frac{3}{5}$. 19. 7. 20. 4. 21. 4. 22. 5.

EXAMPLES 158. PAGE 290.

1. $\frac{1}{8}$ yr. 2. $7\frac{1}{2}$ yrs. 3. $2\frac{1}{2}$ per cent. 4. $3\frac{3}{4}$ yrs.
5. $6\frac{1}{4}$. 6. £100. 7. £266. 10*s*. 8. £1479. 12*s*. 9*d*.
9. £25. 10. £270. 6*s*. 11. £574. 13*s*. ; $6\frac{1}{4}$ per cent.

EXAMPLES 159. PAGE 292.

1. £31. 17*s*. 4*d*. 2. £5842. 13*s*. 4*d*. 3. £60. 4. 2*s*. 4*d*.
5. 17*s*. 10*d*. 6. £6. 9*s*. 4*d*. 7. 5*s*. 2*d*. 8. 1*s*. 4*d*.
9. £1. 4*s*. 5*d*. 10. 19*s*. 5*d*. 11. 18*s*.
12. £100. 10*s*. 4*d*. ; £12. 3*s*. 1*d*.

EXAMPLES 160. PAGE 294.

1. £36. 3*s*. 1*d*. 2. £8. 2*s*. 11*d*. 3. £4. 5*s*. 2*d*. 4. £12. 4*s*. 5*d*.
5. £11. 19*s*. 5*d*. 6. £35. 8*s*. 8*d*. 7. £83. 19*s*. 10*d*. 8. £468.
9. £25. 10*s*. 10. £6. 11*s*. 1*d*. 11. £6. 18*s*. 3*d*. 12. £8. 11*s*. 3*d*.
13. £2. 1*s*. 5*d*. 14. £6. 0*s*. 2*d*. 15. £13. 4*s*. 10*d*. 16. £1895. 14*s*. 9*d*.
17. £12. 1*s*. 4*d*. 18. £24. 0*s*. 6*d*. 19. £1. 2*s*. 10*d*. 20. £18. 19*s*. 7*d*.

EXAMPLES 161. PAGE 297.

1. £138. 18*s*. 4*d*. ; £12. 18*s*. 4*d*. 2. £1428. 10*s*. 2*d*. ; £194. 10*s*. 2*d*.
3. £689. 12*s*. 3*d*. ; £64. 2*s*. 3*d*. 4. £771. 6*s*. 6*d*. ; £55. 1*s*. 6*d*.
5. £649. 11*s*. 5*d*. ; £88. 8*s*. 11*d*. 6. £994. 18*s*. 3*d*. ; £71. 0*s*. 9*d*.
7. £343. 8*s*. 0*d*. ; £85. 8*s*. 0*d*. 8. £3978. 3*s*. 6*d*. ; £1184. 3*s*. 6*d*.
9. £3727. 18*s*. 9*d*. ; £1109. 13*s*. 9*d*. 10. £476. 11*s*. 3*d*. ; £101. 14*s*. 8*d*.
11. £3819. 4*s*. 9*d*. ; £219. 4*s*. 9*d*. 12. £4058. 3*s*. 3*d*. ; £306. 3*s*. 3*d*.
13. £1333. 0*s*. 5*d*. : £76. 10*s*. 5*d*. 14. £7868. 6*s*. 3*d*. ; £663. 1*s*. 3*d*.

15. £553. 12*s.* 0*d.* ; £46. 19*s.* 6*d.* **16.** £413. 7*s.* 2*d.* ; £35. 1*s.* 6*d.*
17. £3541. 7*s.* 7*d.* ; £276. 16*s.* 0*d.* **18.** £3664. 9*s.* 1*d.* ; £406. 15*s.* 4*d.*
19. £5794. 10*s.* 7*d.* ; £530. 2*s.* 9*d.* **20.** £3174. 16*s.* 4*d.* ; £205. 0*s.* 1*d.*
21. £1810. 13*s.* 2*d.* ; £223. 19*s.* 8*d.* **22.** £2067. 8*s.* 3*d.* ; £170. 13*s.* 1*d.*
23. £1853. 11*s.* 7*d.* ; £175. 1*s.* 8*d.* **24.** £4089. 7*s.* 3*d.* ; £399. 19*s.* 0*d.*
25. £2554. 0*s.* 5*d.* ; £239. 3*s.* 3*d.* **26.** 61759. **27.** 407635.

EXAMPLES 162. PAGE 299.

1. £338. 7*s.* 6*d.* **2.** £520. 10*s.* 3*d.* **3.** £718. 16*s.* 11*d.*
4. £1299. 17*s.* 8*d.* **5.** £2788. 15*s.* 3*d.*

EXAMPLES 163. PAGE 301.

(In Answers 1 to 6 a fraction of a penny is reckoned as a whole penny.)

1. £90. 14*s.* 1*d.* **2.** £340. 2*s.* 9*d.* **3.** £399. 9*s.* 3*d.* **4.** £460. 14*s.* 6*d*
5. £656. 0*s.* 3*d.* **6.** £351. 7*s.* 1*d.* **7.** £431. 18*s.* 4*d.*; £68. 1*s.* 8*d.*
8. £1098. 9*s.* 7*d.*; £101. 10*s.* 5*d.* **9.** £800. 14*s.* 11*d.*; £74. 5*s.* 1*d.*
10. £1903. 15*s.* 3*d.*; £146. 4*s.* 9*d.* **11.** £304. 2*s.* 3*d.*; £23. 7*s.* 9*d.*
12. £755. 0*s.* 9*d.*; £106. 11*s.* 9*d.* **13.** £399. 0*s.* 1*d.*; £63. 8*s.* 5*d.*
14. £1197. 9*s.* 1*d.*; £79. 8*s.* 1*d.* **15.** £1974. 11*s.* 11*d.*; £110. 11*s.* 10*d.*
16. £6942. 2*s.* 9*d.*; £679. 9*s.* 7*d.* **17.** £4701. 12*s* 0*d.*; £311. 19*s.* 3*d.*

EXAMPLES 164. PAGE 302.

1. £224. 12*s.* 6*d.* **2.** £577. 10*s.* 11*d.* **3.** £1695. 16*s.* 7*d.*
4. £2830. 14*s.* 4*d.* **5.** £5262. 13*s.* 7*d.* **6.** £7603. 0*s.* 0*d.*
7. £5974. 11*s.* 3*d.* **8.** £12250. 0*s.* 2*d.* **9.** £3843. 6*s.* 8*d.*
10. £2562. 2*s.* 8d.

EXAMPLES 165. PAGE 303.

1. 1·19. **2.** 4·52. **3.** 4·47. **4.** 3·8. **5.** 4·39.
6. 21·2. **7.** 5·8. **8.** 5·2.

EXAMPLES 166. PAGE 303.

1. 126 f. 56 c. **2.** 14 f. 68 c. **3.** $3\frac{3}{4}$ %. **4.** £1540. 5*s.* 10½*d.*
5. 2. **6.** £590. 12*s.* 6*d.* **7.** $2\frac{1}{3}$ years. **8.** £225. 13*s.* 4*d.*
9. $1\frac{1}{4}$ years. **10.** £886. 10*s.* 9*d.* **11.** £3267. 18*s.* 4*d.* **12.** £185. 12*s.* 6*d.*
13. $2\frac{1}{2}$. **14.** 1026 f. **15.** £345. **16.** 4. **17.** $2\frac{1}{7}$ years.
18. 3 %. **19.** £925. 16*s.* 8*d.* **20.** 3 years. **21.** $4\frac{1}{2}$.
22. 11 dol. 25 c. **23.** $3\frac{1}{4}$. **24.** $7\frac{1}{2}$ %. **25.** £15. **26.** £39. 0*s.* 9*d.*
27. £409. 8*s.* 4*d.* **28.** £5. 9*s.* 4*d.* **29.** £193. 0*s.* 6*d.* **30.** £10. 14*s.* 7*d.*
31. £84. 19*s.* 5*d.* **32.** £319. 13*s.* 7*d.* **33.** £616. 9*s.* 1*d.*
34. £284. 19*s.* 4*d.* **35.** £134. 12*s.* $7\frac{1}{2}$*d.* **36.** £1616. 14*s.* 7*d.*
37. £1. 13*s.* $8\frac{1}{2}$*d.* **38.** £300. 9*s.* 11*d.* ; £306. 6*s.* 9*d.*
39. £216. 13*s.* 4*d.* **40.** £520. 16*s.* 8*d.* **41.** £800.
42. $6\frac{7}{9}$ %. **43.** £222. **44.** £172. 4*s.* 8*d.*

EXAMPLES 167. PAGE 311.

1. £165. **2.** £171. **3.** £28. **4.** 450 f. 75 c. **5.** £281. 18*s.*
6. 323 dol. 25 c. **7.** 2186 f. 73 c. **8.** 205 dol. 62½ c. **9.** £344. 5*s.*
10. £32. 10*s.* **11.** £108. 3*s.* 3*d.* **12.** £125. 6*s.* 3*d.* **13.** £238. 14*s.* 5¼*d.*
14. £194. 10*s.* 3*d.* **15.** £768. **16.** £2812. 10*s.* **17.** £300.

EXAMPLES 168. PAGE 311.

1. £5000. **2.** £2280. **3.** 4200 f. **4.** £3610.
5. £7200. **6.** £14400. **7.** 14400 dol. **8.** £1482. 2*s.* 10*d.*
9. £11363. 12*s.* 8*d.* **10.** 7716 f. 49 c. **11.** £561. 9*s.* 11*d.*

EXAMPLES 169. PAGE 312.

1. 1350 f. **2.** 4231 f. 50 c. **3.** £5062. 10*s.* **4.** £2092. 10*s.*
5. £1710. **6.** 3273 dol. 20 c. **7.** £8308. 15*s.* **8.** 1045 dol. 80 c.
9. £6909. 18*s.* **10.** £6897. 16*s.* **11.** £2314. **12.** £1096. 14*s.* 5*d.*
13. £1329. 7*s.* 6*d.* **14.** £4209. 5*s.* **15.** £13953. 5*s.* 7½*d.* **16.** £7169. 15*s.*

EXAMPLES 170. PAGE 313.

1. £124. **2.** £49. **3.** 864 dol. 50 c. **4.** 168 f.
5. 140 dol. **6.** £126. **7.** £240. **8.** £107.
9. £2. 1*s.* 8*d.* **10.** £11. 7*s.* **11.** Greater by 1*s.* 6*d.*

EXAMPLES 171. PAGE 313.

1. 85½. **2.** 93. **3.** 84. **4.** 97⅝. **5.** 104.
6. 94½. **7.** 90. **8.** 90½. **9.** 80. **10.** 91½.

EXAMPLES 172. PAGE 315.

1. £3600; £126. **2.** £159. 12*s.* **3.** £64. 6*s.* 10*d.*
4. £500; £541. 13*s.* 4*d.* **5.** £166. 4*s.* 5*d.* **6.** 72.
7. £3 $\frac{7}{4}\frac{1}{3}$, or £3. 5*s.* 10·12*d.* **8.** $3\frac{9}{17}$ %. **9.** $3\frac{4}{7}$ %. **10.** $3\frac{11}{13}$ %.
11. Bank stock. **12.** £19508. 4*s.*; $3\frac{3}{4}\frac{1}{8}$. **13.** £1660. **14.** £52. 10*s.*
15. £41. 13*s.* 4*d.* **16.** $10\frac{2}{3}$. **17.** £15. 10*s.* **18.** £19 increase.
19. £48. 15*s.* increase. **20.** £20. **21.** £19. 6*s.* 4*d.* increase.
22. £1250; £2. 10*s.* less. **23.** Greater by £20. **24.** No change.
25. No change. **26.** £285. 4*s.* increase. **27.** £60 increase.
28. £75 loss. **29.** £1942. 10*s.* **30.** The latter; £15000.
31. £130. **32.** 90. **33.** £127. 10*s.* **34.** The latter; £1. 19*s.* 4*d.*
35. £5 loss. **36.** 95$\frac{1}{8}$; £156. 18*s.*; £4108. 16*s.* 4$\frac{1}{2}$*d.*
37. £40. 16*s.*; £27. 4*s.* **38.** £17. 17*s.* 1$\frac{5}{7}$*d.* **39.** 77$\frac{1}{2}$. **40.** 94$\frac{1}{2}$. **41.** $103\frac{8}{9}$
42. $3\frac{1}{4}$. **43.** 90. **44.** £17135; £147. 4*s.* **45.** £1. 12*s.* increase.
46. £864. 10*s.*; £921. 4*s.* **47.** 6*d.* gain. **48.** £1300; £4 loss.
49. £8. 12*s.* 4*d.* increase. **50.** £31. 5*s.* **51.** £4725. **52.** £14520.

53. A's income is the greater by £4. 16*s.*; 35 : 34. **54.** 86. **55.** 5002 : 5005.
56. The latter; £136762. 10*s.* **57.** Anything less than $148\frac{5}{8}$. **58.** £48000.
59. (i) The rate % obtained from the second is $\frac{25}{4828}$ greater than that from the first, (ii) $\frac{100}{7759}$ in favour of second stock.
60. 10 %. **61.** £10166. 13*s.* 4*d.*; £6000.
62. £561. 9*s.* 11*d.*; £22. 9*s.* 1*d.* **63.** 156.

EXAMPLES 173. PAGE 321.

1. £9, £15, £21. **2.** 156; 234; 312. **3.** 162; 108; 81.
4. £900, £1120, £750. **5.** £335. 4*s.* **6.** £150, £180, £240, £300.
7. 113; 339; 678; 791. **8.** 29167; 13739; 3686; 3408.
9. £14583. 10*s.*; £6869. 10*s.*; £1843; £1704.
10. £212. 2*s.*; £353. 10*s.*; £388. 17*s.* **11.** 560; 420; 480.
12. £5. 17*s.* $6\frac{1}{4}d.$; £11. 15*s.* $0\frac{1}{2}d.$; £17. 12*s.* $6\frac{3}{4}d.$ **13.** £120; £40; £20.
14. £1003. 13*s.*; £501. 16*s.* 6*d.*; £167. 5*s.* 6*d.*; £41. 16*s.* $4\frac{1}{2}d.$
15. $14\frac{2}{3}$ cwt.; $3\frac{1}{9}$ cwt.; $2\frac{2}{9}$ cwt. **16.** £955. 4*s.*; £636. 16*s.*
17. A and C give to B 2*s.* 6*d.* and 3*s.* respectively.
18. £480; £440; £165; £605. **19.** 66; 44; 22.
20. £15; £7. 10*s.*; £5. **21.** $4\frac{41}{110}d.$ **22.** 19 : 17.
23. 12 dol. 25 c.; 14 dol. **24.** £298. 8*s.*; £373. **25.** £287. 10*s.*; £322.

EXAMPLES 174. PAGE 323.

1. £138. 17*s.* $9\frac{1}{3}d.$; £236. 2*s.* $2\frac{2}{3}d.$ **2.** £11333. 6*s.* 8*d.*; £5666. 13*s.* 4*d.*
3. £1000; £1500; £500. **4.** £960; £1440; £480. **5.** £360; £144; £240.
6. £503. 8*s.*; £587. 6*s.*; £671. 4*s.* **7.** £457. 10*s.* $10\frac{1}{4}d.$; £374. 7*s.* $0\frac{3}{4}d.$; £291. 3*s.* $3\frac{1}{4}d.$ **8.** £93. 2*s.* $3\frac{3}{11}d.$; £141. 0*s.* $5\frac{5}{11}d.$; £182. 7*s.* $3\frac{3}{11}d.$
9. £11000; £8250; £8250. **10.** £329; £121. **11.** £350; £450.
12. A, £1228; B, £1542.

EXAMPLES 175. PAGE 326.

1. £23. 5*s.* 9*d.*; £30. 14*s.* 3*d.* **2.** £63; £218. 8*s.*; £138. 12*s.* **3.** £20; £21; £25. **4.** £525. **5.** £240; £135. **6.** £240; £220; £200.; £158. **7.** 60. **8.** £15. 6*s.* 8*d.*; £17. 5*s.* **9.** £225. **10.** In 8 months.
11. 864 acres, 1440 acres, 2304 acres. **12.** £265.

EXAMPLES 176. PAGE 328.

1. (i) $5\frac{1}{2}$, (ii) 22, (iii) 23·9554, (iv) $6\frac{121}{150}$, (v) $11\frac{169}{400}$. **2.** 41·7987$\dot{4}$28571$\dot{4}$.
3. $21990\frac{1}{3}$. **4.** $57\frac{5}{7}$. **5.** 185901. **6.** £2. 14*s.* $9\frac{39}{7}d.$ **7.** £17. 6*s.* 8*d.* profit.
8. £107. 9*s.* $9\frac{11}{21}d.$ **9.** 10 st. 13 lb. **10.** 11 st. 13 lb. 11 oz. **11.** 1*s.* $5\frac{3}{4}d.$
12. £19. 4*s.* 11*d.* **13.** 62·7. **14.** 1 ton 2 qrs. **15.** £71. 10*s.*; 14*s.* 3·6*d.*
16. £19. 4*s.* 2*d.* **17.** 11 st. 1 lb. **18.** 78. **19.** 771. **20.** $3\frac{1}{2}$ per cent.
21. 1182720. **22.** 26719. **23.** 15. **24.** 37. **25.** $18\frac{1}{8}$ %. **26.** 85.
27. $3\frac{1}{8}$ %. **28.** 24 miles per hour.

EXAMPLES 177. PAGE 331.

1. 15*s.* 10*d.* **2.** 3 f. 90 c. **3.** $6\frac{2}{5}d.$ **4.** 3 f. 90 c. **5.** £1. 5*s.* 4*d.*
6. 2*s.* $2\frac{1}{2}d.$ **7.** 19*s.* 2·86*d.* **8.** At not less than $3\frac{1}{1}\frac{1}{2}d.$
9. At not less than $1\frac{3}{4}d.$ **10.** 7*s.* 10*d.* **11.** 1*s.* $6\frac{1}{2}d.$ **12.** 17*s.* 6*d.*
13. $12\frac{6}{8}\frac{6}{5}d.$ **14.** £1. 9*s.* $11\frac{1}{1}\frac{}{8}d.$ **15.** $8\frac{1}{3}$ %. **16.** $118\frac{4}{7}.$ **17.** 1*s.* 3*d.*
18. $4\frac{4}{4}\frac{3}{4}\frac{3}{8}d.$; $30\frac{7}{7}\frac{0}{1}\frac{1}{8}\frac{0}{3};$ $23\frac{1}{2}\frac{5}{3}\frac{2}{5}\frac{9}{2}.$ **19.** $25\frac{1}{8}$ %. **20.** 70 %.

EXAMPLES 178. PAGE 333.

1. 3 : 1. **2.** 5 : 9. **3.** 1 : 2. **4.** 7 : 2. **5.** 1 : 3 : 3.
6. 1 : 3 : 1 : 2. **7.** 4 : 5 : 1. **8.** 1 : 3 : 4, or 2 : 3 : 6. **9.** 1 : 1 : 1 : 1.
10. 3 : 5 : 1 : 2. **11.** 10 gallons. **12.** 25 cl. **13.** $\frac{1}{2}$ pint.
14. 144. **15.** Town, $66\frac{2}{3}$ %; country, $33\frac{1}{3}$ %. **16.** 3 : 2.

EXAMPLES 179. PAGE 335.

1. $11\frac{1}{9}$ days. **2.** $4\frac{4}{9}$ days. **3.** $1\frac{3}{7}\frac{4}{1}$ hrs. **4.** $2\frac{1}{2}\frac{4}{9}$ days. **5.** $1\frac{1}{4}\frac{3}{7}$ days; $\frac{2}{4}\frac{0}{7}$, $\frac{1}{4}\frac{5}{7}$, $\frac{1}{4}\frac{2}{7}.$ **6.** $\frac{2}{2}\frac{0}{1}$ hrs. **7.** 12 minutes. **8.** 10 dys. **9.** *A*, 50 dys.; *B*, $21\frac{3}{7}$ dys. **10.** $8\frac{2}{3}$ dys. **11.** $2\frac{1}{2}$ min. **12.** $1\frac{1}{1}\frac{1}{9}$ hrs. **13.** $\frac{7}{8}.$
14. 60 dys. each. **15.** 24 hrs. **16.** 4 min. 42 seconds. **17.** 5 p.m.
18. 3 men. **19.** 24 hrs. **20.** $22\frac{1}{2}$ ft.; 3 hrs. 36 min.

EXAMPLES 180. PAGE 339.

1. 48 yds. **2.** 25. **3.** *B* is better than *C* in the ratio of 799 : 798; can give him no points in a game of 500. **4.** 5. **5.** 10. **6.** 18.
7. $106\frac{2}{3}$ yds. **8.** 39 yds. **9.** *C* will win by $31\frac{3}{7}$ yds. **10.** 38.
11. 5 min. 15 sec. **12.** 88 yds. **13.** 21 miles. **14.** $293\frac{1}{3}$ ft.; $586\frac{2}{3}$ ft.
15. 8 sec. **16.** 54 miles per hour. **17.** $3\frac{1}{2}$ min. **18.** 30 miles per hour; 170 yds. **19.** 2 miles per hour. **20.** 2 hrs. **21.** $1\frac{1}{4}$ miles per hour.
22. 2 miles per hour. **23.** $88\frac{1}{2}$ miles. **24.** $69\frac{3}{5}$ sec. **25.** 13 min.; 14 min.
26. 58 miles. **27.** 18 miles per day, 10 miles per day.

EXAMPLES 181. PAGE 342.

In 1–6 the numbers denote minutes past the hour.

1. (i) $5\frac{5}{1}\frac{}{1}$, (ii) $38\frac{2}{1}\frac{}{1}$, (iii) $21\frac{9}{1}\frac{}{1}$; $54\frac{6}{1}\frac{}{1}$, (iv) $16\frac{4}{1}\frac{}{1}$; 2 o'clock, (v) $32\frac{8}{1}\frac{}{1}$; $43\frac{7}{1}\frac{}{1}.$
2. (i) $16\frac{4}{1}\frac{}{1}$, (ii) $49\frac{1}{1}\frac{}{1}$, (iii) 3 o'clock; $32\frac{8}{1}\frac{}{1}$, (iv) $5\frac{5}{1}\frac{}{1}$; $27\frac{3}{1}\frac{}{1}$, (v) $43\frac{7}{1}\frac{}{1}$; $54\frac{6}{1}\frac{}{1}.$
3. (i) $21\frac{9}{1}\frac{}{1}$, (ii) $54\frac{6}{1}\frac{}{1}$, (iii) $5\frac{5}{1}\frac{}{1}$; $38\frac{2}{1}\frac{}{1}$, (iv) $10\frac{1}{1}\frac{0}{1}$; $32\frac{8}{1}\frac{}{1}$, (v) $49\frac{1}{1}\frac{}{1}$; 5 o'clock.
4. (i) $32\frac{8}{1}\frac{}{1}$, (ii) 6 o'clock, (iii) $16\frac{4}{1}\frac{}{1}$; $49\frac{1}{1}\frac{}{1}$, (iv) $21\frac{9}{1}\frac{}{1}$; $43\frac{7}{1}\frac{}{1}$, (v) $5\frac{5}{1}\frac{}{1}$; 7 o'clock.
5. (i) $38\frac{2}{1}\frac{}{1}$, (ii) $5\frac{5}{1}\frac{}{1}$, (iii) $21\frac{9}{1}\frac{}{1}$; $54\frac{6}{1}\frac{}{1}$, (iv) $27\frac{3}{1}\frac{}{1}$; $49\frac{1}{1}\frac{}{1}$, (v) 7 o'clock; $10\frac{1}{1}\frac{0}{1}.$
6. (i) $54\frac{6}{1}\frac{}{1}$, (ii) $21\frac{9}{1}\frac{}{1}$, (iii) $5\frac{5}{1}\frac{}{1}$; $38\frac{2}{1}\frac{}{1}$, (iv) 10 o'clock; $43\frac{7}{1}\frac{}{1}$, (v) $16\frac{4}{1}\frac{}{1}$; $27\frac{3}{1}\frac{}{1}.$
7. 40 min. 46 sec. past 10 a.m. **8.** 6 a.m. Friday. **9.** 9 p.m.
10. 6.45 a.m. next day. **11.** 12 midnight. **12.** 10.54$\frac{1}{2}$ a.m.

13. At 4.48 p.m., April 20. **14.** Noon, Dec. 31; 7.30. **15.** 44 times.
16. $36\frac{12}{13}$ min. past 4, and $23\frac{1}{13}$ min. past 7. **17.** Noon on Tuesday, Dec. 18, 1906. **18.** $18\frac{6}{23}$ min. and $49\frac{18}{23}$ min. past 7.
19. At points distant 240, 40, 280, 80, 320, 120 and 360 yards from start, measured in A's direction; B has 293 yds. 1 ft. still to run.
20. A, 5368 yds.: B, 4488 yds.; C, 3168 yds.

EXAMPLES 182. PAGE 345.

1. 32. **2.** 70. **3.** 32. **4.** 190.
5. 40. **6.** 2. **7.** $16\frac{4}{11}$. **8.** $11\frac{3}{7}$.
9. $18\frac{6}{11}$. **10.** 15 lb. 10 oz. **11.** 10. **12.** 14 min.

EXAMPLES 183. PAGE 349.

1. 2700 Kg. **2.** 270 grams. **3.** 70. **4.** 3380. **5.** 14·1804.
6. 7·68 c.m.; 7680 litres. **7.** 45 min. 41·2 sec. **8.** 7·92 Km. per hr.
9. 68·15187 Kg. **10.** 330 mi. 4 fur. 6 ch. (to nearest chain).
11. 10441. **12.** 320 metres. **13.** 1·1 metres. **14.** 1·0936.
15. ·4125 f. per litre. **17.** 929. **18.** 6·45148. **19.** 600 f.
20. 503·3. **21.** 1·274. **22.** 6562·5 litres; 5998·125 Kg.
23. 750 litres: 1380 Kg. **24.** 1·452 Kg.
25. Olive oil, 91·5 Kg.; sea water, 102·6 Kg. **26.** 1024.
27. 883 Kg. nearly. **28.** ·032 mm. **29.** 7·3. **30.** 2125. **31.** 158·27735.
32. 2·471 ac. **33.** 14390·63 cu. in. **34.** 4·575. **35.** 49·8 litres.
36. 600 f. 94 c. **37.** 640·84 yds. **38.** 3 f. 78 c. **39.** 161 c. cm.
40. ·568 litre. **41.** 6·070 Ha. **42.** 15·45 lb. **43.** 28349·8 c. cm.

EXAMPLES 184. PAGE 356.

1. £182. 7*s.* 2*d.* **2.** £20. 2*s.* 6*d.* **3.** 6211·70 f.
4. £500. 19*s.* 9*d.* **5.** 16827 f. to nearest franc. **6.** 600 Rupees.
7. 25·58 f. **8.** 57·58 f. **9.** 26·59 kr. **10.** 21·36 m.
11. 15*s.* 6*d.* **12.** £4. 17*s.* 6*d.* **13.** £36. 4*s.* $11\frac{1}{2}d.$
14. 9*s.* $4\frac{1}{4}d.$; 11 f. 70 c. **15.** 37·40. **16.** £6. 14*s.* 3*d.*
17. Gains £11. 5*s.* **18.** Gains £2. 3*s.* 2·62*d.* **19.** 3*s.* $7\frac{1}{8}d.$
20. £21. 10*s.* 5*d.* **21.** 1899 f. 58 c. **22.** 396 dol. 83 c.
23. £1011. 12*s.* 6·7*d.* **24.** £500. 15*s.* 1·52*d.* **25.** $1\frac{9}{10}\frac{1}{09}$ per cent. premium.
26. 44989·78 f. **27.** £972. 4*s.* 5*d.* **28.** £1 = 25·23 f. **29.** 580.
30. $127\frac{17}{29}\%$. **31.** 4 dol. 87 c. **32.** £2595. 13*s.*

EXAMPLES 185. PAGE 361.

1. Period = 2·866 months. **2.** Period = 4·833 months.
3. Period = 3·75 months. **4.** Period = 2·844 months.
5. Period = 15·818 months. **6.** Period = 4·239 months.

7. Period, 294·3 days; Date, 23 March, 1905.
8. Period, 314·1 days; Date, 13 Dec., 1903.
9. Period, 389·3 days; Date, 26 Jan., 1902.

EXAMPLES 186. PAGE 365.

1. $\pm$·96*d*. 3. $-2{\cdot}4d$. 4. $\pm$·015. 5. $\pm$·012.
6. 2 figures; prod. = 19, correct to nearest unit.
7. 3 figures; prod. = 3930, correct to nearest 10.
8. 3 figures; prod. = 2150, correct to nearest 10.
9. 1 figure; prod. = 6100, correct to nearest 100.
10. 4 figures; prod. = 14267, correct to nearest unit.
11. 5 figures; prod. = 13231, correct to nearest unit.
12. 1 figure; true value lies between 2·39 and 2·57.
13. No figures; quot. = 7, correct to nearest unit.
14. 2 figures; quot. lies between 29 and 30.
15. 1 figure; quot. = 70000, correct to nearest 10000.
16. 3 figures; quot. = 22·4, to 1 decimal place.
17. 3 figures; quot. = 25·2, to 1 decimal place.
18. 2. 19. 4000. 20. 100. 21. 3 figures; area = 14·6 sq. m. to 1 place.
22. $\pm$·0245 of a sq. m. 23. $4\frac{7}{9}\%$. 24. 122·6 g. 25. $\pm$·4 % nearly.
26. 39·9 sq. m. and 40·2 sq. m. 27. 61·7 m. and 61·8 m.
28. Area lies between 3100 sq. in. and 2971 sq. in.; $\pm$64·4 sq. in.
29. 4 figures; 1 lb. = 453·6 g. 30. Sp. G. lies between 7·75 and 7·84.

EXAMPLES 187. PAGE 370.

1. ·14286. 2. ·71828. 3. 1·41069.
4. ·49182. 5. ·042. 6. ·78540.
7. (i) 20·12 m., (ii) 51·82 m., (iii) 66·06 m.
8. (i) 66·03 pts., (ii) 892·7 pts., (iii) 15·97 pts.
9. (i) 104·7 sq. yds., (ii) 965·2 sq. yds., (iii) 60640 sq. yds.
10. (i) 57·32 lb., (ii) 787·0 lb., (iii) ·5512 lb. 11. 40·04 sq. ft.
12. 9·96 m. 13. (i) 7·55552 mi., (ii) 1·90459 Km.

EXAMPLES 188. PAGE 373.

1. 1·414.	2. 2·236.	3. 3·464.	4. 3·606.
5. 1·265.	6. ·632.	7. ·707.	8. ·837.
9. ·045.	10. 2·134.	11. 1·805.	12. 1·537.
13. ·072.	14. 2·239.	15. ·173.	16. 1·732.
17. 8·021.	18. 3·162.	19. ·042.	20. ·548.
21. 4·151.	22. ·935.	23. ·756.	24. ·853.
25. ·933.	26. 2·059.	27. 3·191.	28. 4·615.
29. 3 ft. 9 in.	30. 3.	31. 6 ft.	32. 43·84 m.
33. 25·456 in.	34. 271·53 m.		

EXAMPLES 189. PAGE 375.

1. 2·414. **2.** 1·366. **3.** ·309. **4.** ·290.
5. ·236. **6.** 2·225. **7.** 3·146. **8.** ·318.
9. ·252. **10.** ·732. **11.** 2·732. **12.** 1·618.
13. 4·882. **14.** ·822. **15.** 1·932. **16.** 1·809.
17. 5·449. **18.** ·564. **19.** 1·410. **20.** ·764.
21. 5·828. **22.** ·268. **23.** 2·618. **24.** ·098.
25. 9·899. **26.** ·127. **27.** 1·336. **28.** ·748.
29. 4·828. **30.** 4·051. **31.** 3·146. **32.** 3·646.
33. 4·828. **34.** 1·366. **35.** 1·932. **36.** ·457.

EXAMPLES 191. PAGE 389.

1. 65%. **2.** 48%; 59%; 31%; 69%; 41%; 100%; 79%.
3. 3%; £52. **4.** £129; $7\frac{1}{2}$%. **5.** 3*s.* 10*d.*; £3. 9*s.*; 2*s.* 6*d.*; 15*s.*
6. $3\frac{3}{4}$ miles an hour. **7.** 70 miles; 2 hrs. 20 min. and 2 hrs.
8. 36 metres; A's rate : B's rate = 17 : 13. **9.** 12 : 5.
10. Ratio = 4 : 3; value per lb. is a little less than 1*s.* $8\frac{1}{2}$*d.*
11. B, $11\frac{2}{5}$ sec. nearly; C, $10\frac{1}{2}$ sec. nearly.
12. 6 hrs. from start; 8 miles from Q. **13.** At 11.18 a.m.; 63 miles from P.
14. At 3 p.m. **15.** 45 miles. **16.** 12 days; 24 miles. **17.** 10*s.*; 20%.
18. 3·3 miles per hour. **19.** 3 hrs. **20.** £60; 3%.
21. $31\frac{1}{2}$ yrs; £253. **22.** 5 hrs. **23.** 6.

EXAMPLES 192. PAGE 394.

1. 13. **2.** 17. **3.** 19. **4.** 43. **5.** 59. **6.** 61. **7.** 83.
8. 97. **9.** 101. **10.** 109. **11.** 113. **12.** 137. **13.** 197. **14.** 223.
15. 281. **16.** 293. **17.** 307. **18.** 389. **19.** 461. **20.** 571. **21.** 599.
22. 643. **23.** 691. **24.** 769. **25.** 3261. **26.** 5018. **27.** 12143. **28.** 23014.

EXAMPLES 193. PAGE 396.

1. ·2. **2.** ·3. **3.** ·4. **4.** ·8. **5.** ·9. **6.** 1·1. **7.** 1·2. **8.** 1·6.
9. 2·1. **10.** 2·3. **11.** 7·8. **12.** 8·2. **13.** 11·1. **14.** 2·17.
15. 2·34. **16.** 4·06. **17.** 5·91. **18.** 76·3. **19.** ·074. **20.** 1·314.
21. 132·5. **22.** 50·23. **23.** 8·004. **24.** 2·6. **25.** 2·224. **26.** 1·426.
27. ·520. **28.** ·278. **29.** ·830. **30.** ·613. **31.** ·498. **32.** ·339.

EXAMPLES 194. PAGE 397.

1. 3368·4 in. **2.** 8 ft.
3. (i) 7 ft. 8 in., (ii) $11\frac{1}{2}$ ft., (iii) 15 ft. 4 in., (iv) 34 ft. 6 in., (v) 148 ft., (vi) 16 ft. **4.** 1 ft. 7 in. **5.** 7 ft. **6.** 570·7321 sq. m.
7. 151·2 in. **8.** 20·785 in. **9.** 27 cm.; 46·765 cm
10. 60 ft.; 103·923 ft. **11.** $3460\frac{4\,1}{5\,1\,2}$ cu. ft.; $60\frac{1}{2}$ ft.
12. £4. 9*s.* 5*d.* **13.** £1. 4*s.* 5*d.*

EXAMPLES 195. PAGE 398.

1. 18. **2.** 35. **3.** $2\frac{2}{3}$. **4.** 6. **5.** 12. **6.** 13. **7.** 15. **8.** 11. **9.** 7. **10.** 5.

EXAMPLES 196. PAGE 404.

1. $\cdot\dot{8}57142\dot{8}$. **2.** $5\cdot4\dot{3}86\dot{2}$. **3.** $305\cdot18\dot{2}7\dot{5}$. **4.** $4\cdot2\dot{3}0466\dot{8}$.
5. $8\cdot1474019\dot{}$. **6.** $31\cdot0\dot{8}\dot{1}$. **7.** $12\cdot\dot{9}65223\dot{6}$. **8.** $9\cdot750486039594\dot{9}$.
9. $395\cdot0\dot{1}$. **10.** $4125\cdot7\dot{8}\dot{5}$. **11.** $5\cdot87\dot{7}2\dot{3}$. **12.** $3\cdot9914\dot{2}74\dot{8}$.
13. $\cdot000058507962\dot{}$. **14.** $\cdot0\dot{7}14285\dot{}$. **15.** $\cdot9\dot{4}\dot{5}$. **16.** $189\cdot0\dot{3}\dot{7}$.
17. $3\cdot7\dot{8}\dot{3}$. **18.** $\cdot\dot{0}76923\dot{}$; yes; 13 is prime.
19. $\cdot\dot{0}00\dot{9}$; no; denr. $= 11 \times 101$, and factor 11 gives a 2-figure period.
20. $\cdot\dot{0}0699\dot{3}$; yes; denr. $= 11 \times 13$, and neither factor gives a 3-figure period.
21. $\cdot\dot{0}01245\dot{3}3\dot{}$; no; denr. $= 11 \times 73$, and 11 can give a 4-figure period.
22. $\cdot\dot{0}0042936882782\dot{3}\dot{1}$; no; denr. $= 17 \times 137$, and the latter gives an 8-figure period.
23. (i) 3; (ii) 11; (iii) 37; (iv) 101; (v) 41 or 271; (vi) 7 or 13.

EXAMPLES 197. PAGE 409.

1. 375 sq. ft. **2.** 150 sq. ft. **3.** 33·6 yds. **4.** 6 ft.
5. 165 ac. **6.** 17 ft. **7.** 11 ft. **8.** £240.
9. 16 ft. **10.** £75. 17*s.* 4*d.* **11.** £60. 15*s.* **12.** 2500.
13. £1. 2*s.* 6*d.* **14.** 535·67 sq. ft. **15.** £2. 13*s.* 3*d.* **16.** £7. 17*s.* 1*d.*
17. £55. 19*s.* 1*d.* **18.** 18. **19.** 50 ft. **20.** 84·85 yds.
21. 192·25 yds. **22.** 3546 sq. yds. **23.** $194\frac{1}{2}$ sq. ch. **24.** 13 a. 51·5 ca.

EXAMPLES 198. PAGE 413.

1. 60 cu. ft. **2.** 9 ft. **3.** 12 sq. ft.; 6 ft. **4.** 634·98 cu. in.
5. 7700. **6.** 2·45 inches per minute nearly. **7.** 15 ft.
8. 3*s.* 6·6*d.* **9.** 133·52 sq. in. **10.** 25·13 cu. ft. **11.** 22·7 oz.
12. 80·56 sq. in.; $28\frac{2}{7}$ sq. in. **13.** 113·1 cu. in.; 9·05*d.* **14.** 7 in.
15. 54 lb. **16.** 7·8 in. **17.** $6\sqrt[3]{13}$ or 14·1 in. **18.** 4·84 ft.

EXAMPLES 201. PAGE 420.

1. $\overline{1}3\cdot4870$. **2.** $1\cdot3521$. **3.** $\bar{3}\cdot5135$. **4.** $2\cdot0362$.
5. $\bar{9}\cdot6252$, $\overline{1}4\cdot0420$, $\overline{2}8\cdot0840$. **6.** $\overline{2}1\cdot9388$, $\overline{4}6\cdot8623$, $\overline{24}1\cdot2656$.
7. $\bar{1}\cdot2316$, $\bar{1}\cdot4024$, $\bar{1}\cdot5110$. **8.** $\bar{2}\cdot1085$, $\bar{2}\cdot8178$, $1\cdot2434$.

EXAMPLES 202. PAGE 421.

1. $\cdot8330929$. **2.** $\cdot3223192$. **3.** $7\cdot9360196$.
4. $\cdot4971498$. **5.** $\bar{4}\cdot6364625$. **6.** $4\cdot865071$.
7. $847\cdot9382$. **8.** $70\cdot30548$. **9.** $\cdot0241108$.

EXAMPLES 203. PAGE 424.

1. 1·442. **2.** 19·91. **3.** 1·470. **4.** ·1544. **5.** ·6933.
6. 16390. **7.** 12440. **8.** ·5555. **9.** 62·34. **10.** 11140.
11. 845·7. **12.** 5·838. **13.** 1·395. **14.** ·08634. **15.** ·4662.
16. 3·260. **17.** ·007176. **18.** 3·473. **19.** 15·81. **20.** 4467.
21. 5·113 sq. in. **22.** (i) ·5641 ft., (ii) 2·821 ft. **23.** 14·69 in.
24. ·9642 dm. **25.** £1050. **26.** £101. 8*s*. **27.** 4·4 %.
28. 18 yrs. nearly. **29.** 22200. **30.** 5·125 in.

EXAMPLES 204. PAGE 424A.

1. 2·297. **2.** 19·85 %. **3.** 60. **4.** ·003347. **5.** 53·31.
6. 10·19. **7.** 0·2515. **8.** 3·048. **9.** 982·8. **10.** 31590.
11. 470400. **12.** 214·8. **13.** 38·53. **14.** 18180. **15.** 387100000
16. 0·1421. **17.** 23·15. **18.** 1·103. **19.** 139·2. **20.** 2·934.
21. 56·11. **22.** 0·2324. **23.** 5·342. **24.** 12040. **25.** 467·1.
26. 654·21. **27.** 2. **28.** 0·5537. **29.** 1. **30.** 58·26.
31. 4·31. **32.** 4·92. **33.** 1·99. **34.** 51·551. **35.** 7·04.
36. 19 and 20. **37.** 10·08 in.

EXAMPLES 205. PAGE 424B.

1. £273. 10*s*. **2.** £728. 10*s*. **3.** £968. 14*s*. **4.** £668. 4*s*.
5. £1084. **6.** £699. 16*s*. **7.** £417. 6*s*. **8.** £207. 6*s*.
9. £343. 2*s*. **10.** £493. 16*s*. **11.** £455. 12*s*. **12.** £319. 12*s*.
13. £277. 14*s*. **14.** £160. 14*s*. **15.** £627. 14*s*. **16.** £379. 18*s*.
17. £340. 2*s*. **18.** £460. 18*s*. **19.** £304. 4*s*. **20.** £755. 2*s*.
21. £258. 8*s*. **22.** £469. 14*s*. **23.** £251. 2*s*. **24.** £353. 6*s*.
25. 4 %. **26.** 4·5 %. **27.** 9·9 %. **28.** 4·4 %. **29.** 3·2 %.
30. 28·1 years. **31.** 28·0 years. **32.** 35·8 years. **33.** £12150.

EXAMINATION PAPERS. PAGE 425.

1. Ninety thousand nine hundred and ninety-nine million, ninety thousand, and nine; 2000400060.

2. 656100810001; 450 tons 5 lb. **3.** 160401; £3. 3*s*. $3\frac{3}{4}d$. **4.** 63309 lb. **5.** $\frac{1}{9\,2\,2\,4}$. **6.** 4; ·06. **7.** 54.

8. Eighteen million, eight hundred and forty.

9. 212066284500; 30203791800; 32·5. **10.** 360. **11.** 435.

12. 4 qrs. 7 bush. 3 pks. 1 gal. 3 qts. 1 pt. **13.** $\frac{1}{2\,0}$; 5 lb.

14. ·0075. **15.** $\frac{4\,3\,5}{1\,1\,7\,2\,0}$. **16.** £1. 16*s*. 10*d*.; 51 f. 30 c. **17.** 815

18. ·02$\dot{6}$. **19.** 41. **20.** 12·745 cm. **21.** ·03493.

22. 2407861. **23.** $\frac{5\,2\,3}{6\,4\,5\,2}$; $1\frac{9\,0\,1}{1\,9\,0\,0}$. **24.** $4\frac{2\,9}{1\,0\,4}$; 10*s*. 7*d*.

25. 365 f. 85 c. **26.** 875. **27.** 7. **28.** 72.

29. 82775. **30.** 22. **31.** 1009. **32.** $\frac{2}{3}$; $1\frac{7}{1\,5}$.

33. $\frac{3}{4}$; $3\frac{1\,1\,5}{1\,6\,8}$. **34.** ·549. **35.** 1*s*. $0\frac{3}{4}d$. **36.** 1861.

37. $2^2 . 3 . 5 . 11 . 13^2$; $2 . 3 . 5 . 17 . 83$; $2^6 . 3^4 . 13$; 67989430080.

38. 37. **39.** $\frac{7\,1}{8\,0\,0\,0\,0}$. **40.** $72\frac{1\,9\,0\,1}{2\,4\,6\,4}$. **41.** 625.

42. 168. **43.** 38115. **44.** 48 f. 79 c. **45.** 145.

46. £7. 8*s*. $4\frac{3}{4}d$.; 1*s*. $5\frac{1}{4}d$. over. **47.** £1. 16*s*.; 67 m. 65 cm.

48. 2. **49.** Thursday, 5.12 p.m. **50.** $1\frac{2\,9}{4\,5}$; $4\frac{1}{9}$.

51. $\frac{3\,1}{4\,8}$. **52.** $\frac{7}{8}$; $\frac{3}{1\,0\,0}$. **53.** ·8272; ·$\dot{4}\dot{5}$; $\frac{1\,0\,6}{1\,2\,5}$.

54. $3\frac{1}{2}d$. **55.** 3917. **56.** 1, 2, 3, 6, 37, 74, 111, 222.

57. Yes; $\frac{3}{4}d$. **58.** 794997. **59.** $3\frac{3}{8}$.

60. ·529296875; ·95$\dot{9}\dot{4}\dot{1}$; $\frac{1\,4\,4\,1}{3\,3\,3\,0}$. **61.** 8302626 f. **62.** 1071, 1547, 4641.

63. £105. 16*s*. 10*d*. **64.** £2. 15*s*. $4\frac{1}{2}d$.; $376\frac{1}{2}$ cm.

65. $\frac{9\,0}{7\,1\,4}$; $\frac{9\,0}{7\,1\,5}$; the former. **66.** ·8984375; ·$\dot{4}$28571$\dot{4}$. **67.** $\frac{1\,1}{1\,2}$.

68. 600 wks. **70.** 3248208000. **71.** 10586; £8. 8*s*. $2\frac{1}{2}d$. **72.** $\frac{1}{4\,4}$.

73. ·0020304; ·000080223. **74.** 3000; 408000.

75. 2 mi. 7 fur.; 2 ac. 8 sq. ch. 363 sq. yds. **76.** 3 min. 43 sec.

77. $\frac{2}{3}$. **78.** 2504. **79.** 12514·47745; 1234·16255.

80. $\frac{7}{1\,2\,0}$. **81.** ·828869. **82.** £1. 5*s*. 1*d*.

83. 252 lb. S.; 378 lb. C.; 1890 lb. N. **84.** 6 d. 4 h. 20 min.

85. 562918400; 7040000. **86.** £8794. 12*s*. $7\frac{3}{4}d$. **87.** $1\frac{1\,0\,7}{1\,6\,5}$; ·00467.

88. 144·3 *d*., 770 c. **89.** 11. **90.** 35 sq. Km. **91.** £2350. 5*s*. $2\frac{1}{2}d$.

92. 148·280. **93.** £4. 3*s*. 4*d*. **94.** 149328 oz. **95.** £17. 19*s*. $9\frac{1}{2}d$.

96. £7. 12*s.* $4\frac{3}{4}d.$ **97.** £7. 8*s.* $6\frac{1}{2}d.$ **98.** £2. 10*s.* $3\frac{3}{4}d.$
99. 48 times; £22. 15*s.* 9*d.* **100.** 227 f. 9 c. **101.** $9\frac{1}{8}$.
102. £47. 16*s.* 3*d.* **103.** ·35. **104.** 40 f. 95 c. **105.** £2884. 19*s.* $0\frac{1}{2}d.$
106. £536581. **107.** £108. 4*s.* $2\frac{3}{4}d.$ **108.** 166666 f. 67 c.
109. £63. 6*s.* $5\frac{3}{4}d.$ **110.** $1\frac{267}{280}$; 3; ·0415. **111.** £1. 9*s.*; $\frac{1}{3}$.
112. 2464 sq. yds. **113.** 3485 guineas. **114.** 150.
115. £3600. **116.** £10473. 18*s.* $2\frac{1}{2}d.$ **117.** $\frac{13}{18}$; $\frac{11}{27}$; $\frac{17}{54}$; $\frac{1}{4}$.
118. 13*s.* 7*d.*; $\cdot\dot{3}$. **119.** 183 ac. 6 sq. ch. **120.** 2184.
121. £543. 2*s.* 1*d.* **122.** 3. **123.** $\cdot4\dot{5}\dot{9}$; ·046875. **124.** £3. 6*s.* 8*d.*
125. 247 ac. **126.** £163. 2*s.* 4*d.* **127.** £9. 5*s.* $2\frac{1}{2}d.$; £37. 0*s.* 10*d.*
128. $\frac{1}{2}$; $\frac{16}{37}$. **129.** 51; ·00121 m. **130.** ·140625.
131. 8*s.* 3*d.* **132.** Man 16*s.* $3\frac{3}{4}d.$; woman 10*s.* $10\frac{1}{2}d.$; boy 5*s.* $5\frac{1}{4}d.$
133. £248. 8*s.* $3\frac{1}{2}d.$ **134.** 11441476. **135.** 2 ft. **137.** $\frac{223}{360}$.
138. ·0005; 500000. **139.** £20. 2*s.* $7\frac{1}{5}d.$ **140.** 29.
141. 75 Km. 22 m. **142.** ·508857. **143.** 49007; 72·015.
144. 77. **145.** 60. **146.** $9\frac{155}{273}\frac{51}{7}$. **147.** £39. 16*s.* $7\frac{1}{5}d.$; $\frac{1}{4}d.$
148. $51\frac{1}{3}$. **149.** 7 horses, 7 pigs, 21 cows, 105 sheep. **150.** 126.
151. 9. **152.** £11666. 13*s.* 4*d.* **153.** £7117. 18*s.* $8\frac{3}{4}d.$
154. $536\frac{1}{4}$ Kg. **155.** 42. **156.** 139. **157.** £2. 7*s.* 3*d.*
158. £70. 12*s.* 11*d.* **159.** $\frac{11}{17}$; $\frac{7}{11}$. **160.** £58. 15*s.* 4*d.* **161.** 2004 sq. cm.
162. $1\frac{17}{1685}$; ·000571; 5·71. **163.** 2 fur. 9 ch. 13 yds. 1 ft. 4 in.
164. 5*s.*, 3*s.*, 2*s.* **165.** 5. **166.** £7. 11*s.* **167.** 60 cm.
168. 119, or 595. **169.** 31016 qts. **170.** 248 m. **171.** 24; $\frac{58}{115}$.
172. 561 Kg. **173.** $\frac{6}{11}$. **174.** £2. 3*s.* 7*d.*
175. 5 tons. 4 cwt. 1 qr. $20\frac{2}{4}$ lb. **176.** $38\frac{8}{9}$. **177.** ·1015625 of a ton.
178. 1·032. **179.** 148 min. **180.** £5393. 1*s.* 11*d.* **181.** £324. 6*s.* 8*d.*
182. £12. 12*s.* **183.** £100. 10*s.* $10\frac{3}{4}d.$; 7 sq. yds. 6 sq. ft. 39 sq. in.
184. £5 gain. **185.** $1\frac{5}{252}$. **186.** £1658. 12*s.* $6\frac{3}{4}d.$
187. 1228800 times. **188.** 22. **189.** £21. 14*s.* 11*d.*
190. 763. **191.** £562. 3*s.* 2*d.* **192.** 4828 m.
193. 1824, 1852, 1880. **194.** 64 qrs. 7 bush. 3 pks. **195.** £2. 1*s.* 3*d.*
196. 5·237. **197.** £261. 11*s.* $4\frac{1}{2}d.$; divisor, yes; multiplier, no.
198. 7200. **199.** 2·5; 4. **200.** $8\frac{2}{5}$ and $8\frac{13}{252}$.
201. 20715·34. **202.** 14 d. 7 h. 11 min. 17 sec. **203.** £197. 0*s.* 2*d.*
204. $5\frac{5}{11}$. **205.** 137696 c. cm. **206.** 1 lb.
207. £8. 1*s.* $6\frac{3}{4}d.$ **208.** £560. **209.** £10. 5*s.*
210. £35. 3*s.* 10*d.* **211.** 3155673600 sec. **212.** $\frac{9}{100}$.
213. £7700. **214.** 1 yd. 4 in. **215.** 300; 101 cm. by 101 cm.
216. £50. **217.** £433. 4*s.* 10*d.* **218.** 10715820.
219. ·2034375. **220.** $\cdot\dot{4}$; ·0794. **221.** $\frac{16}{165}$.
222. £62. 18*s.* $1\frac{1}{2}d.$ **223.** 87120. **224.** £1540. 5*s.* $10\frac{1}{2}d.$
225. 60. **226.** 3 qrs. 7 lb. **227.** ·00176; $\frac{10}{37}$.
228. 150·1. **229.** £14. 3*s.* 6*d.* **230.** £48. 16*s.* 5*d.*

231. £40. 13*s.* 4½*d.* **232.** 76; $13\frac{1}{14}$. **233.** £3. 6*s.* 9½*d.*; £1. 13*s.* 2½*d.*
234. 270 cu. ft. **235.** $\frac{2}{27}$; $4\frac{15}{99}$. **236.** £4532. 19*s.* 4½*d.*
237. 40. **238.** 5¼. **239.** 460. **240.** $339\frac{31}{66}$.
241. 11 miles an hour. **242.** £300. 2*s.* 2¼*d.* **243.** 504·26 yds.
244. £56. 1*s.* 8*d.* **245.** £324. 18*s.* 7½*d.* **246.** $\frac{26}{277}$.
247. $1\frac{1}{66}$. **248.** 3 dwt. 6·4 grs. **249.** 6015; ·012.
250. £26. 15*s.* 11½*d.*; £252. 9*s.* 3½*d.* **251.** 21.
252. 50%, 16%, 13%. **253.** 221. **254.** 425·0896 pints.
255. 3141·4118 oz. **256.** £2. 2*s.* 11*d.* **257.** 100.
258. 9*s.* 9*d.* **259.** 4½ years. **260.** £160. 6*s.* 5½*d.*
261. 10*s.* 5*d.* **262.** 1 ft. 2 in. **263.** 4⅙% gain.
264. 876 d. 14 h. 24 min. **265.** 30. **266.** £558. 9*s.* 11*d.*
267. 3 tons 13 cwt. 14 lb. **268.** 2·65$\dot{9}\dot{0}$; 10. **269.** £5. 11*s.* 7*d.*
270. 11 days. **271.** £10. 5*s.* 2½*d.* **272.** 2½ min. **273.** £297. 1*s.* 8*d.*
274. Horses, £94. 13*s.* 7*d.* and £68. 6*s.* 5*d.*; carriage £87. **275.** $99\frac{3}{11}$.
276. £4477. 18*s.* 6*d.* **277.** 135 Km. **278.** 3⅓ days.
279. £11. 12*s.* 7*d.* **280.** £18524225. 10*s.*; £1684020. 10*s.*
281. 206·574 min. **282.** $\frac{3}{4}$; $\frac{4}{325}$. **283.** 31·353; ·745; 6·154.
284. £5479. 17*s.* 8¼*d.* **285.** 67200 litres. **286.** 900 f.
287. £1822. 18*s.* 4*d.* **288.** 192 yds. **289.** 1.
290. £28. 17*s.* 6*d.* **291.** 12 c. 5 m.; 4·1$\dot{6}$ m. **292.** $29\frac{1}{8}$.
293. £5. 2*s.* 7¼*d.* **294.** 8*s.* 5*d.*; $2\frac{7}{9}$%. **295.** 309.
296. 38·94%, 61·06%. **297.** £80. **298.** 3·$\dot{4}$61538$\dot{}$.
299. 18 ac. 3 ro. **300.** 60 f. **301.** £165. 11*s.* 5¼*d.*
302. 700 litres. **303.** To $\frac{575}{576}$ of orig. vol. **304.** 2·0685.
305. 13. **306.** 15*s.* 6*d.* **307.** 10101; 4662; 3108.
308. £1000. **309.** 18; 3*s.* 9*d.* **310.** 251.
311. 4·4 m. and 8·8 m. **312.** 2¼. **313.** £50. 12*s.* 6½*d.*
314. £88. 11*s.* **315.** £1500. **316.** $\frac{21255}{29904}$.
317. ⅛ of a day. **318.** $\frac{913}{60036}$. **319.** £2103. 14*s.* 1*d.*
320. £33. 13*s.* 0¾*d.*; £39. 15*s.* 5¼*d.*; £52. 0*s.* 2¼*d.* **321.** £9. 14*s.*
322. £600. **323.** ·4885416$\dot{}$. **324.** 10.
325. $\frac{1591}{31486}$. **326.** $3\frac{1}{33}$%. **327.** £775.
328. 1*d.* **329.** £280; 4%. **330.** ·003003....
331. 516. **332.** 13*s.* 6½*d.* **333.** Twice.
334. £185. 12*s.* $10\frac{3}{4}$*d.* **335.** 60%. **336.** £150. 14*s.* 10*d.*
337. 567342. **338.** 367. **339.** $59\frac{3}{8}$ sq. ft.
340. 12 min. **341.** 14. **342.** £1524. 12*s.*
343. 2¼%. **344.** Wednesday; Tuesday.
345. £29170800. **346.** 16. **347.** £274. 13*s.* 2·90*d.*
348. £12. 14*s.* 2*d.* **349.** £5. 14*s.* **350.** £8700.
351. £5. 4*s.* 6*d.*; ·25. **352.** 8 m. 74 pf. **353.** £313. 3*s.* 3*d.*
354. £174. 10*s.* 1¼*d.*; £1572. 1*s.* 4½*d.*
355. 9 hrs. **356.** £502. 10*s.* 3*d.*; £12. 11*s.* 3*d.* **357.** 9·6 c.

358. 1057. **359.** ·4; 17578·54627. **360.** ·0000815625.
361. £722. 13*s.* 4*d.* **362.** 5 yrs. **363.** £9776.
364. 4 min. **365.** £552. **366.** ·000000142857̇; 11·319.
367. 31. **368.** £208. 8*s.* 9$\frac{1}{2}$*d.* **369.** £305. 17*s.* 9*d.*
370. £7. 7*s.* **371.** £28. **372.** £182. 7*s.* 2*d.*
373. ·28845. **374.** £95. 4*s.* 5$\frac{1}{2}$*d.* **375.** 3.
376. 20·12 metres per sec. **377.** 100 f. **378.** The latter.
379. 1 cwt. 2 qrs. 8 lb. 12 oz. **380.** $\frac{1}{4}$. **381.** £4441. 16*s.* 4*d.*
382. 20$\frac{1}{2}$; ·0372. **383.** £295. 16*s.* 8*d.* **384.** £620.
385. £140. **386.** 116. **387.** 12240. **388.** £20.
389. 20$\frac{4}{7}$ miles. **390.** $3\frac{11}{23}$. **391.** £14668. 18*s.* **392.** $91\frac{3}{7}$.
393. £48. 12*s.* 6*d.*; £81. 0*s.* 10*d.*; £113. 9*s* 2*d.* **394.** 32.
395. £15. 15*s.* **396.** 6*s.* 6*d.* **397.** 1449 m. 26 pf. **398.** £25. 10*s.*
399. 79·4 cl. liquid; 20·6 cl. water. **400.** $28\frac{2}{3}\frac{5}{8}$. **401.** 12*s.* 6*d.*
402. $9\frac{7}{5}\frac{}{5}$ min. **403.** £16. 10*s.* 9*d.* **404.** £810. 12*s.* 6*d.*
405. 3 dwt. $8\frac{1}{4}\frac{9}{0}$ grs. **406.** £1. 5*s.* **407.** 22·88 sq. yds.
408. ·2$\dot{5}$. **409.** 85812 f. 70 c. **410.** £2710. 6*s.* 3*d.*
411. £17. 16*s.* $1\frac{3}{3}\frac{5}{7}$*d.*; £23. 7*s.* $5\frac{4}{7}\frac{1}{4}$*d.*; £41. 3*s.* $7\frac{1}{2}$*d.* **412.** £1073. 4*s.* $0\frac{3}{4}$*d.*
413. £15.; £30.; £40. 10*s.* **414.** 1753. **415.** 8*s.* per yd. **416.** $\frac{9}{80}$.
417. 20$\frac{1}{2}$ hrs. **418.** 3721756. **419.** 324 bush.
420. £4108. 16*s.* 4$\frac{1}{2}$*d.* **421.** £6. 6*s.* **422.** £191. 16*s.*
423. 100203; 999663. **424.** 425 qrs. of each. **425.** 63 sq. m.
426. £46. 13*s.* 4*d.*; 90. **427.** £540. **428.** 4·63294. **429.** $\frac{14}{19}$.
430. 2904 tons. **431.** A, 50·46 %; B, 17·69 %; C, 31·85 %.
432. £55. 10*s.* **433.** 560 lb. **434.** £448. **435.** 24754.
436. 389 yds. **437.** Coffee by 2 lb. **438.** £611. 5*s.* 10*d.*
439. 11*s.* 4$\frac{1}{2}$*d.* **440.** £352. 5*s.* 4*d.*; £8. 16*s.* 1*d.* **441.** 25·22 f.
442. ·0575; 42. **443.** £3. 11*s.* 3*d.*; £1. 11*s.* 8*d.*; 19*s.* **444.** 37·19; $1\frac{4}{7}$.
445. £13. **446.** £11. 9*s.* 5·66*d.* **447.** £254. 1*s.* 2*d.* **448.** £864. 10*s.*
449. $656\frac{1}{4}$. **450.** 6*s.* $3\frac{1}{4}$*d.*; $\frac{7}{15}$. **451.** $312\frac{1}{2}$ ares.
452. £357. 2*s.* 5*d.*; £190. 9*s.* 3*d.*; £444. 8*s.* 4*d.* **453.** ·005$\dot{3}$.
454. 4*s.* 8*d.* **455.** $95\frac{1}{8}$; £130. 15*s.* **456.** ·119140625.
457. 409. **458.** 46 Km. per hour. **459.** 1763; $1\frac{5}{7}$. **460.** £1. 19*s.* 0*d.*
461. 125 lb. **462.** £104. **463.** 7. **464.** Tuesday.
465. 3*s.* 4*d.* in £; £29167; £13739; £3686; £3408.
466. Coffee : chicory = 7 : 5. **467.** 176 yds. and 616 yds. **468.** £49. 10*s.*
469. $97\frac{5}{4}$; £126. **470.** 741; $2\frac{1}{3}$. **471.** $4\frac{1}{4}$ %. **473.** £387. 3*s.* 7*d.*
474. 48. **475.** 104 million gallons. **476.** £1000; £7000. **477.** £338.
478. 984 g. per sq. cm. **479.** £619. 10*s.*; £604. 0*s.* 3*d.* **480.** £3.7*s.*6*d.*
481. £4. 10*s.* **482.** Length, 14 ft.; width, 5·6 ft.; height, 2·8 ft.
483. $1\frac{1}{2}$ ft. **484.** 1·2586805̇. **485.** ·3025. **486.** £6. 17*s.* 8·05*d.*
487. milk : water = 28 : 5. **488.** $4\frac{1}{2}$*d.* nearly. **489.** $33\frac{1}{3}$ ft.
490. 300 lb.

PROBLEMS. PAGE 449.

1. £275. **2.** $4\frac{323}{1288}$. **3.** 1344 lb.; $179\frac{1}{5}$ lb.; $268\frac{4}{5}$ lb. **4.** 960.
5. $300\frac{25}{32}$. **6.** $6\frac{1}{4}$. **7.** (i) 223·359; (ii) 20·057 oz.
8. 1·25 lb. **9.** 250 lb. **10.** 9·15 m. **11.** 24000. **12.** 15 hrs.
13. $34\frac{2}{3}$. **14.** 5 tons 7 cwt. 16 lb. **15.** 342·17. **16.** £650.
17. £725. **18.** £2232. 5*s*. 10*d*. **19.** 1 mil = $54\frac{87}{172}d$.
20. 45. **21.** 1088. **22.** 9*s*. $3\frac{4}{11}d$. **23.** 81097 sq. mi.
24. 7·82 g. **25.** ·29. **26.** 21·46. **27.** ·816.
28. (i) + ·06 %; (ii) − ·21 %. **29.** 24 m. **30.** 20 mi. **31.** 721.
32. £634. 16*s*.; $9\frac{1}{5}$. **33.** 50 gallons. **34.** 12300 f.
35. 80. **36.** $3\frac{1}{8}$ per cent. **37.** £4. 10*s*. gain.
38. Captain, £250; officer, £62. 10*s*.; man, £15. 12*s*. 6*d*. **39.** 100.
40. £17. 10*s*.; £70; £157. 10*s*. **41.** 3 %. **42.** £194. 5*s*.; £224. 14*s*.
43. The latter. **44.** 361 sq. m. **45.** 5 h. 46 min. 44 sec. p.m.
46. (i) $\frac{19}{24}$; (ii) 105. **47.** 750. **48.** 2256.
49. 14, 7, $10\frac{1}{2}$ days. **50.** 24*s*., 15*s*., 1*s*. **51.** £78. 2*s*. 6*d*.
52. 5 sq. m.; 112 cm. **53.** 4331 days. **54.** 769·7.
55. $4\frac{1}{2}$ Km. per hr.; 6 Km. per hr. **56.** 1421. **57.** 6·3 %.
58. 232 cu. in. **59.** 39·769. **60.** £41. 13*s*. 4*d*.
61. 15·03. **62.** 63, 105, 120. **63.** 91·5064 inches.
64. Value lies between 2112·63 and 2113·75; error, ± ·56 nearly.
65. 22 cwt. 26 lb. **66.** 24. **67.** £559. 13*s*. 9*d*. **68.** + ·58%, + ·58%, − ·61%.
69. $1633\frac{1}{2}$ tons. **70.** 6336000 cu. ft.; $176785\frac{5}{7}$ tons.
71. £480, £440, £165, £605. **72.** 1 lb.; 36 min. **73.** 300.
74. Loses £25 capital; gains £81 income.
75. $\frac{8}{9}$ ton alloy, $202\frac{2}{3}$ lb. lead, $46\frac{2}{3}$ lb. tin. **76.** £1330800. **77.** 36 %.
78. $5\frac{1}{2}$ yds. **79.** £72. 6*s*. 8*d*. **80.** 35. **81.** 564.
82. $\frac{1}{3}$. **83.** 138. **84.** £3. 7*s*. 9*d*.; 364 of each. **85.** £7500.
86. 168 lb. **87.** 679 g. 45 cg. **88.** A : B = 40 : 53. **89.** 5^{m}. 15^{s}.
90. 2. **91.** 5 %. **92.** $5\frac{173}{180}$ grs. **93.** $15\frac{685}{1408}$. **94.** £962. 10*s*.
95. 20 carats. **96.** 14. **97.** $5\frac{6}{283}$ oz.; $4\frac{16}{283}$ oz.
98. 1100 ft. per second. **99.** $12\frac{1}{2}$ miles per hour.
100. £15000; £9400; £5600. **101.** The latter. **102.** £552 loss.
103. 999. **104.** 5 min. fast. **105.** $11\frac{1}{13}$ min. **106.** 9 hrs. $43\frac{3}{4}\frac{6}{3}\frac{6}{1}\frac{7}{1}$ min. p.m.
107. In 120 days; *i.e.* on April 24th or April 23rd; 1.46 p.m.; 2.16 p.m.
108. A, 63; B, 35. **109.** 102 cg. **110.** 1848; $14\frac{2}{3}$ min.
111. A, 280, B, 315, C, 336 out of 420. **112.** 12*s*. 11*d*.
113. A, £36. 9*s*.; B, £67. 10*s*.; £5. 8*s*. per week. **114.** £83080.
115. £15, £7. 10*s*., £5. **116.** Loses $16\frac{2}{3}$ per cent.
117. 1 min. 55 sec. **118.** £75. **119.** $\frac{81}{256}$.
120. The one, $21\frac{35}{144}$ gall. of water and $2\frac{109}{144}$ gall. of wine; the other, $2\frac{109}{144}$ gall. of water and $21\frac{35}{144}$ gall. of wine.
121. 4*is*. 3*d*. per qr. **122.** £106. 13*s*. 4*d*. **123.** $7\frac{1}{5}$ in.

124. 18·4325. **125.** 57. **126.** A, 35; B, 25.
127. 11 dwt. $1\frac{7}{17}$ grs. **128.** $322\frac{2}{3}$ yds.
129. 567. **130.** £17.10*s*. **131.** $12\frac{1}{2}$ %.
132. £1666. 13*s*. 4*d*. **133.** 7 ft. **134.** (*a*) $55\frac{1}{2}$ sec.; (*b*) $7\frac{1}{2}$ sec.
135. 55 min. **136.** 50 mi. and 30 mi. per hr.
137. $32\frac{4}{13}$ min. past 2 a.m. Wednesday. **138.** $1166\frac{2}{3}$, 1169, 1000, 1002.
139. $9\frac{2}{3}\frac{0}{5}\frac{7}{2}$; $9\frac{3}{8}$. **140.** 84 min. **141.** 480 m. **142.** $3\frac{1}{2}$ hrs.
143. 4 Km. 180 m. **144.** $2\frac{2}{3}$ days after 2nd starts; at the end of 124 miles.
145. $3\frac{3}{4}$ miles. **146.** 70; $23\frac{1}{3}$ % gain. **147.** 300 for £4. 5*s*. 1*d*.
148. 800 yds. **149.** $7\frac{1}{2}$ min.; $77\frac{1}{2}$ min. **150.** 3*s*. 6*d*., 2*s*., 1*s*. 6*d*.
151. 1000. **152.** 15 yds. from it. **153.** 3 : 7. **154.** $2\frac{3}{1}\frac{8}{2}\frac{8}{1}$ hrs.
155. £192000. **156.** £1199. 11*s*. 6*d*. **157.** £2010. **158.** $61\frac{2}{3}\frac{9}{6}d$.
159. $123\frac{1}{5}$. **160.** £65. 2*s*. 1*d*. **161.** $49\frac{9}{3}\frac{3}{4}\frac{3}{3}$ %. **162.** 41·28 gall.; 18·72 gall.
163. $10\frac{2}{2}\frac{2}{6}$ hrs. **164.** At 9.15. **165.** 360. **166.** 98.
167. $4\frac{1}{5}$ mi. per hr. **168.** $2\frac{7}{8}$ miles. **169.** £4. 3*s*. 4*d*. **170.** 2*s*. 6*d*.
171. £3990. **172.** $\frac{1}{3}\frac{3}{3}$; 2 : 11. **173.** 13*s*. 4*d*.
174. £7500; £35. 5*s*. **175.** 208 days.
176. A, £18400; B, £12266. 13*s*. 4*d*. **177.** A, £51. 12*s*.; B, £81.
178. $\frac{5}{8}$. **179.** $\frac{1}{7}\frac{}{9}\frac{}{1}$ or ·001264; The latter. **180.** ·24.
181. £1016. 2*s*. 11*d*.; £1015. 12*s*. 6*d*. **182.** 25; 50. **183.** £500.
184. Error is ·0000214 nearly. **185.** £705. **186.** £3484800.
187. 1*s*. 8*d*. **188.** 250 strokes. **189.** $17\frac{9}{7}$; $\frac{\text{superior}}{\text{inferior}} = \frac{1}{4}$.
190. Man, £1. 2*s*.; woman, 12*s*. 6*d*.; boy, 4*s*. 7*d*.; girl, 2*s*. 11*d*. **191.** £1 per share.
192. £1030. **193.** A, £156; B, £121. 6*s*. 8*d*.; C, £39. **194.** 60.
195. $66\frac{2}{3}$ yds. from A's starting point; A has passed his starting point 3 times, B twice. **196.** The latter by 4*s*. 2*d*. **197.** 125.
198. B wins taking 5 min. 16 sec.; A takes 5 min. 20 sec. **199.** 11*s*.
200. $2\frac{1}{5}$ ft. **201.** 2 h. 40 min. **202.** $77\frac{1}{7}$. **203.** ·384 in. nearly.
204. 3 mi. per hr. **205.** $11\frac{7}{3}\frac{1}{0}\frac{}{8}$ sec. **206.** 15 min. 18 sec. past 1.
207. $49\frac{1}{2}$. **208.** 58800; £10; 1*d*. **209.** 40·72. **210.** 49·2.
211. The former; Customer loses 1·12 oz. in 1 lb. **212.** 17 times; 158 yds.
213. £3840. **214.** $113\frac{1}{6}\frac{}{2}\frac{}{3}$; $22\frac{2}{9}\frac{9}{9}\frac{5}{6}\frac{9}{8}\frac{3}{0}d$., or $22\frac{1}{2}d$. practically. **215.** £412. 10*s*.
216. £2000. **217.** (i) $2\frac{1}{4}$ min.; (ii) $2\frac{1}{3}$ min. **218.** 13·82 ft. per second.
219. 6 in. **220.** 124 fathoms. **221.** 6·339 in. **222.** 4 miles 1270 yds.
223. $6\frac{8}{3}\frac{6}{4}\frac{}{5}$. **224.** $27\frac{2}{5}\frac{9}{5}$. **225.** £456976. **226.** £102.14*s*.6*d*. to next penny.
227. 230 min., the boys bringing pint and gallon coming up 1 min. too late.
228. (i) A : B : C = 266 : 315 : 379; (ii) $8\frac{8}{1}\frac{8}{2}\frac{}{9}$ days. **229.** 12 more.
230. 8 min. 4 sec.; 8 min. 15 sec.; 8 min. 26 sec. **231.** 2 miles per hour.
232. £79. 1*s*. $0\frac{3}{4}d$. nearly. **233.** 1st class, 122; 2nd class, 366; 3rd class, 524.

PRINTED IN GREAT BRITAIN BY ROBERT MACLEHOSE AND CO. LTD.
THE UNIVERSITY PRESS, GLASGOW.